

Calculation of the Uncertainty in the Variogram for More Realistic Reservoir Forecasting

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Abstract

There are often limited data available for reservoir characterization. This leads to considerable uncertainty in statistical parameters including the variogram. This paper presents an approach to calculate the uncertainty in the variogram. We discuss the methodology to transfer this uncertainty through geostatistical simulation and flow modeling in order to achieve more realistic predictions.

The variogram value $2\gamma(\mathbf{h})$ for a separation lag vector \mathbf{h} is the mean squared difference between two data separated by \mathbf{h} . The variance of a mean can be calculated with a model of the correlation between the pairs of data used in the calculation. MultiGaussian theory is used to calculate this four-point correlation. Multiple plausible variograms may be fit knowing the uncertainty in each point, $2\gamma(\mathbf{h})$. Multiple geostatistical realizations may then be constructed and subjected to flow simulation to assess the impact of this uncertainty on reservoir performance predictions.

The theoretical approach is presented together with a number of examples. The theoretical results are validated by numerical simulation. The simulation approach permits generalization to nonGaussian situations.

Introduction

Variogram modeling is a critical step in any geostatistical study; however, a reliable variogram is difficult to infer in presence of sparse data. This is particularly true in the early stages of reservoir development. It would be beneficial to have a quantitative model of the uncertainty in the variogram.

We show how to calculate the pointwise uncertainty in the variogram. This pointwise uncertainty must be translated to the joint uncertainty, that is, into variogram modeling. Within the bounds of pointwise uncertainty, we propose to establish different scenarios, ranging from small continuity to great continuity. These “scenarios” can be used in simulation to generate a number of realizations. These realizations can be used to determine the sensitivity of the flow response to variogram uncertainty.

Pointwise Variogram Uncertainty

The variogram $2\gamma(\mathbf{h})$ is the average of squared differences between data separated by that distance vector \mathbf{h} :

$$2 \cdot \gamma(\mathbf{h}) = \frac{1}{n(\mathbf{h})} \cdot \sum_{i=1}^{n(\mathbf{h})} [z(\mathbf{u}_i) - z(\mathbf{u}_i + \mathbf{h})]^2 \quad (1)$$

where $n(\mathbf{h})$ is the number of data pairs approximately \mathbf{h} apart.

Consider $x_i = [z(\mathbf{u}_i) - z(\mathbf{u}_i + \mathbf{h})]^2$, the squared difference between the value at locations \mathbf{u}_i and $\mathbf{u}_i + \mathbf{h}$. The mean of the x_i 's is the variogram:

$$\bar{x} = 2 \cdot \gamma(\mathbf{h}) = \frac{1}{n(\mathbf{h})} \cdot \sum_{i=1}^{n(\mathbf{h})} x_i \quad (2)$$

From classical statistics, we know that the uncertainty in the mean \bar{x} is defined as:

$$Var\{\bar{x}\} = E\{(\bar{x} - E\{\bar{x}\})^2\} = E\{\bar{x}^2\} - (E\{\bar{x}\})^2 \quad (3)$$

Now, using expression (3) we can calculate the uncertainty in the variogram assuming that we have a ‘‘reference’’ variogram model fitted to the experimental points. \bar{x} is replaced by $2 \cdot \gamma(\mathbf{h})$ and the variance of squared differences around this value is calculated as follows:

$$\begin{aligned} \sigma_{2 \cdot \gamma(\mathbf{h})}^2 &= E\{2 \cdot \gamma(\mathbf{h})^2\} - (E\{2 \cdot \gamma(\mathbf{h})\})^2 \\ &= E\left\{\left(\frac{1}{n(\mathbf{h})} \cdot \sum_{i=1}^{n(\mathbf{h})} [z(\mathbf{u}_i) - z(\mathbf{u}_i + \mathbf{h})]^2\right)^2\right\} - (E\{2 \cdot \gamma(\mathbf{h})\})^2 \\ &= E\left\{\left(\frac{1}{n(\mathbf{h})^2} \cdot \sum_{i=1}^{n(\mathbf{h})} \sum_{j=1}^{n(\mathbf{h})} [z(\mathbf{u}_i) - z(\mathbf{u}_i + \mathbf{h})]^2 \cdot [z(\mathbf{u}_j) - z(\mathbf{u}_j + \mathbf{h})]^2\right)\right\} - (2 \cdot \gamma(\mathbf{h}))^2 \end{aligned} \quad (4)$$

$$\sigma_{2 \cdot \gamma(\mathbf{h})}^2 = \frac{1}{n(\mathbf{h})^2} \cdot \sum_{i=1}^{n(\mathbf{h})} \sum_{j=1}^{n(\mathbf{h})} E\{[z(\mathbf{u}_i) - z(\mathbf{u}_i + \mathbf{h})]^2 \cdot [z(\mathbf{u}_j) - z(\mathbf{u}_j + \mathbf{h})]^2\} - (2 \cdot \gamma(\mathbf{h}))^2 \quad (5)$$

This can be simplified by using the definition of the covariance:

$$\begin{aligned} C_{ij} &= Cov\{x_i, x_j\} = E\{(x_i - E\{x_i\}) \cdot (x_j - E\{x_j\})\} \\ &= E\{x_i \cdot x_j\} - E\{x_i\} \cdot E\{x_j\} \\ &= E\{x_i \cdot x_j\} - \bar{x}^2 \end{aligned} \quad (6)$$

Now, replacing x_i and x_j by the squared differences $[z(\mathbf{u}_i) - z(\mathbf{u}_i + \mathbf{h})]^2$ and $[z(\mathbf{u}_j) - z(\mathbf{u}_j + \mathbf{h})]^2$ respectively, and \bar{x} by the variogram $2 \cdot \gamma(\mathbf{h})$:

$$C_{ij} = E\{[z(\mathbf{u}_i) - z(\mathbf{u}_i + \mathbf{h})]^2 \cdot [z(\mathbf{u}_j) - z(\mathbf{u}_j + \mathbf{h})]^2\} - (2 \cdot \gamma(\mathbf{h}))^2 \quad (7)$$

A simple formula for the variance of a particular variogram value is obtained replacing the covariance (7) in expression (5):

$$\sigma_{2 \cdot \gamma(\mathbf{h})}^2 = \frac{1}{n(\mathbf{h})^2} \cdot \sum_{i=1}^{n(\mathbf{h})} \sum_{j=1}^{n(\mathbf{h})} C_{ij} \quad (8)$$

where C_{ij} is calculated as in (7). To avoid confusion, note that C_{ij} is covariance between pair i $[z(\mathbf{u}_i) - z(\mathbf{u}_i + \mathbf{h})]^2$ and j $[z(\mathbf{u}_j) - z(\mathbf{u}_j + \mathbf{h})]^2$.

Expression (8) tells us that the uncertainty in the variogram at a distance \mathbf{h} is the average covariance between ‘‘pairs of pairs’’ used to calculate the variogram for that particular lag.

The covariance between ‘‘pairs of pairs’’ can be calculated theoretically under a multi-Gaussian assumption. The following section presents this approach. The next sections present the Local and Global Simulation Methods to check the results given by the Theoretical Approach. The Global Simulation Method is more general in the sense that it gives

the whole distribution of uncertainty in the variogram values for each lag. Although the theory says that the distribution of uncertainty in a variogram point should be Gaussian, we know that the variogram values must be non-negative and therefore, the distribution is asymmetric; this is demonstrated by the Global Simulation Method.

The following steps are required for all three methodologies,

1. **Transform data to normal space:** Any data distribution can be easily transformed to a Gaussian univariate distribution. In the following examples the program `nscore` in `GSLIB` was used to perform the transformation. This transformation is commonly done to allow Gaussian simulation.
2. **Check multigaussianity:** To fulfil the multiGaussian condition, one should assure that not only the univariate distribution is Gaussian, but also the bivariate and all multivariate distributions. In practice, some tests can be done to the transformed distribution in order to prove the bigaussianity, however they are not often applied, especially in presence of sparse data. The higher order distributions are assumed Gaussian, without any test.
3. **Calculate the experimental variogram:** The location of the sampled points and the values of the variable under study at these locations are used to calculate the experimental variogram.
4. **Fit a variogram model.** The fitted variogram model defined in this step is critical for subsequent stages of uncertainty evaluation. The uncertainty assessment depends on the variogram model. Nevertheless, we require a model assumption regarding the variogram.

The difference between the Theoretical Approach and the Numerical methods lies in how the variance for each lag is calculated.

Theoretical Approach

The variogram uncertainty can be calculated through a theoretical approach, assuming that the regionalized variable is multigaussian.

Expanding expression (7), the covariance can be written as a sum of fourth order moments:

$$\begin{aligned}
C_{ij} &= E\{[z(\mathbf{u}_i) - z(\mathbf{u}_i + \mathbf{h})]^2 \cdot [z(\mathbf{u}_j) - z(\mathbf{u}_j + \mathbf{h})]^2\} - (2 \cdot \gamma(\mathbf{h}))^2 \\
&= E\{z(\mathbf{u}_i)^2 \cdot z(\mathbf{u}_j)^2 - 2 \cdot z(\mathbf{u}_i)^2 \cdot z(\mathbf{u}_j) \cdot z(\mathbf{u}_j + \mathbf{h}) + z(\mathbf{u}_i)^2 \cdot z(\mathbf{u}_j + \mathbf{h})^2 \\
&\quad - 2 \cdot z(\mathbf{u}_i) \cdot z(\mathbf{u}_i + \mathbf{h}) \cdot z(\mathbf{u}_j)^2 + 4 \cdot z(\mathbf{u}_i) \cdot z(\mathbf{u}_i + \mathbf{h}) \cdot z(\mathbf{u}_j) \cdot z(\mathbf{u}_j + \mathbf{h}) \\
&\quad - 2 \cdot z(\mathbf{u}_i) \cdot z(\mathbf{u}_i + \mathbf{h}) \cdot z(\mathbf{u}_j + \mathbf{h})^2 + z(\mathbf{u}_i + \mathbf{h})^2 \cdot z(\mathbf{u}_j + \mathbf{h})^2\} - (2 \cdot \gamma(\mathbf{h}))^2
\end{aligned} \tag{9}$$

This covariance is called a *quadratic covariance* (Matheron, 1965) and it can be calculated if $z(\mathbf{u}_i)$, $z(\mathbf{u}_i + \mathbf{h})$, $z(\mathbf{u}_j)$, and $z(\mathbf{u}_j + \mathbf{h})$ have a multivariate Gaussian distribution. In such case, any fourth order moment can be calculated using the pairwise covariance values as follows:

$$E\{z_1 \cdot z_2 \cdot z_3 \cdot z_4\} = C_{12} \cdot C_{34} + C_{13} \cdot C_{24} + C_{14} \cdot C_{23} \tag{10}$$

Then, the variogram variance is calculated as a sum of fourth order moments minus two times the variogram squared.

To perform this calculation, the program `gamu2` was prepared. For each lag, the location of pairs considered in the experimental variogram calculation is used to determine the fourth order moment as follows:

$$\begin{aligned}
 E\{z(\mathbf{u}_i) \cdot z(\mathbf{u}_i + \mathbf{h}) \cdot z(\mathbf{u}_j) \cdot z(\mathbf{u}_j + \mathbf{h})\} &= C(z(\mathbf{u}_i), z(\mathbf{u}_i + \mathbf{h})) \cdot C(z(\mathbf{u}_j), z(\mathbf{u}_j + \mathbf{h})) \\
 &+ C(z(\mathbf{u}_i), z(\mathbf{u}_j)) \cdot C(z(\mathbf{u}_i + \mathbf{h}), z(\mathbf{u}_j + \mathbf{h})) \\
 &+ C(z(\mathbf{u}_i), z(\mathbf{u}_j + \mathbf{h})) \cdot C(z(\mathbf{u}_i + \mathbf{h}), z(\mathbf{u}_j))
 \end{aligned}
 \tag{11}$$

Simulation Alternative to Estimate C_{ij} .

Local Simulation Method

This method permits the estimation of all the four order moments presented in expression (9) by simulation of all “pairs-of-pairs” or four-point locations. Again, the assumption of multigaussianity will be made to simplify simulation. To calculate the variance for each lag, multiple simulations are generated at the same locations of pairs used to evaluate the variogram. All fourth order moments in expression (9) are estimated as averages using the simulated values, and the variogram variance is calculated with formula (8). The program `gamu` calculate the uncertainty in the variogram using LU simulation. This approach is efficient because only four points are considered at a time and there are no conditioning data.

Global Simulation Method

The basic idea is to generate several non-conditional realizations of the entire domain using the variogram model, extract the values at the sampled locations and then calculate the variogram. The variance between the variogram values at each lag calculated using these realizations should converge to the same value obtained through any of the other approaches. Moreover, the entire uncertainty distribution is given through this method.

This approach was implemented using the GSLIB program `sgsim`. The variogram using all the simulated nodes must be checked to see if it honors the input variogram model. Ergodic fluctuations should be expected. For each realization, the closest values to the original sample locations are kept. Then the variogram is calculated using the same parameters. With multiple realizations a histogram for each lag of variogram values can be constructed.

Validation of Theoretical Approach by Simulation

The Theoretical Approach has the following advantages over the two simulation-based methods:

- Implementation is easier since the fourth order moments are calculated analytically and directly. In the Local Simulation Method, after the calculation of the covariance matrix, several realizations of a Gaussian variable must be performed. The generation

of a random number and calculation of the inverse of the normal distribution are required. The Global Simulation Method requires simulation of several realizations; then, the locations are extracted and the experimental variograms are calculated for each realization.

- The program to calculate the variogram variance through the Theoretical Approach, is faster than the program designed to calculate the variogram variance using the Local Simulation Method, due to the lower number of calculations involved. It is also faster than the Global Simulation Method.
- The Theoretical Approach gives the correct result if the variable is multigaussian, while the numerical methods give an approximation, but should always converge to this result.

The Global Simulation Method shows the entire distribution of uncertainty, which is an advantage since, even if the theory says that the uncertainty for each lag is normally distributed, one should expect that the values are always greater than or equal to zero.

Example 1: Cluster.dat

Consider the database `cluster.dat` available in GSLIB. The sample locations are in a pseudo-regular grid, with clusters in the high value zones, see Figure 1. After normal score transformation, the north-south variogram is calculated for five lags, using a lag separation distance of 4.0 and a lag tolerance of 2.0.

An isotropic spherical variogram model with range 15 m and 90% of variance contribution is fitted to the experimental variogram. The nugget effect is 0.1 (10% of variance contribution):

$$\gamma(\mathbf{h}) = 0.1 + 0.9 \cdot Sph\left(\frac{h}{15}\right) \quad (12)$$

The variogram uncertainty is assessed using both `gamu` and `gamu2` and the results are displayed using `gamuplt`. The parameters for each program are presented in Appendix. The variance has been calculated for each lag using the theoretical approach and the local simulation approach. In the local simulation approach (using LU simulation), 100 realizations were performed. The results are presented in Table 1.

The results show that with a reasonable number of LU simulations, the Local Simulation Method gives a variance very close to the theoretical result. Assuming normality in the uncertainty distribution, the confidence intervals can be calculated. The variogram, its model and the central confidence intervals at 95%, 75%, 50% and 25% for each lag are shown in **Figure 2**.

Example 2: Amoco.dat

This database contains porosity data and the variogram uncertainty is calculated using the Theoretical Approach and both numerical methods. There are no clusters in the data locations, see Figure 3. The normal score transformation is performed over the data. A

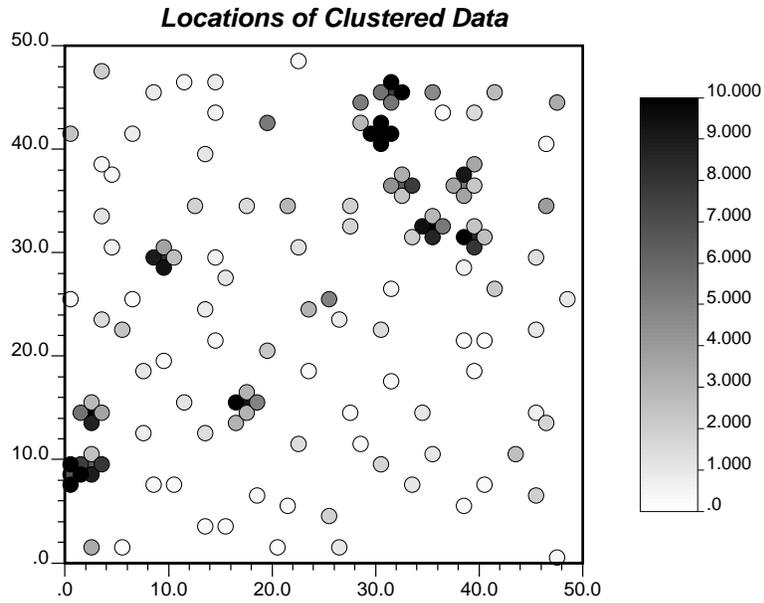


Figure 1: Location map of samples - Cluster.dat

Lag	Lag Distance	Experimental Variogram	Fitted Variogram	Theoretical Variance	Simulation Variance
2	1.395	0.262	0.225	0.016	0.015
3	4.361	0.431	0.481	0.085	0.077
4	7.906	0.716	0.746	0.182	0.164
5	11.876	1.191	0.946	0.318	0.269
6	15.796	1.198	1.000	0.383	0.333

Table 1: Theoretical approach to calculate the variogram confidence intervals

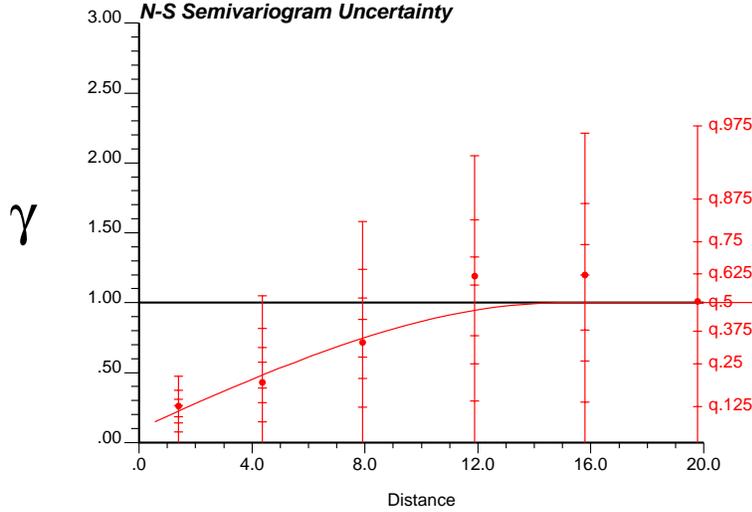


Figure 2: Experimental variogram, model and confidence intervals.

zonal anisotropy was found using the variogram map presented in Figure 4. The anisotropy directions are North-South and East-West. The variogram was calculated for the North-South and East-West directions and the following 2-dimensional model was fitted:

$$\gamma(\mathbf{h}) = 0.15 + 0.30 \cdot Sph\left(\sqrt{\left(\frac{h_x}{3700}\right)^2 + \left(\frac{h_y}{2000}\right)^2}\right) + 0.55 \cdot Sph\left(\sqrt{\left(\frac{h_x}{3700}\right)^2}\right) \quad (13)$$

where h_x is the distance measured toward the East direction and h_y is the distance measured toward the North direction.

The calculation of confidence intervals was performed using `gamu` and `gamu1t`, and the results are shown in Figure 5.

The Global Simulation Method was used to obtain the entire uncertainty distribution for each lag. 100 non-conditional realizations of a Gaussian random variable were generated using `sgsim`. The simulated values at the sampled locations (obtained from the database `amoco.dat`) were extracted for each realization. The experimental variogram was calculated using the simulated values at the sampled locations and the same parameters were used to find the experimental points shown in Figure 5.

The experimental variogram using only the simulated data at the sample locations is calculated for each realization (now considering the effect of the ergodic fluctuations and the “sampling fluctuations”). The results are shown in Figure 6.

Table 2 shows the variogram variance for each direction and lag, calculated using `gamu2` (Theoretical Approach), `gamu` (Local Simulation Method) and `sgsim` (Global Simulation Method). 100 realizations were generated for the numerical methods.

The results obtained from the Theoretical Approach and the Local Simulation Method are similar. However, the Global Simulation Method gives lower variances for all the lags. The main difficulty of this approach is to generate realizations that honor the variogram model used as input. Ergodic fluctuations should be accepted. The variogram calculated for

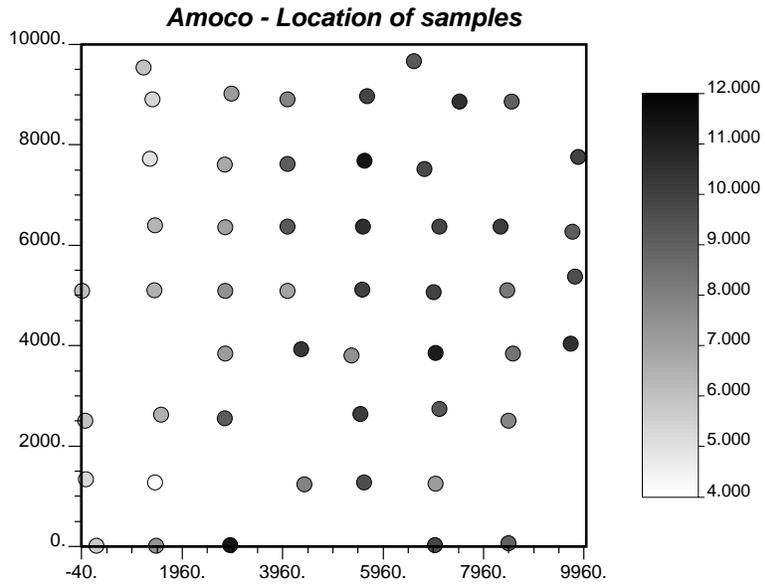


Figure 3: Location map of samples - Amoco database.

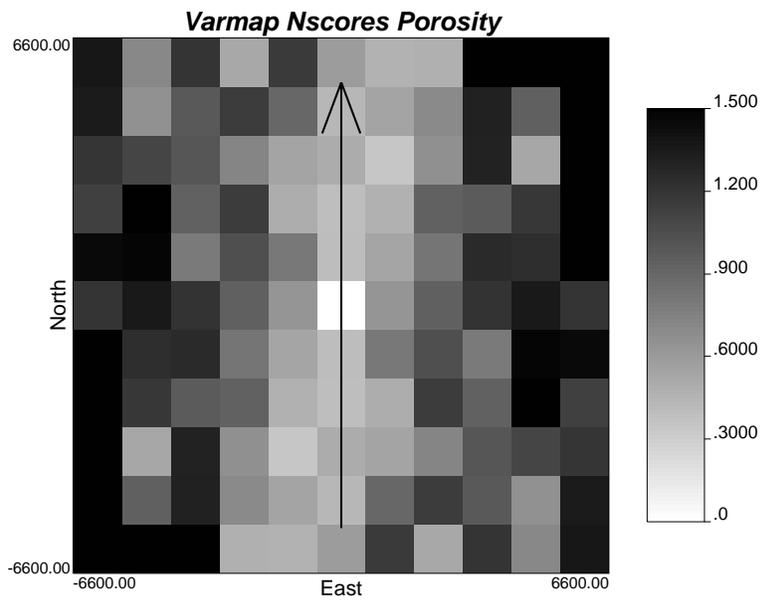


Figure 4: Variogram map - Amoco database.

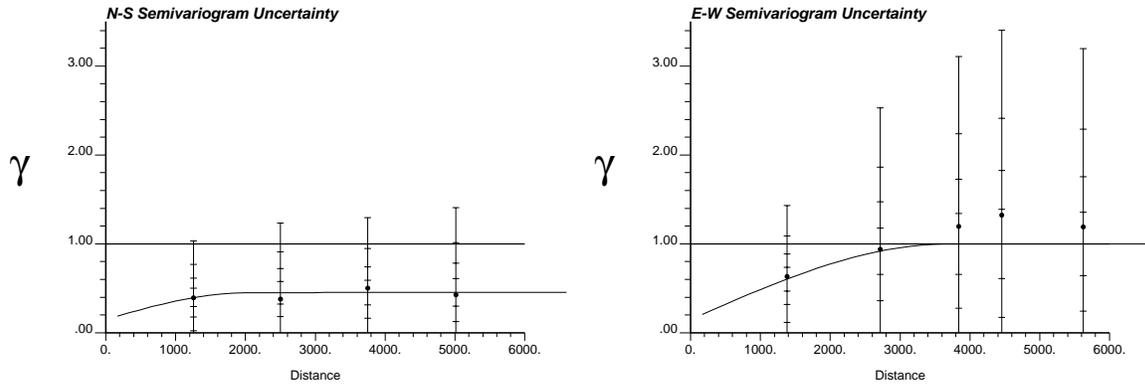


Figure 5: Confidence intervals for each lag. Left: North-South direction; Right: East-West direction.

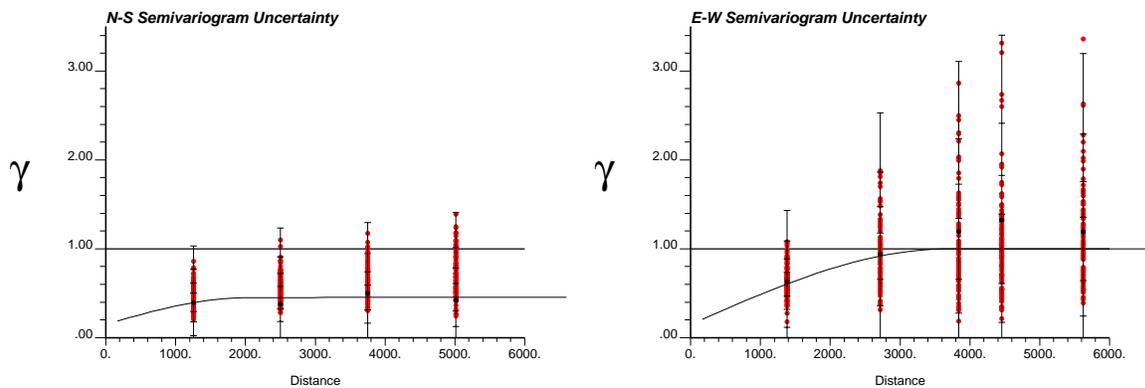


Figure 6: Experimental variograms calculated using the simulated data at sampling locations.

Direction 1: North-South						
Lag	Lag Distance	Experimental Variogram	Fitted Variogram	Variance Theo. App.	Var. Local Sim. Meth.	Var. Global Sim. Meth.
2	1256.382	0.392	0.397	0.100	0.105	0.015
3	2497.855	0.381	0.452	0.165	0.159	0.018
4	3750.738	0.501	0.453	0.188	0.184	0.024
5	5013.975	0.426	0.454	0.230	0.236	0.045
6	6308.414	0.585	0.455	0.255	0.277	0.086
Direction 2: East-West						
Lag	Lag Distance	Experimental Variogram	Fitted Variogram	Variance Theo. App.	Var. Local Sim. Meth.	Var. Global Sim. Meth.
2	1382.653	0.631	0.604	0.178	0.177	0.023
3	2715.147	0.937	0.918	0.678	0.703	0.092
4	3840.735	1.195	1.000	1.157	1.177	0.210
5	4459.274	1.323	1.000	1.505	1.278	0.258
6	5624.451	1.193	1.000	1.255	1.257	0.213

Table 2: Theoretical approach to calculate the variogram confidence intervals.

each realization (using all the simulated nodes) is presented in **Figure 7**. The variability in the variograms calculated using all the nodes in the grid is lower than the expected variability at the experimental lags (calculated using only the samples, not the entire grid). This can be due to the “sampling fluctuations”, i.e. the variability in the variogram due to the scarce data available to do the calculation. However, this missed variability should be recovered when the locations values are extracted and the variogram is calculated only with these data, but that does not occur. The variable may not be multigaussian and therefore, the Theoretical Approach and the Local Simulation Method, based on the same assumptions would fail.

Histograms showing the entire uncertainty distribution for the corresponding lags are presented in **Figure 8**. All the histograms generated through the Global Simulation Method are asymmetric with a tail to the right. This asymmetry was expected since the variogram must be a non-negative function.

Comments

In all approaches, a variogram model is required. Ideally, one could determine the uncertainty using the experimental points, before fitting the model. The assessment of uncertainty, however, requires a positive definite covariance model (i.e. a non-negative variogram model), therefore a variogram must be fitted before evaluating the uncertainty. This seems circular, however, it is the only way to solve the problem: the authorized model is assumed as the expected value of the variogram at each lag and then the variance is calculated.

The variogram uncertainty assessment allows uncertainty in the variogram to be transferred to subsequent stages of a geostatistical study. The Theoretical Approach and the

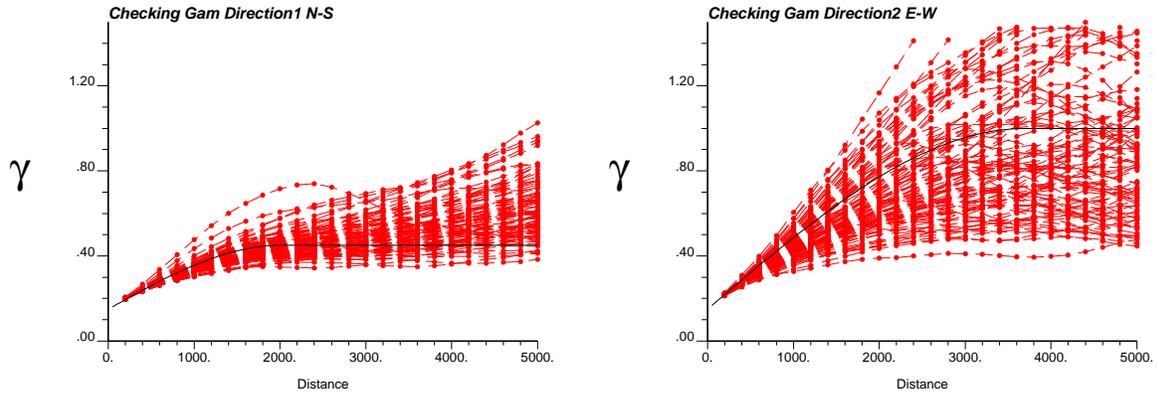


Figure 7: Simulated Variogram and Model used as Input.

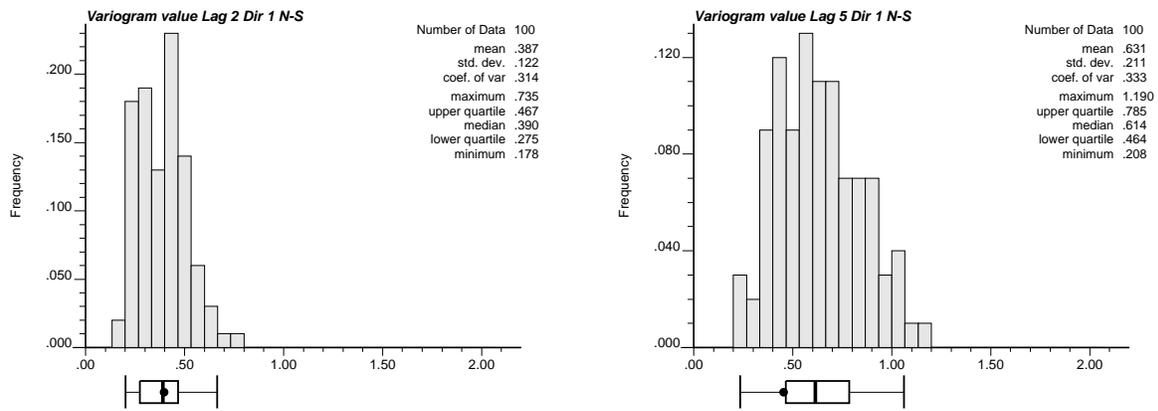


Figure 8: Histograms for lags 2 and 5 in direction N-S.

Local Simulation Method presented in this paper generate the same result. The Global Simulation Method requires more computer time and should give the same result, since the idea is basically the same than the Local Method; however, it is difficult to honor the variogram between the sampling locations and consequently, the results are quite different. Another reason to justify these differences is that the multiGaussianity can not be checked. An important result from the Global Simulation Method is that one can observe the shape of the entire distribution of uncertainty in the variogram for each lag. As it was expected, the distribution is not normal (the variogram cannot take negative values), so one should be cautious when applying the theory.

The confidence intervals were calculated using the experimental variogram value as a mean for each lag and the standard deviation assuming normality. This assumption is not totally correct since the histogram of variogram values obtained for each lag is asymmetric and constrained by the condition of non-negativity of the variogram.

The difference between the point uncertainty and the joint uncertainty must be addressed: the procedures presented in this paper allows the calculation of the pointwise uncertainty. Within this uncertainty, several variogram models (joint models) can be fitted. The confidence intervals for the joint model will be different since one is interested in finding the uncertainty in the continuity of the variable. Several joint models with different degrees of continuity (e.g. characterized by a Gaussian model the more continuous and by a spherical model the less) should be used in the subsequent stages of the study (simulations, flow modeling) to account for the joint uncertainty of the variogram model.

Future Work / Conclusions

The objective in evaluating the variogram variance at each lag is to propagate a joint uncertainty (i.e. the uncertainty in the whole model) through the next steps in the geostatistical study. The relation between the ergodic fluctuations and the variance calculated for each lag must be studied. The use of the joint uncertainty (fitting a model using the variances at each lag) is an issue that must be considered in future research.

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Appendix

`gamu` allows the calculation of the variogram uncertainty at each lag through a local LU simulation of each pair of pairs used in the calculation of the experimental variogram. The parameters are basically the same than those of `gamv` in `GSLIB`, however a variogram model must be specified. The number of LU simulations and the seed number must also be given. Figure 9 shows the parameter file to run `gamu`. The parameter file requires the following variables:

- **datafl**: File with the input normal transformed data.
- **icolx**, **icoly** and **icolz**: Columns for x, y, and z coordinates.
- **nvar** and **ivar(1),...**, **ivar(nvar)**: Number of variables and their column order in the data file.
- **tmin** and **tmax**: Minimum and maximum value to trim data.
- **outfl**: File for variogram output. In this file the experimental variogram and the model are listed.
- **outfl2**: File for variogram uncertainty. This file contains columns with the lag number, lag distance, experimental variogram value, variogram model and variance of the model for each lag.
- **nlag**: Number of lags to calculate.
- **xlag**: Lag separation distance.
- **xltol**: Lag tolerance.
- **azm**, **atol**, **bandwh**, **dip**, **textbftol**, and **bandwd**: The azimuth angle, the half-window azimuth tolerance, the azimuth bandwidth, the dip angle, The half-window dip tolerance, and the dip bandwidth.
- **standarize**: If set to 1, the variogram values will be divided by the variance.
- **ivtail**, **ivhead**, and **ivtype**: The variable for the tail and the head of the vector used to calculate the variogram, and the type of variogram (refer to `GSLIB` for the list of possible variograms).
- **nst** and **c0**: The number of structures and the nugget effect.
- **it**, **cc**, **ang1**, **ang2**, and **ang3**: For each structure, the type of structure, the c parameter, and the angles defining the geometric anisotropy must be specified.
- **aahmax**, **aahmin**, and **aavert**: For each structure, the maximum horizontal range, the minimum horizontal range and the vertical range must be specified.
- **seed**: Random number seed (a large odd integer).

Parameters for GAMU

START OF PARAMETERS:

nscore.out	-file with normal data
1 2 0	-columns for X, Y, Z coordinates
1 7	-number of variables,col numbers
-1.0e21 1.0e21	-trimming limits
gamv0.out	-file for variogram output
gamu0.out	-file for var. uncertainty output
4	-number of lags
4.0	-lag separation distance
2.0	-lag tolerance
0.0 22.5 15.0 0.0 90.0 50.0	-azm,atol,bandh,dip,dtol,bandv
0	-standardize sills? (0=no, 1=yes)
1 1 1	-tail var., head var., variogram type
1 0.10	-nst, nugget effect
1 0.90 0.0 0.0 0.0	-ivt, cc, ang1, ang2, ang3
15.0 15.0 15.0	-a_hmax, a_hmin, a_vert
69069	-random number seed
100	-number of realizations

Figure 9: Parameters for gamu.

Parameters for GAMU2

```

START OF PARAMETERS:
nscore.out          -file with normal data
1 2 0              -columns for X, Y, Z coordinates
1 7                -number of variables,col numbers
-1.0e21 1.0e21     -trimming limits
gamv0.out           -file for variogram output
gamu0.out           -file for var. uncertainty output
4                  -number of lags
4.0                -lag separation distance
2.0                -lag tolerance
1                  -number of directions
0.0 22.5 15.0 0.0 90.0 50.0 -azm,atol,bandh,dip,dtol,bandv
0                  -standardize sills? (0=no, 1=yes)
1 1 1              -tail var., head var., variogram type
1 0.10             -nst, nugget effect
1 0.90 0.0 0.0 0.0 -ivt, cc, ang1, ang2, ang3
15.0 15.0 15.0    -a_hmax, a_hmin, a_vert

```

Figure 10: Parameters for gamu2.

- **nsim**: The number of realizations to generate.

The parameter file for **gamu2** is presented in Figure 10. The variables are exactly the same than those for **gamu**, except for **seed** and **nsim**, which are not needed in this case.

Finally, the parameter file for **gamuplt** is given in Figure 11. The parameters needed to run **gamuplt** are:

- **outfl**: File for PostScript output.
- **xmin** and **xmax**: Minimum and maximum plotting limits for the x axis (distance).
- **ymin** and **ymax**: Minimum and maximum plotting limits for the y axis (variogram).
- **title**: A 40-character title for the top of the plot.
- **datafl**: File with model data. Recall that the model was stored in the second variogram in the first output file (outfl) of **gamu** or **gamu2**.
- **vnum** and **color**: The position of the model in the file datafl and the color to plot it.
- **datafl2**: File with the experimental variogram and uncertainty for each lag.
- **vnum** and **color**: The position of the uncertainty data in the file datafl2 and the color to plot it.

Parameters for GAMUPLT

START OF PARAMETERS:

gamuplt.ps	-file for PostScript output
0.0 20.0	-distance limits (from data if max<min)
0.0 3.0	-variogram limits (from data if max<min)
N-S Semivariogram Uncertainty	-Title for variogram
gamv1.out	-file with model data
2 1	-variogram #, color
gamu1.out	-file with variogram uncertainty
1 1	-variogram #, color

Figure 11: Parameters for gamuplt.