Sensitivity Calculation and Derived Information

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Abstract

An important element of any study on dynamic data integration is the computation and investigation of the sensitivity coefficients of reservoir parameters to the simulation variables. Proper understanding of these sensitivity coefficients can steer subsequent course of the research. In this study, some important aspects of the sensitivities have been explored. Beyond the expectation of the authors, a great deal of information has been elicited from the study.

Introduction

An important constituent of a research on dynamic data integration is the computation and investigation of the sensitivities of reservoir parameters to the simulation responses. Understanding the behavior of these sensitivity coefficients is critical to dynamic data integration in reservoir characterization. Much of future courses of the research may hinge on a good understanding of these coefficients.

Originally, two options were considered for sensitivity computation. One choice would be to develop a reservoir simulator with a sensitivity computation module. The second is to use some widely accepted reservoir simulator which computes these sensitivity coefficients or provides sufficient output to permit calculation. The latter was found to be the viable option. Achieving the required level of sophistication for realistic features in simulator would itself take up immense man-hours. `Eclipse` is probably the most widely used reservoir simulation program today [2]. It has been used, verified and validated for numerous reservoir scenarios, and many research studies have been based on the responses of this reservoir simulator. `Eclipse` has the feature of sensitivity computation.

Sensitivity Computation

For the sake of completeness, the mathematics involved in the gradient computation is summarized here. The finite-difference formulation of the governing flow equations for 3D 3-phase reservoir simulation can be represented as:

$$F(U^{n+1}, U^n, \theta) = 0 \tag{1}$$

where $U^{n+1} = [P_o \; S_w \; S_g \; \ldots]^T$ at time step $(n+1)$, i.e. the response variables to be simulated. Let $[\Theta] = [\theta_1 \; \theta_2 \; \cdots \theta_M] = [\overline{k} \; \phi \; \ldots]^T$ be the set of reservoir simulation parameters, which may be the permeability vectors, porosity at all grid location. These algebraic equations are strongly coupled nonlinear ones. Gradient based iterative techniques used for the solution of the above equations involves:

$$\frac{\partial f}{\partial U^{n+1}}(U^{(k+1)} - U^{(k)}) = -f(U^{(k)}, U^n, \Theta) \tag{2}$$

where the superscript k is the iteration index, while n refers to the time step.

The sensitivity coefficients of the response variables with respect to any parameter, θ, can be written as

$$\frac{\partial U^{n+1}}{\partial \theta}$$

which can be computed indirectly from Equation 1 or 2. Differentiation with respect to parameter θ of the flow equation (Equation 1) leads to:

$$\frac{\partial f}{\partial U^{n+1}} \frac{\partial U^{n+1}}{\partial \theta} + \frac{\partial f}{\partial U^n} \frac{\partial U^n}{\partial \theta} + \frac{\partial f}{\partial \theta} = 0 \tag{3}$$

which can be rearranged to obtain

$$\frac{\partial U^{n+1}}{\partial \theta} = -\left[\frac{\partial f}{\partial U^{n+1}}\right]^{-1} \left(\frac{\partial f}{\partial U^n} \frac{\partial U^n}{\partial \theta} + \frac{\partial f}{\partial \theta}\right) \tag{4}$$

At each time step, discretized flow equations are solved once. In other words, the Jacobian of the flow equations, $\left[\frac{\partial f}{\partial U^{n+1}}\right]$, is inverted only once. The sensitivity coefficient with respect to any reservoir parameter θ is obtained using Equation 4. This is an efficient approach as flow equations are solved only once regardless of the number of reservoir parameters. However, the computation time increases by a factor of (0.1-0.2) times the usual run for each sensitivity parameter depending on the complexity of the problem.

Base Case Reservoir Description

In order to investigate the behavior of computed sensitivities, a “base case” (labeled `MOD5U`) reservoir scenario is studied. A regular grid of 64×64×16 is considered with dimensions 500 ft × 500 ft × 7.5 ft for the reservoir overlying an aquifer (grid: 64×64×2; dimensions: 500 ft × 500 ft × 15 ft). A single structure variogram

Variable	Variogram
Porosity	$\gamma_\phi(h) = 0.1 + 0.9Sph_{\substack{a_x=a_y=5000ft \\ a_z=15ft}}(h)$
Permeability	$\gamma_k(h) = 0.3 + 0.7Sph_{\substack{a_x=a_y=5000ft \\ a_z=15ft}}(h)$

Table 1: Porosity and permeability variograms used to generate the reservoir model

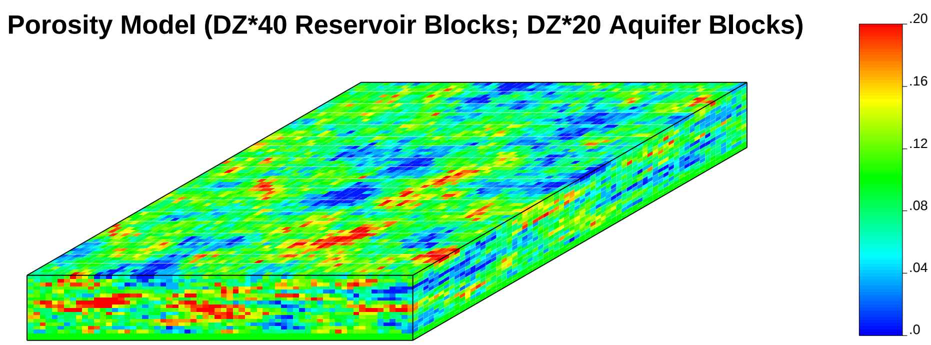

Figure 1: Porosity model for the "base case" study.

is used to generate a porosity model. Permeability is generated using collocated cokriging using the porosity model and correlation coefficient of 0.7. Variogram models used for porosity and permeability are shown in Table 1. To simplify the problem, anisotropy is considered only for the vertical direction by a multiplier of 0.1. Porosity is considered to have truncated normal distribution with a mean of 10% and variance 25%, while permeability a log-normal distribution with a mean 100 mD and variance 1000 mD^2. Aquifer properties are homogeneous with a porosity of 0.1 and a permeability of 100 mD. The idea is to model a reservoir with a moderate bottom-water drive. This emulates a realistic reservoir fluid flow situation.

To create the reservoir models, two `FORTRAN` programs `normsim` and `lognsim` are written which generate normally and log-Normally distributed values from simulated normally distributed values using `sgsim` [1]. Parameter files for these programs are shown in Figures 28 and 29 in the Appendix. Figure 1 shows the isometric view of the porosity model.

A simplified two-phase oil-water system is employed for the simulation. The capillary pressure and the relative permeability curves for the "base case" are shown in Figure 2. There is only one transition zone (layer 16) over the aquifer. Four producing wells are considered at (X,Y) grid locations (16, 17), (45, 15), (14, 40) and (39, 47). Top 12 grids of each are completed. The ECLIPSE data file is shown in Figure 30 in the Appendix. For the limits of well controls, maximum oil production rate, maximum water production rate, maximum reservoir fluid volume flow rate and minimum bottom-hole pressure are set at 5000 STBD, 1500 STBD, 5000 RBD and 1000 PSI, respectively. It should be noted that no artificial well control change (e.g.

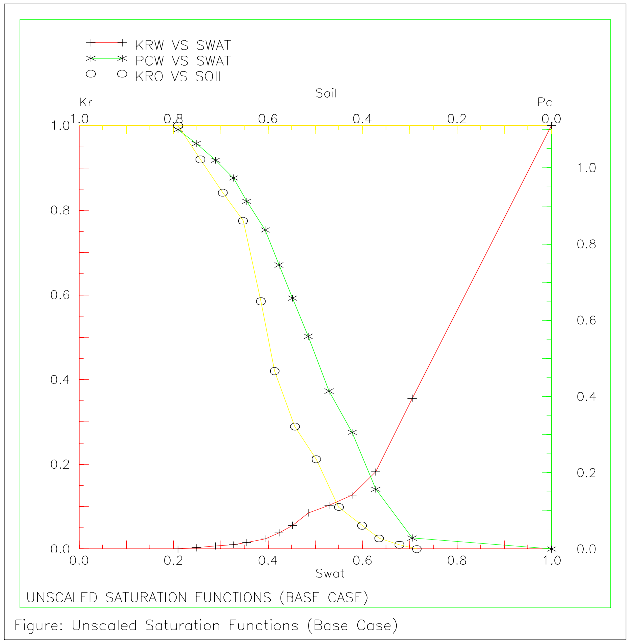

Figure 2: Relative permeability and capillary pressure curves used for the "base case" study.

well recompletions, plugging, etc.) has been activated for the base case simulation.

Base Case Flow Responses

For the base case simulation run, the flow responses are obtained with `ECLIPSE 100`. Bottom-hole pressure, oil production rate and water-cut are shown here only for well 1 and well 2 in Figures 3. The history of well control changes are shown in Table 2.

Event ID	**Days**	**Event**
A	50	Well 1 and Well 4 change from oil rate control to reservoir fluid rate control
B	74.5	Well 3 changes to reservoir fluid rate control
C	100	Well 2 changes to reservoir fluid rate control
D	450	Well 1 changes to water rate control
E	550	Well 4 changes to water rate control
F	600	Well 3 changes to water rate control
G	1150	Well 2 changes to water rate control

Table 2: Well control history for the base case simulation run.

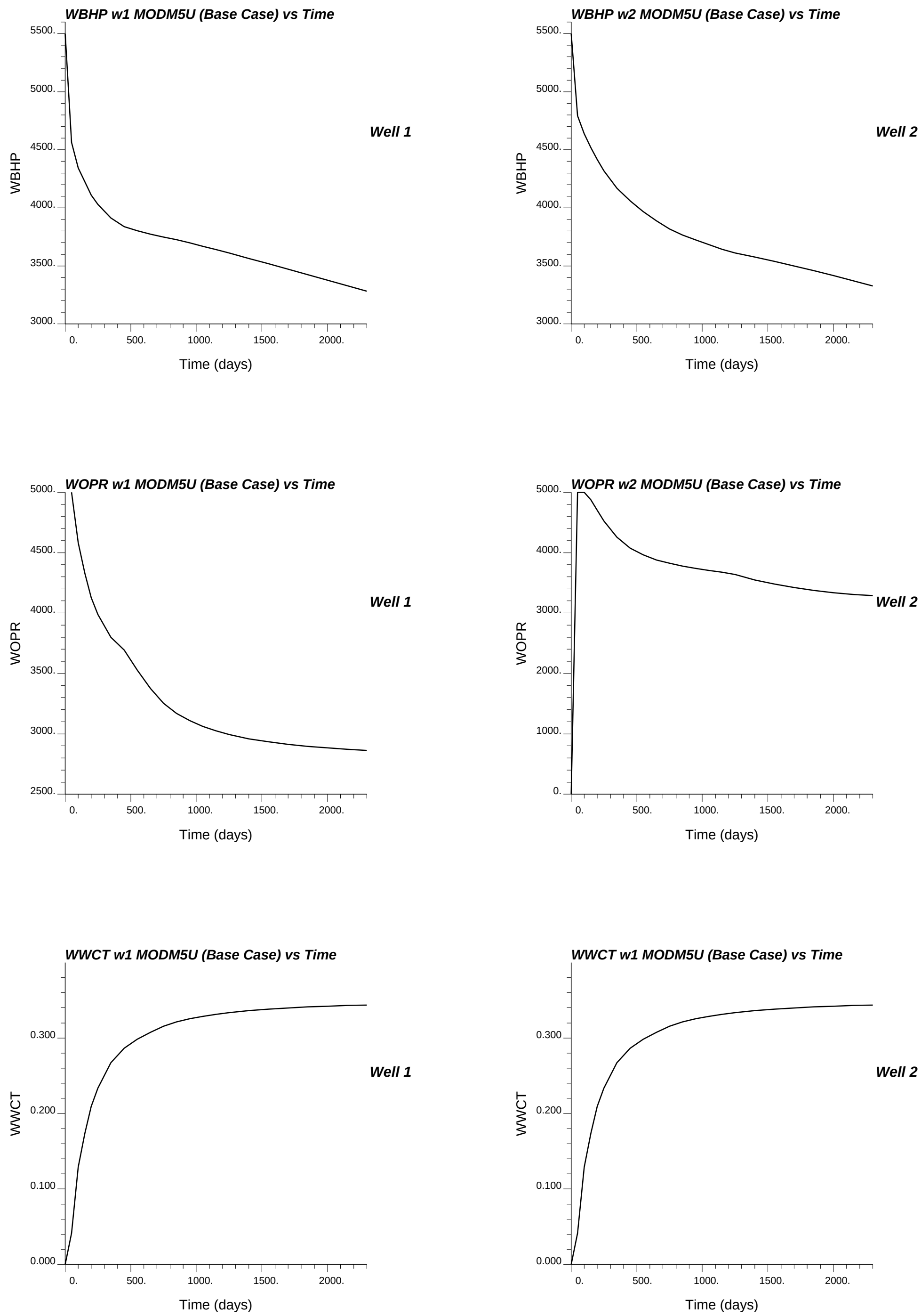

Figure 3: Flow responses for the "base case" study.

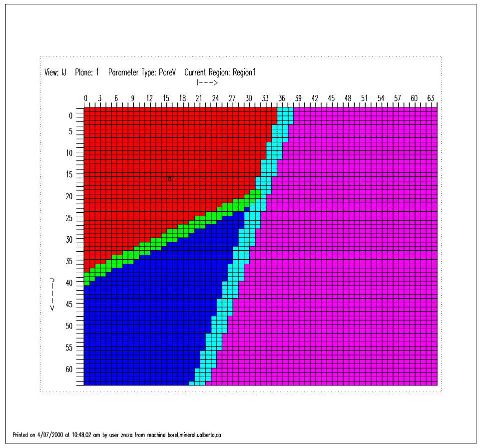

Figure 4: Parameter regions definition for the base case study (XY direction). (Region 1: red, Region 2:green, Region 3: blue, Region 4: light blue; Region 5: purple)

Region Specification for Sensitivity Computation

To compute the sensitivities, reservoir parameters must be identified and their regions defined. The focus here is only on the permeabilities and porosities. Originally, the objective of the present study is to acquire as much information as possible from the computed gradients. Their pattern, behavioral changes in time, particularly due to some 'event' during the simulation period, may capture the influence of reservoir heterogeneity. Regions definition can be 'ad hoc', suitable to specific goals of the study. Regions can be chosen to be oriented along some geological features that are hypothesized to be present in the reservoir. Sensitivity of the response variables with respect to the parameters are computed for the defined regions and investigated. For the base case sensitivity calculation, five regions are defined for each parameter: transmissibility (TRANSX) in the $x-$direction and pore volume (PORV). Figures 4, 5 and 6 show the region definition for the gradient parameters.

Depending on shape and complexity of the parameter regions, few `FORTRAN` programs are coded to define the regions appropriate to specific goals in this study. Two typical parameter files for these programs are shown in the Figures 7 and 8 for regularly shaped region definition, and region definition along presumed fault planes, respectively.

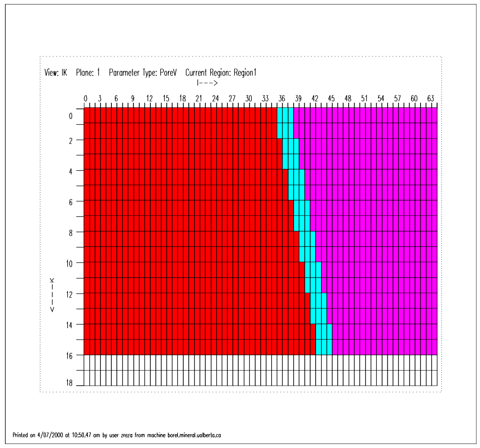

Figure 5: Parameter regions definition for the base case study (ZX direction). (Region 1: red, Region 2:green, Region 3: blue, Region 4: light blue; Region 5: purple)

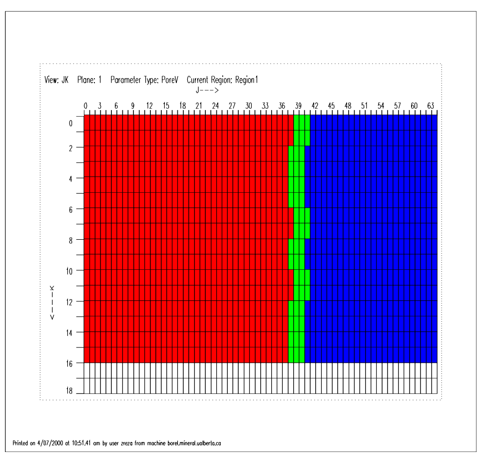

Figure 6: Parameter regions definition for the base case study (YZ direction). (Region 1: red, Region 2:green, Region 3: blue, Region 4: light blue; Region 5: purple)

```
                   Parameters for PARREGPAR
                   ************************
START OF PARAMETERS:
parregpar.data     !file for output
64  64  16         !nx, ny, nz
1  1  0  0         !option for HMPORVM,HMTRANX,HMTRANY,HMTRANZ (1=Yes)
32  32  16         !region specification
2                  !no. of bottom layers for region 0
```

Figure 7: A typical parameter file for parameter region definition for a regular shaped region.

```
                   Parameters for MAKREGHOM
                   ************************
START OF PARAMETERS:
5                      !number of regions
1  1                   !option for HMPORV,HMTRANX (1=Yes)
0.1                    !PORO Region 1
100                    !PERMX Region 1
0.001                  !PORO Region 2
1                      !PERMX Region 2
0.1                    !PORO Region 3
100                    !PERMX Region 3
0.001                  !PORO Region 4
1                      !PERMX Region 4
0.1                    !PORO Region 5
100                    !PERMX Region 5
ppreghom.data          !file for output
64 64 16               !nx, ny, nz
36 20 1 64 1 16 20 40  !fault plane Specs ixe,ixs,iys,iye,izs,ize,iys2,iye2
0.01                   !fault plane factor
fltind.out             !fault indicator file
2                      !no. of bottom layers
0.1  100.0             !bottom layers PORO,PERMX
0.1  100.0             !bottom layers PORO,PERMX
simopreghom.data       !file for parameter region definition
```

Figure 8: A typical parameter file for parameter region definition for oriented along some presumed fault planes. This program also creates a homogeneous reservoir models that can be used in the sensitivity investigation study.

`ECLIPSE DATA` File for Gradient Computation

`ECLIPSE` is used to obtain the gradients. For gradient computation, the usual `ECLIPSE DATA` file needs to specify the gradient well variables, parameters, parameter regions, and reporting and monitoring options. To generate the modified `DATA` file, a C program AUTOGRAD is written. The rationale to use a C program is the need for greater string manipulation and streaming of input/output records. This creates a few input files to be included in the DATA file. For the minimum specification required for gradient computation are namely: _1.DATA, _1.HMU, _1.HMR, _1.HDR, _1.HMS, _1.HMC files. Each of these file names are prefixed by 'Base name' for gradient simulation run. The parameter file for AUTOGRAD is shown in Figure 9. This program requires the original DATA file, parameter definition file, and well gradient option specification file as inputs to create the modified DATA file gradient computation. A typical input file for well gradient specification ('gradspec.data' in this case) is shown Figure 10 showing gradient computation for just one well.

```
                   Parameters for AUTOGRAD
                   ***********************

START OF PARAMETERS:
BIGMOD1                 !Eclipse base name for gradient computation
BIGMOD1.DATA            !Eclipse DATA file name
5                       !Number of parameter regions
1  1                    !Option for HMPORVM, HMPERMX (1=Yes,0=No)
parreghom.data          !File with parameter region definition
ECLIPSE FILE for GRADIENT COMPUTATION BIGMOD1  !Title for the run
32  32  16  2           !DIMENS nx, ny, nz, nza
1                       !Number of wells
gradspec.data           !File with well gradient specification option
```

Figure 9: AUTOGRAD Parameter file for ECLIPSE gradient computation.

```
! Gradient Specification File
1   ! Option for Well 1 (1=Yes,0=No)
1   ! Gradient Option for WBHP (1=Yes,0=No)
1   ! Gradient Option for WOPR (1=Yes,0=No)
1   ! Gradient Option for WWCT (1=Yes,0=No)
```

Figure 10: A typical input file for well gradient specification options.

Computed gradients are usually obtained in an encoded format in _1.FHMD file. To decode the computed gradients a Fortran code capturegrad is written to obtain the encoded gradients from _1.FHMD format and convert to the GSLIB format. The parameter file for this program is shown in Figure 11.

```
                   Parameters for CAPTUREGRAD
                   **************************
```

```
START OF PARAMETERS:
22                   !number of time steps
2                    !number of parameters
5                    !number of regions: HMTRANX
5                    !number of regions: HMPORVM
4                    !number of wells
3                    !number of variables for sensitivity
grad_a.FHMD          !file with ECLIPSE gradients
10                   !number of header lines in gradient file
tranxgrad.out        !Output file for HMTRANX gradients
porevgrad.out        !Output file for HMPORVM gradients
```

Figure 11: Parameter file for program `capturegrad`.

Typical Sensitivity Coefficient Behavior

For the 'base case' reservoir study, the sensitivity coefficients are computed. Parameters considered are transmissivity in $x-$direction and pore volume. Well variables for the sensitivities are analyzed are well bottom-hole pressure (WBHP), well oil production rate (WOPR), and well water-cut (WWCT). The general behavior of the sensitivity coefficients can be quite complex depending on the flow and reservoir complexity. Signatures of various events are often present in the sensitivity coefficients. However, this depends on numerous factors and can be often masked by the interferences of different concurrent phenomena.

Figure 12 shows the sensitivity coefficients computed for the 'base case' for Well 1 which is located at (16,17) in Region 1. These coefficients are for well bottom-hole pressure, well oil production rate, and well water cut with respect to $x-$direction transmissivity of Region 1. Events **A** @ 50 days and **G** @ 450 days have more pronounced effect on the trends as these events involve Well 1. There may be communication between the wells. The sensitivity coefficients here are reported only at the specified reporting intervals. A detailed investigation is required in order to fully understand these sensitivity curves.

Results with only one well

A logical next approach is to decouple the problem by retaining only one well instead of four wells for the same reservoir description. Thus, there will be no well interference. With only Well 1 in Region 1 active, the sensitivity coefficients are computed (`MOD5UW1`). Figure 13 shows the gradient for Well 1 bottom-hole pressure with respect to $x-$direction transmissivity of Region 1 at specified reporting intervals.

Comparing Figure 13 with Figure 12 (top one - for WBHP), it can be seen that early time and late time trends are quite similar. However, there are significant dissimilarities between the two curves. This suggests some interwell communication

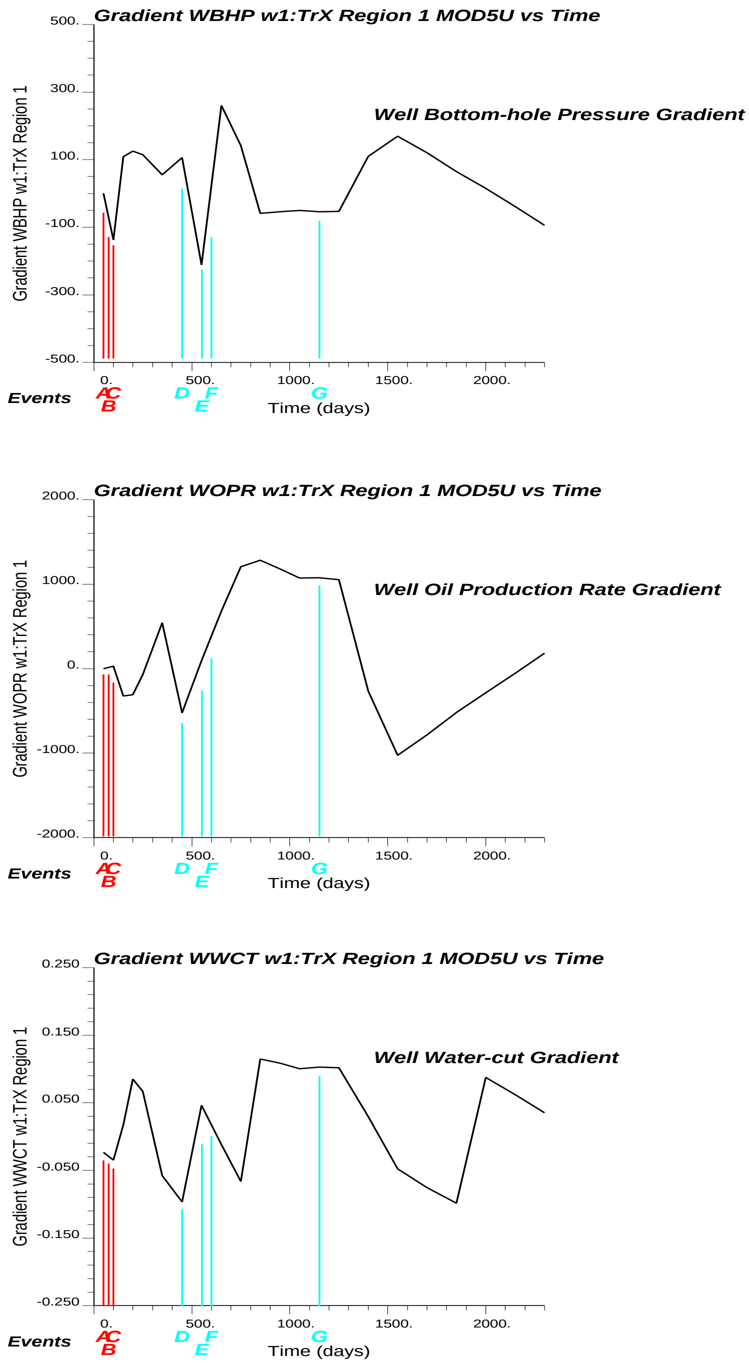

Figure 12: Sensitivity coefficients for the 'base case'. Events labeled (vertical lines) **A** to **G** are those in Table 2.

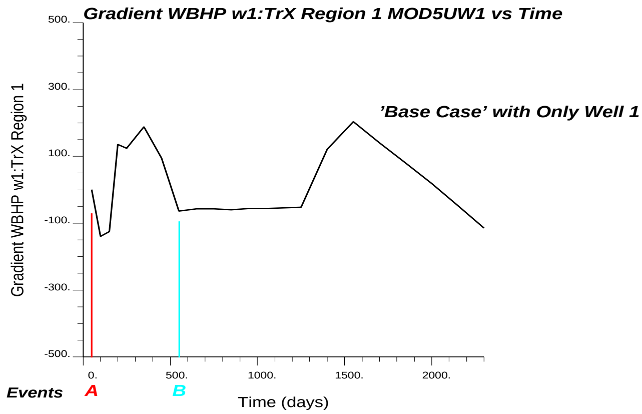

Figure 13: Sensitivity coefficients for the 'base case' reservoir description with only Well 1. Events labeled (vertical lines) are those in Table 3.

Event ID	Days	Event
A	50	Well 1 changes from oil rate control to reservoir fluid rate control
B	450	Well 1 changes to water rate control

Table 3: Well control history for the 'base case' reservoir description but with only Well 1 (`MOD5UW1`).

that affects the sensitivity coefficient behavior as early as about 200 days. Well events have pronounced effect on the coefficients.

Results for different regions

Not all wells are equally sensitive to all the regions. Sensitivity coefficients for regions in which wells are located are orders of magnitude higher than other regions. Figure 14 shows the sensitivity coefficients of well bottom-hole pressure for Region 1, 2 and 5 for both cases `MOD5U` ('base case') and `MOD5UW1`. Well 1 is in Region 1. Thus, sensitivities with respect to parameters in Region 1 are quite large. Whereas, for the parameters in Region 2, which is the narrow strip of region along a presumed fault plane, well variables are almost totally insensitive. While, the variables are only slightly sensitive to parameters in region 5. A close look at the figure says that Well 1 bottom-hole pressure is slightly sensitive to Region 5 transmissivity when well interferences are active in case of `MOD5U`, but it is insensitive in case of `MOD5UW1`. These results apply to this case only.

Effect of reporting specification

The sensitivity coefficients calculated above have been computed at the specified reporting interval only. For a better interpretation, the coefficients may be computed at all time steps (chopped and regular). It is observed that this can be a

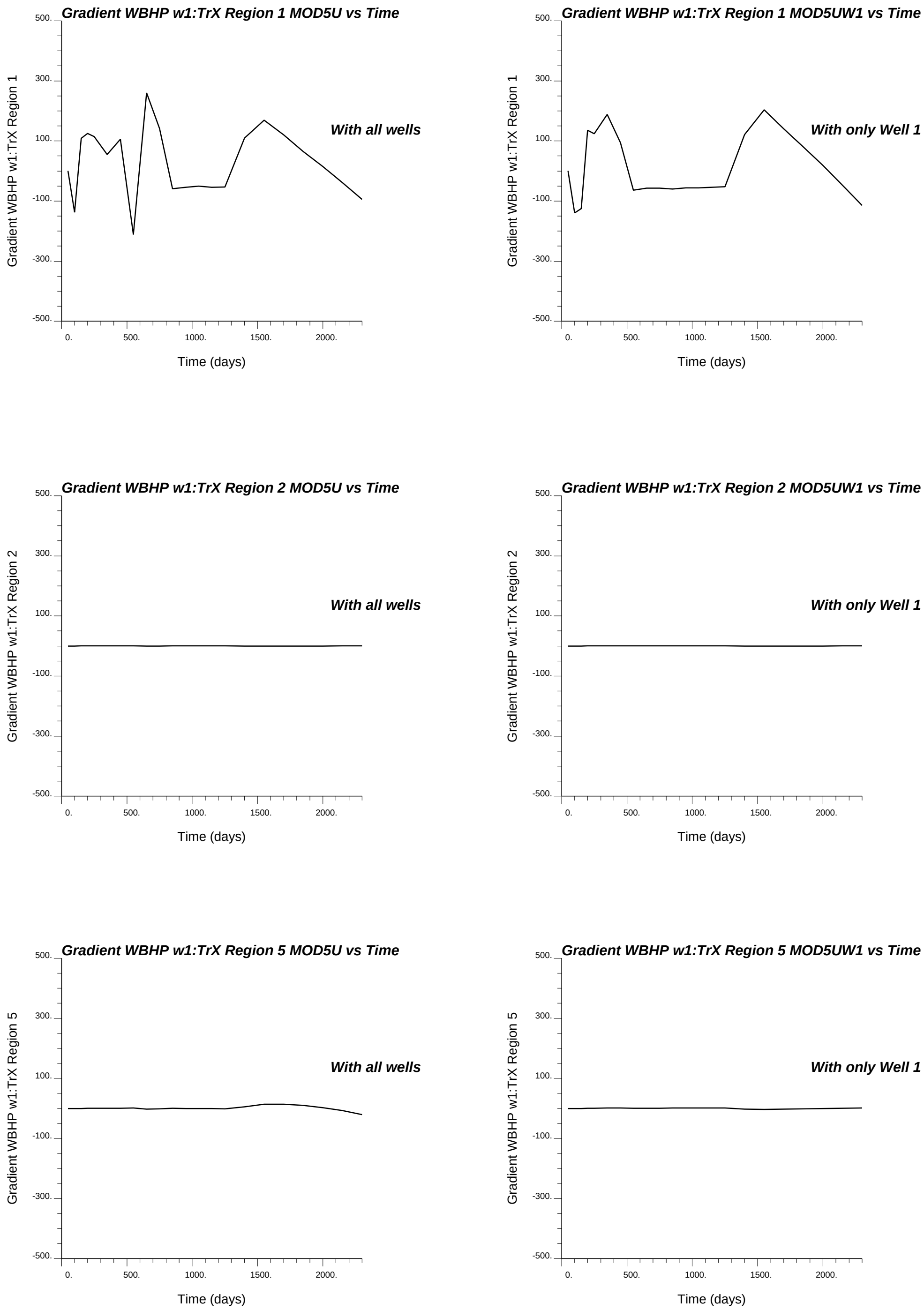

Figure 14: Sensitivity coefficients for Region 1, 2 and 5 for case `MOD5U` and `MOD5UW1` for Well 1 bottom-hole pressure.

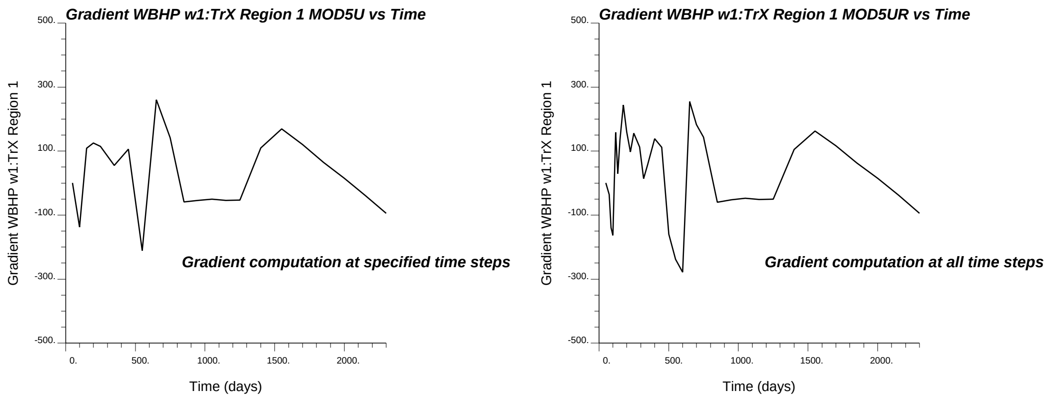

Figure 15: Sensitivity coefficients for Region 1 for case MOD5U and MOD5UR for Well 1 bottom-hole pressure showing the effect of reporting specification.

significant factor particularly when there are many events/phenomena happening in the subsurface flow process or there are some occurrences of non-convergence in the solution process. Figure 15 illustrates this point. Gradients are computed with the same reservoir description MOD5U but at all time steps: case labeled MOD5UR. The figure reveals at early time when there are a few instances of chopped time steps due to some event or problem with solution process, the trends in the gradient curves are dissimilar. Gradients will be computed at all time steps for future investigation and analysis.

Factors Affecting Sensitivity Coefficients

Several investigations were done to determine the changes in the sensitivity coefficient curves. How do these curves change when aquifer strength changes, or artificial well controls are activated ? Does heterogeneity have a significant effect on the gradients ? Are volumes or shapes of parameter regions important in this analysis ? Does well location or grid configuration play a role ? These questions invoke further investigation.

Strength of aquifer

The flow process in a reservoir with a bottom-water drive can be quite complicated with water-coning and other flow phenomena. The volume of an aquifer can be considered as a measure of the strength of the underlying aquifer. To determine the effect of aquifer strength on the sensitivity coefficients, the aquifer thickness is changed.

To simplify the analysis, the reservoir description with only one well is investigated. The 'base case' (MOD5UW1R) has an aquifer thickness of 30 feet. Keeping the reservoir description same as in MOD5UW1R, the aquifer thickness is varied from 5 feet (MOD5UW1AQ2R) to 100 feet (MOD5UW1AQ3R). Figure 16 shows the sensitivity coefficients for Well 1 bottom-hole pressure with respect to $x-$direction transmissivity of Region 1 for these 3 cases. The figure reveals a significant change in the sensitivity coefficients as the aquifer strength changes.

In case of weak aquifer (`MOD5UW1AQ2R`), well control changes only once (at 175 days) from oil rate control mode to reservoir fluid rate control mode. In fact, Well 1 does not attain water rate control during the simulation period. This makes a significant difference in the late time behavior from the other two cases. Strong aquifer case (`MOD5UW1AQ3R`) has similar trends as in the base case `MOD5UW1R`. However, the well bottom-hole pressure is less sensitive to the region transmissibility (Region 1) when the aquifer is stronger. This is expected as pressure support will be higher for a reservoir with a stronger aquifer support or water-drive.

Effect of well control

Due to numerous management decisions, well configurations or modes of operations are changed often within the life of a well. For example, a well connection may be artificially shut off at some depth when water coning occurs or is about to occur. These well control changes affect drastically the sensitivity coefficients.

To illustrate the effect of well controls, reservoir description with a strong underlying aquifer is chosen. The rationale for investigating the strong aquifer is that there are more occurrences of well control changes because of escalated water coning. Figure 17 reveals significant differences in the sensitivity coefficients with and without artificial well controls. Event history for the case with artificial well control is tabulated in Table 4. Figure 17 reveals that before artificial well controls are activated (i.e. before 150 days) the trends are exactly same. Once a 'worst-offending connection' is shut off, subsequent gradient behavior changes significantly.

It is observed that the effect of any particular event, as water rate control mode, is not always the same. It depends on phase saturations, pressure level etc. An event like connection shut-off may cause higher oil production rate if oil phase in the vicinity has sufficient mobility and well bottom-hole pressure will decrease; however, when this is not the situation, flow of both oil and water is hindered and well bottom-hole pressure increases. The gradients show corresponding changes. Similarly, the effect of bottom-hole pressure control mode can be determined. In this case, there will not be any pressure sensitivity, but gradients of oil production rate or water-cut may change significantly.

Effect of time step control

Earlier, it was evident that gradient computation at specified reporting time steps masks information of the gradient behavior. We suggest computing the gradients at all time steps. A study will show whether the time steps significantly change the sensitivity coefficients.

Time steps are varied from 5 days to 200 days. In Figure 18 sensitivity of time steps on the gradients (well bottom-hole pressure) are shown for variation of step sizes from 50 days to 200 days. Table 5 gives the description of the color codes shown in Figure 18. It is apparent from the figure that step increments have significant

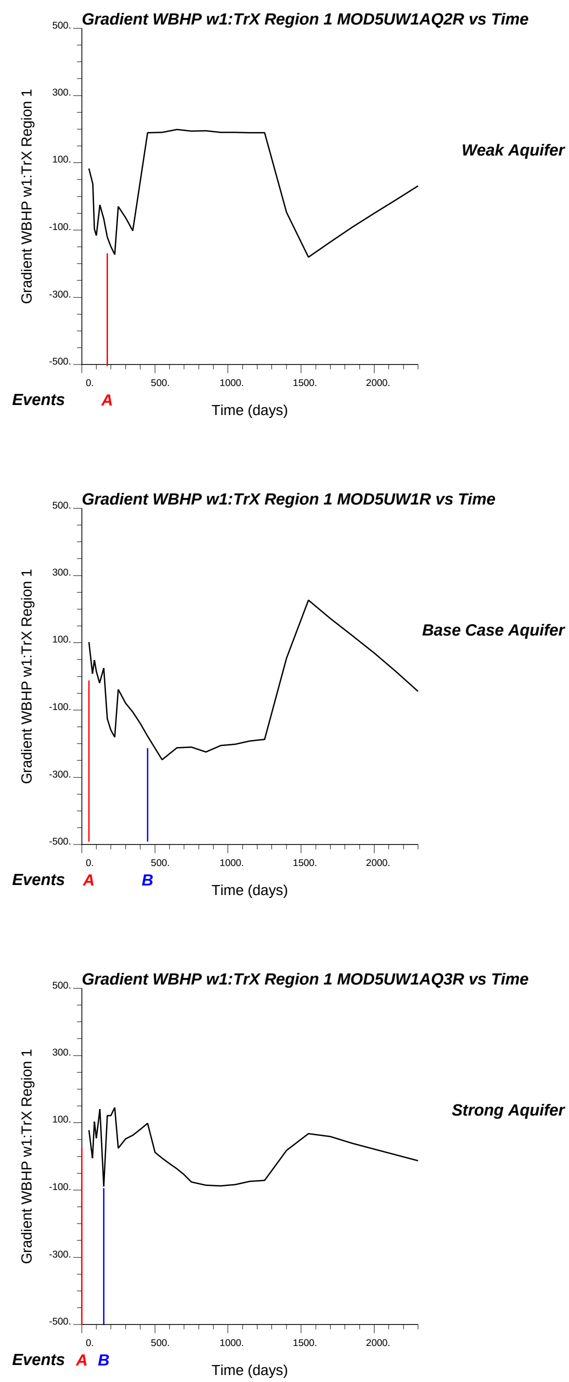

Figure 16: Bottom-hole pressure sensitivity coefficients for cases: `MOD5UW1AQ2R` (weak aquifer), `MOD5UW1R` (base case aquifer) and `MOD5UW1AQ3R` (strong aquifer). Events labeled A (in red vertical lines): well control changes from oil rate control mode to reservoir fluid rate control mode; labeled B (in blue vertical lines): well control changes to water rate control mode.

Event ID	Days	Event
A	0	Changes from oil rate control to reservoir fluid volume rate control mode (`RFVRC`)
B	150	Changes to water rate control mode (`WRC`)
C	175	Worst offending connection 12 shut-off (`WOC 12`) `RFVRC`
D	200	`WRC`
E	225	`WOC 11` and `RFVRC`
F	250	`WRC`
G	280.98	`WOC 10`
H	314.75	`WOC 9`
I	350	`WOC 8`
J	392.45	`WOC 7`
K	421.23	`WOC 6`
L	450	`WOC 5`
M	489.24	`WOC 4` and well changes to bottom-hole pressure control mode (`WBHPC`)
N	519.62	`WOC 3`
O	550	`WOC 2`
P	600	`WOC 1`; Well 1 completely shut-off; Well 1 immediately reopened; Secondary water cut limit (0.5) activated; `WRC`
Q	650	`WOC 12`; `WOC 11`; `WOC 10`
R	1050	`WOC 9`
S	1400	`WOC 8`
T	1550	`WOC 7`
U	1700	`WOC 6`
V	1850	`WOC 5`
W	2000	`WOC 4`; `WBHPC`; Non-linear equation convergence failure
X	2090	`WOC 3`

Table 4: Well control and event history for `MOD5UW1AQ3AR` simulation run.

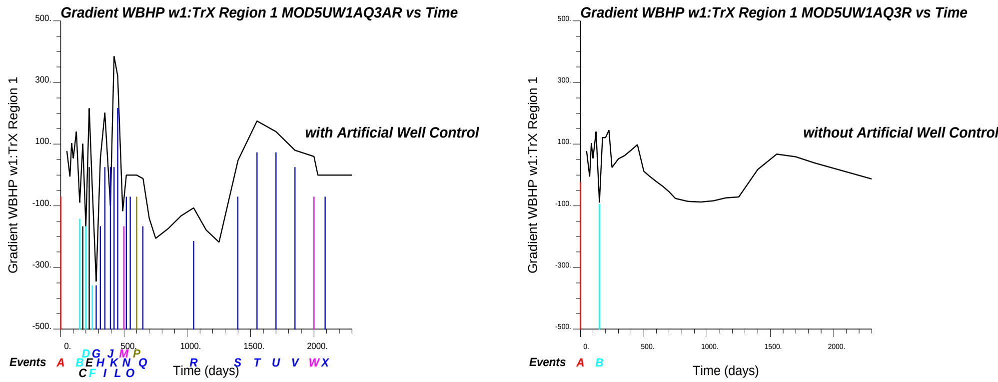

Figure 17: Bottom-hole pressure sensitivity coefficients for cases: `MOD5UW1AQ3AR` (with artificial well controls) and `MOD5UW1AQ3R` (without artificial well control). Events labeled A to X are described in Table 4.

Color Code	Event
`Red`	Change from oil rate control to reservoir fluid volume rate control mode
`Light Blue`	Change to water rate control mode
`Black`	Worst offending connection shut-off and change to reservoir fluid volume rate control mode
`Blue`	Worst offending connection shut-off
`Green`	Worst offending connection shut-off and change to well bottom-hole pressure control mode

Table 5: Event history for 'time steps' sensitivity runs used in Figure 18.

effect on the gradients. The events are shifted forward or backward in time as time steps are varied. One reason for this variation in the trend can be attributed to 'chain effect' that is a shift in any event may trigger different states of fluid saturation or pressure level for the subsequent duration of the flow.

It is also evident from the figure that coarser increments may mask some information. However, it is found that for the reservoir heterogeneity used in these models time steps of 50 days to 100 days will be ideal for future analysis. Less than 50 days step sizes show erratic behavior. A reason for such erratic behavior may be step size approaches the spectral frequency of the noises, which may drastically deteriorate the ill-posedness condition for the optimization problem. In terms of computational efficiency, smaller steps will be quite expensive as opposed to larger time steps.

Effect of heterogeneity

One objective of investigating the sensitivity coefficients is to determine a set of a priori constraints to be used in the optimization process. In an optimization loop,

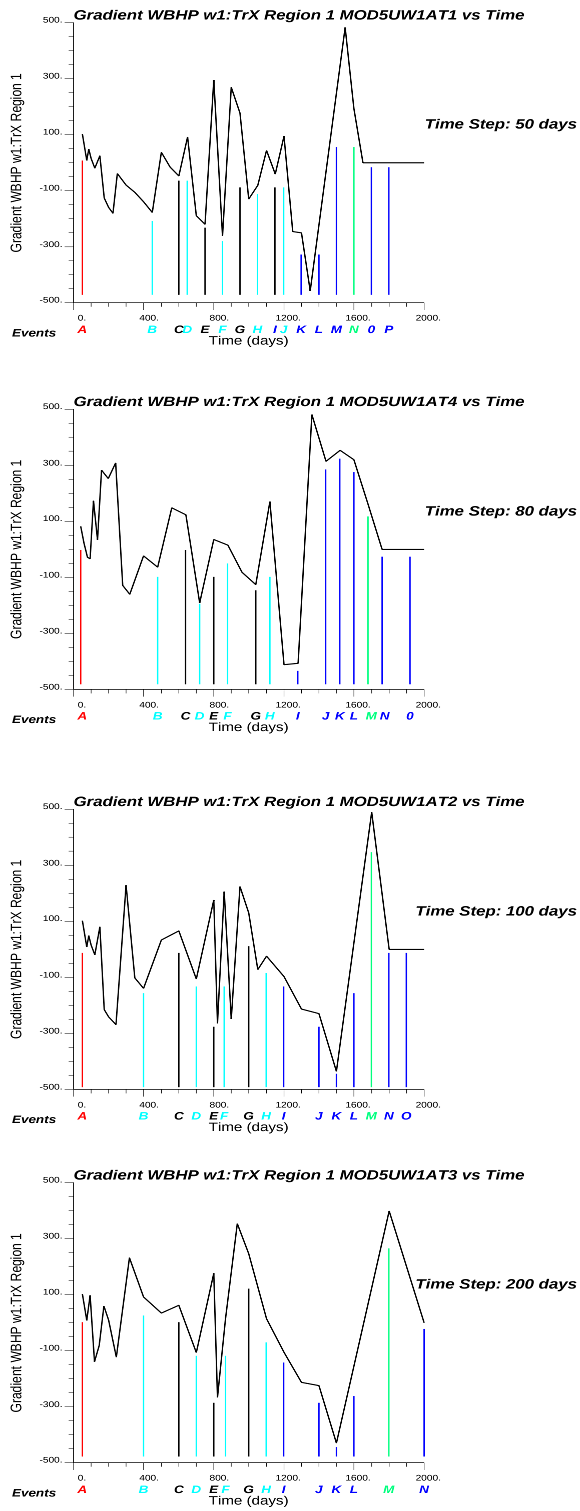

Figure 18: Bottom-hole pressure sensitivity coefficients for cases (time steps in days): `MOD5UW1AT1` (50), `MOD5UW1AT4` (80), `MOD5UW1AT2` (100), and `MOD5UW1AT3` (200). Events color coded are described in Table 5.

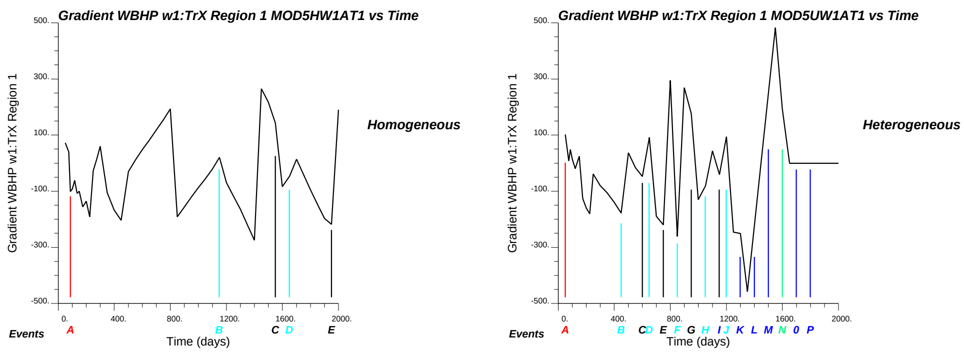

Figure 19: Effect of heterogeneity on the sensitivity coefficients: homogeneous (`MOD5HW1AT1`) and heterogeneous (`MOD5UW1AT1`). Events are color coded in the same manner as in Table 5.

reservoir properties such as grid permeability and porosity values are modified, thus changing the heterogeneity of the reservoir. This optimization process is computationally intensive. To make the algorithm efficient the gradients are frozen or kept unchanged for several iterations, which one could call an inner optimization loop. To ascertain the validity of such approximate technique one should investigate the effect of heterogeneity on the sensitivity coefficients of the flow responses with respect to relevant reservoir properties.

Figure 19 shows the sensitivity coefficients of well bottom-hole pressure with respect to Region 1 transmissivity for a reservoir description with homogeneous properties (`MOD5HW1AT1`) and heterogeneous properties (`MOD5UE1AT1`, base case). It is apparent from the figure that heterogeneity plays a significant role in determining the gradient trends. Although not much can be explained about the effect of heterogeneity, Figure 19 reveals homogeneous properties lead to less frequent changes in well control. However, this does not imply the variability in the gradients will be less for homogeneous reservoirs. Further investigation is needed to quantify the effect of heterogeneity on the sensitivity coefficients.

To quantify the effect of heterogeneity changes in reservoir property models in any optimization procedure, a random permeability ($x-$direction) model is generated with a mean of zero. This model multiplied with a scalar coefficient is added to a 'base case' permeability model. The sensitivity coefficients are computed for the new model. The random model can be considered as a gradient direction in a gradient based optimization technique, and the coefficient can be considered as the distance in this gradient direction. The coefficient is varied from 0.25 to 20. Figure 20 illustrates the effect of such heterogeneity changes. It is evident from the figure that overall trend is quite similar with the exception of some local variation up to a coefficient as high as 10. This affirms the validity of keeping sensitivity coefficients frozen in an inner optimization loop.

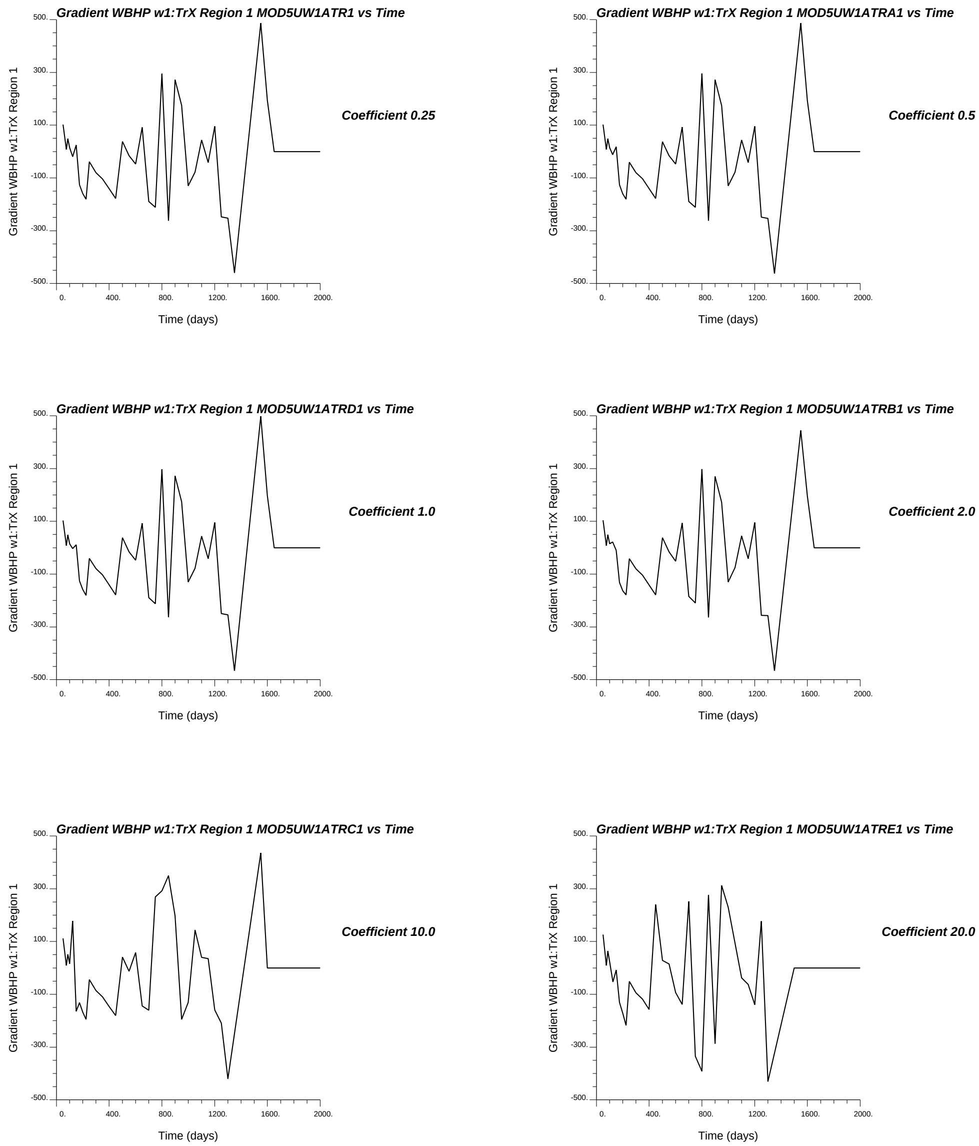

Figure 20: Sensitivity of heterogeneity changes in permeability model effected with a scalar coefficient multiplied random model added to a 'base case' model. The scalar coefficient is varied from 0.25 to 20.

Effect of Region Volume

Question arises if the volume of specified parameter region has any effect on the sensitivity coefficients. Intuition says a parameter region with a larger volume representation has greater effect than a smaller one. In order to investigate this a reservoir model similar to the 'base case' one is considered however with only one well at the central location. Parameter regions are specified with Region 1 encompassing the well and Region 2 rest of the reservoir domain. Originally, the model has 64×64×16 reservoir grids with an underlying aquifer. The volume of Region 1 is varied by including 1×1 to 33×33 grid blocks in $x-$ and $y-$direction. Figure 21 shows the importance of region volume on sensitivity coefficients. There is a significant jump in the gradient absolute values when region volume increased from 1×1 to 3×3. However, any volume increase after that does not have any practical effect on the sensitivity coefficients.

Effect of Well Location and Grid Configuration

Analyses from preceding study suggest that well location may have significant effect on the sensitivity coefficients. To study the effect of well location, 9 concentric parameter regions are specified around a central well. Figure 22 illustrates the specified regions. Figure 23 shows the borehole pressure sensitivity coefficients for the concentric regions. It is evident from the figure that gradient absolute values diminish with the regions further away from the well.

Grid orientation affects the sensitivity coefficients. This is unraveled with the following study. To mimic two different grid orientations, same reservoir model is used but with two different parameter region orientations using square and triangular regions, respectively. The parameter regions for the two cases are shown in Figure 24. There are four regions in each case. The central well is located in Region 1 in both cases. Figure 25 shows borehole pressure sensitivity with respect to transmissibilities of all 4 regions. Due to 7-point finite-differencing scheme of the flow equation, the sensitivity coefficients with respect to Region 4 transmissibility is insignificant in this case. As Region 4 has no neighboring cell with well block. Whereas, in case of triangular regions Region 3 does not have any neighboring cell with the well block. Expectedly, the sensitivities with respect to this region is insignificant compared to other regions.

Sensitivity Coefficients for Regular Parameter Regions

Implementing a multilevel technique in case of regular parameter regions will be more systematic than for irregular regions. These techniques can be implemented at different levels of data integration with minor conforming at each level. It is worth investigating the sensitivity coefficients with regular regions.

Reservoir domain of grids 64×64×16 is subdivided by regular regions of size 8×8×8. Figure 26 shows the sensitivity coefficients of Well 1 borehole pressure with respect to regular regions at different times (50, 74.5, 87.23, 100, 500, 1000, 1500 and 200 days). Only the top regions are shown in the figure. A close inspection of

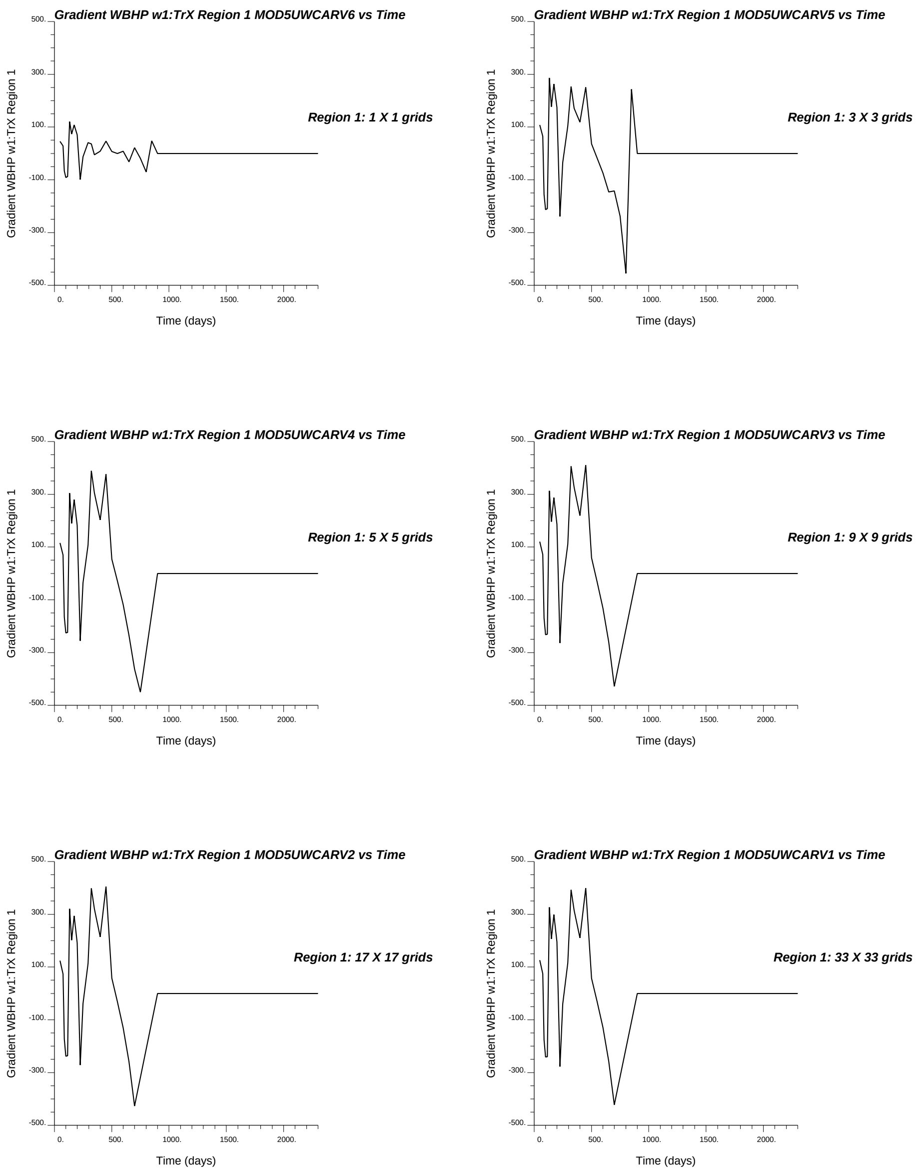

Figure 21: Sensitivity of region volume on the well borehole pressure sensitivity coefficients with respect to transmissibility of Region 1 encompassing a central well.

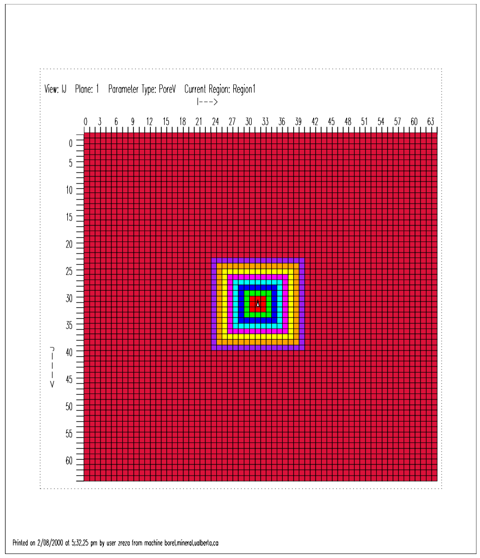

Figure 22: Parameter regions specified to study the effect of well location. There are nine concentric square regions around a central well. Regions are numbered 1 to 9 in an ascending order away from the well.

Figure 26 reveals the magnitudes of the sensitivity coefficients are much less than those observed earlier with only 5 regions. This reaffirms that region volumes have significant effect on the sensitivity coefficients.

Effect of Grid Coarsening

Dynamic data integration is an inverse problem. By nature, any inverse problem suffers from ill-posedness. Forms of regularization are applied in solving these problems. Solution of the inverse problem in a fine grid setup is virtually impossible. One form of regularization is effected through an hierarchical multilevel strategy. Investigating the effect of grid coarsening on the sensitivity coefficients is important to implement this technique.

We started with a 'base case' reservoir description having 128×128×32. It resembled a numerical geological model of the reservoir. Dynamic data integration at this fine resolution model is prohibitive because of the extensive CPU requirement. Five levels of grid coarsening are applied with model sizes 64×64×16, 32×32×16, 32×32×8, 16×16×8 and 16×16×4. Porosity values for these different grids are arithmetically averaged from the finest resolution model. While, power averaging with an index -1 (i.e. harmonic average) is applied to obtain the permeability models. Sensitivity coefficients are computed with respect to 5 regions as used in the earlier studies. Figure 27 shows the sensitivity coefficients of Well 1 borehole pressure with respect to Region 1 transmissibility. It is evident from the figure that with very coarse grid models (as the last one, here) some information is lost. However, a close inspection reveals the overall trend remains same. This is a coign of vantage from

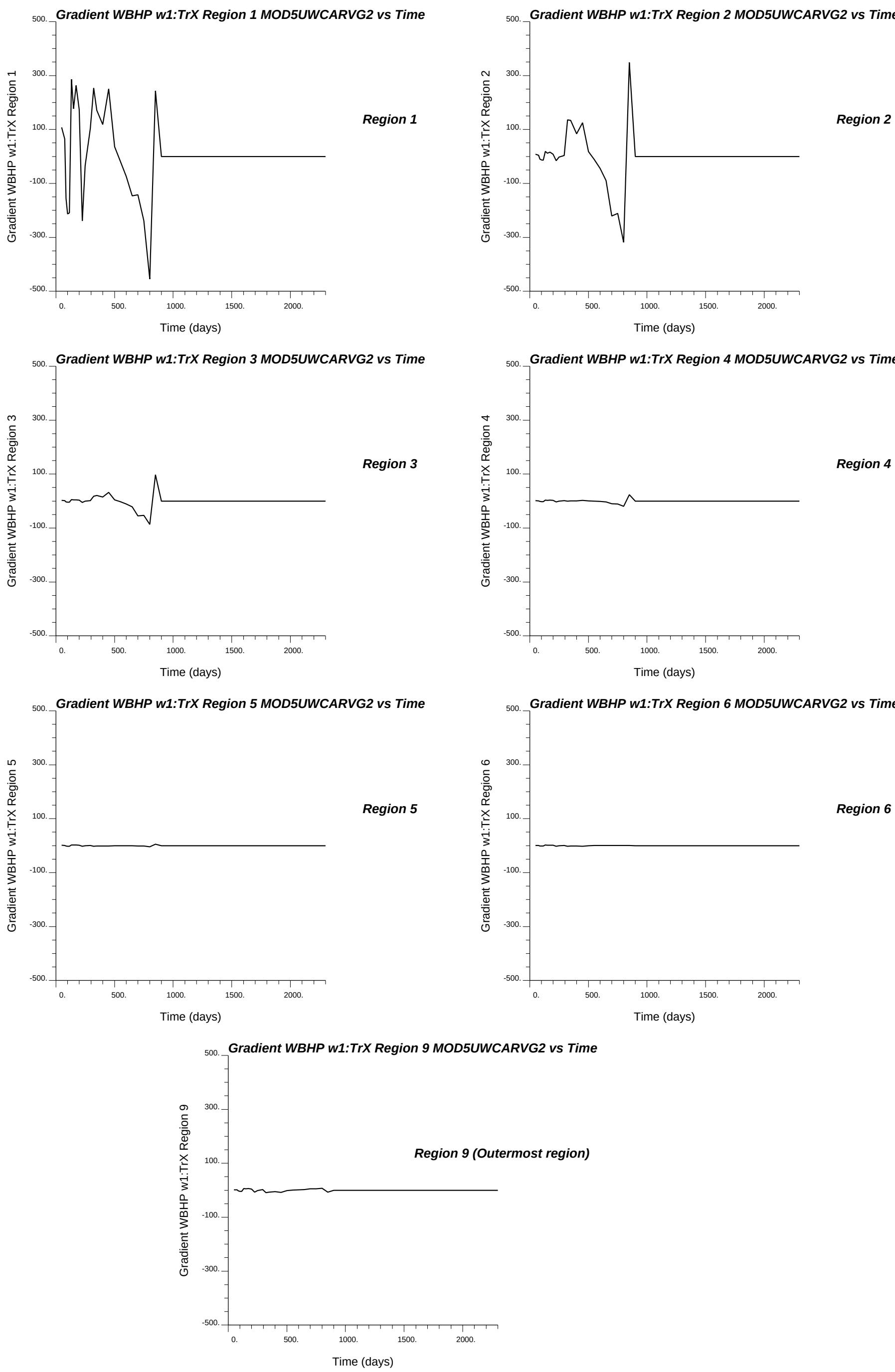

Figure 23: Sensitivity of region volume on the well borehole pressure gradients with respect to transmissibility of concentric regions around a central well. Regions 7 and 8 are not shown. (Regions are shown in Figure 22)

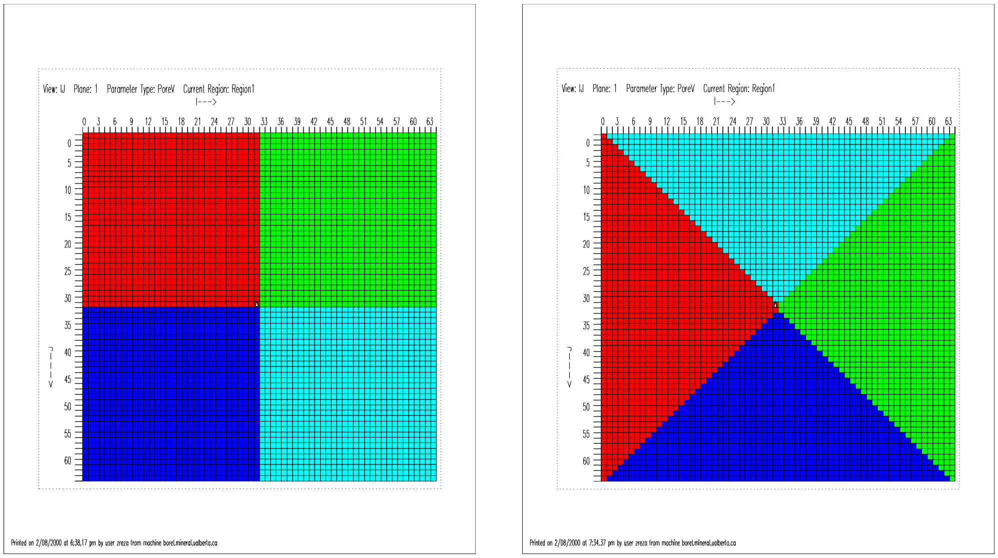

Figure 24: Parameter regions specified to study the effect of grid orientation. Left: squares Right: triangles. Regions are color coded as: red - Region 1; green - Region 2; blue - Region 3; light blue - Region 4. Central Well in Region 1.

the perspective of this research.

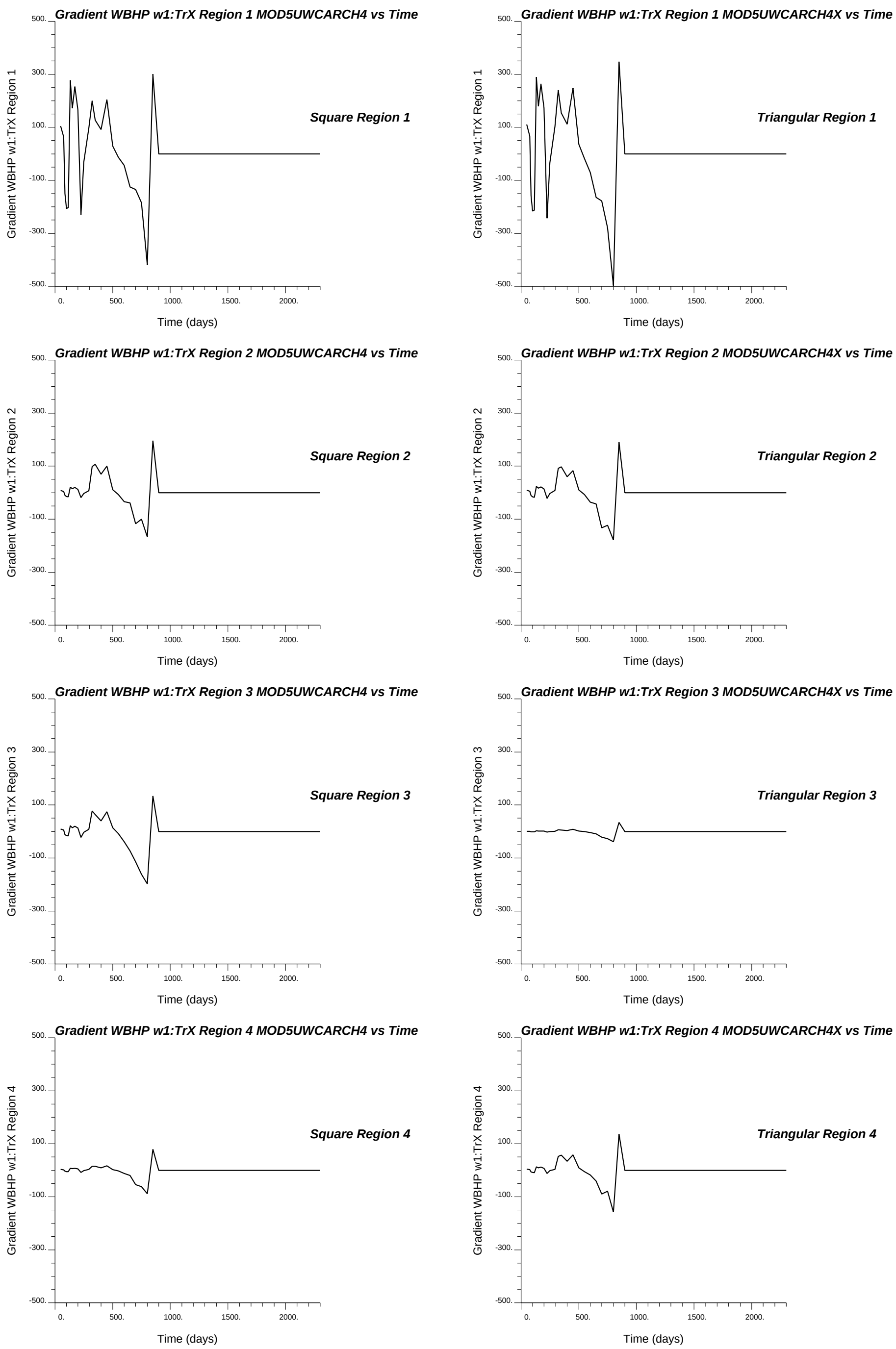

Figure 25: Sensitivity of region volume on the well borehole pressure gradients with respect to transmissibility of Region 1 encompassing a central well. (Regions are shown in Figure 24)

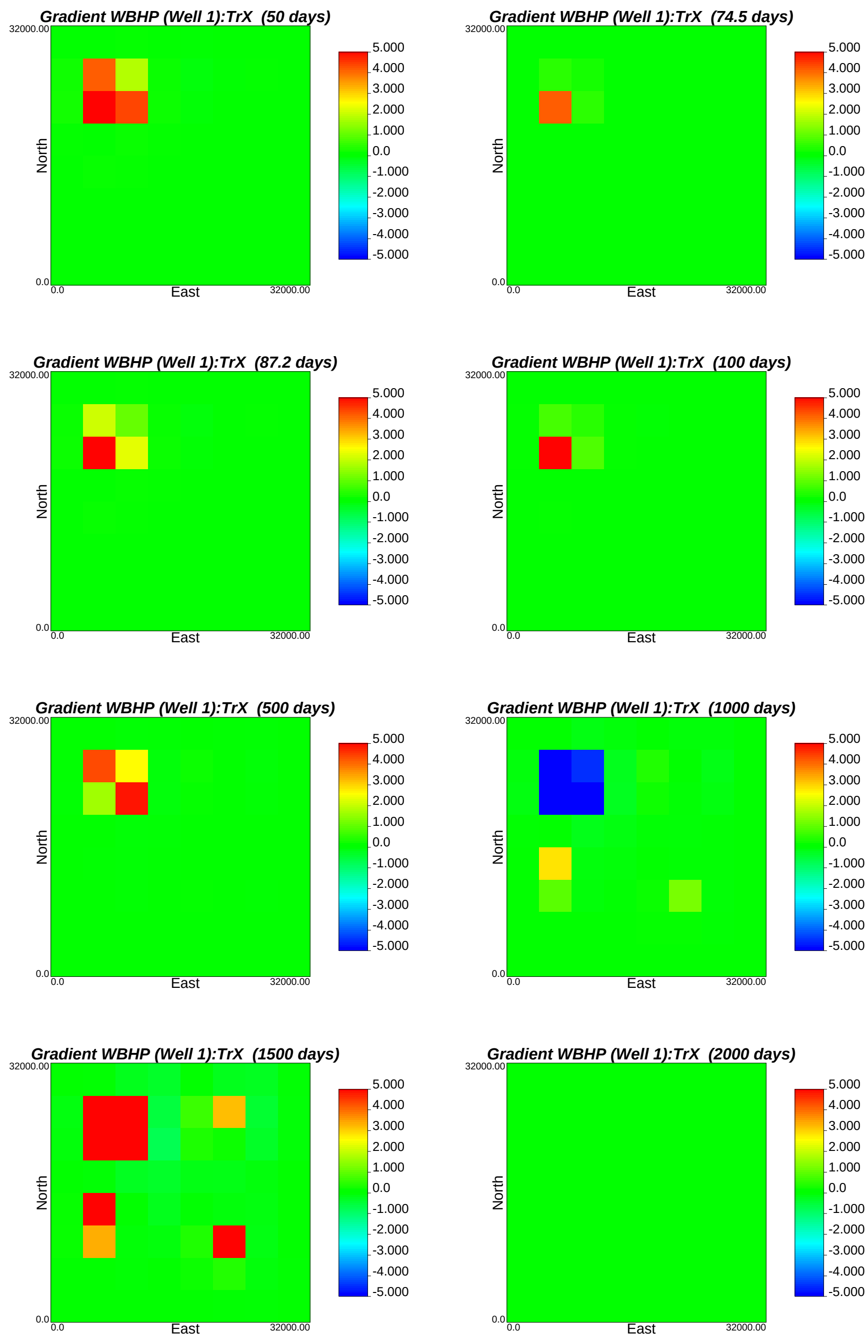

Figure 26: Sensitivity coefficients of the Well 1 borehole pressure gradients with respect to transmissibility of regular regions at time steps of 50, 74.5, 87.23, 100, 500, 1000, 1500 and 200 days. Top 8×8 regions of 8×8×2 are shown in the figure.

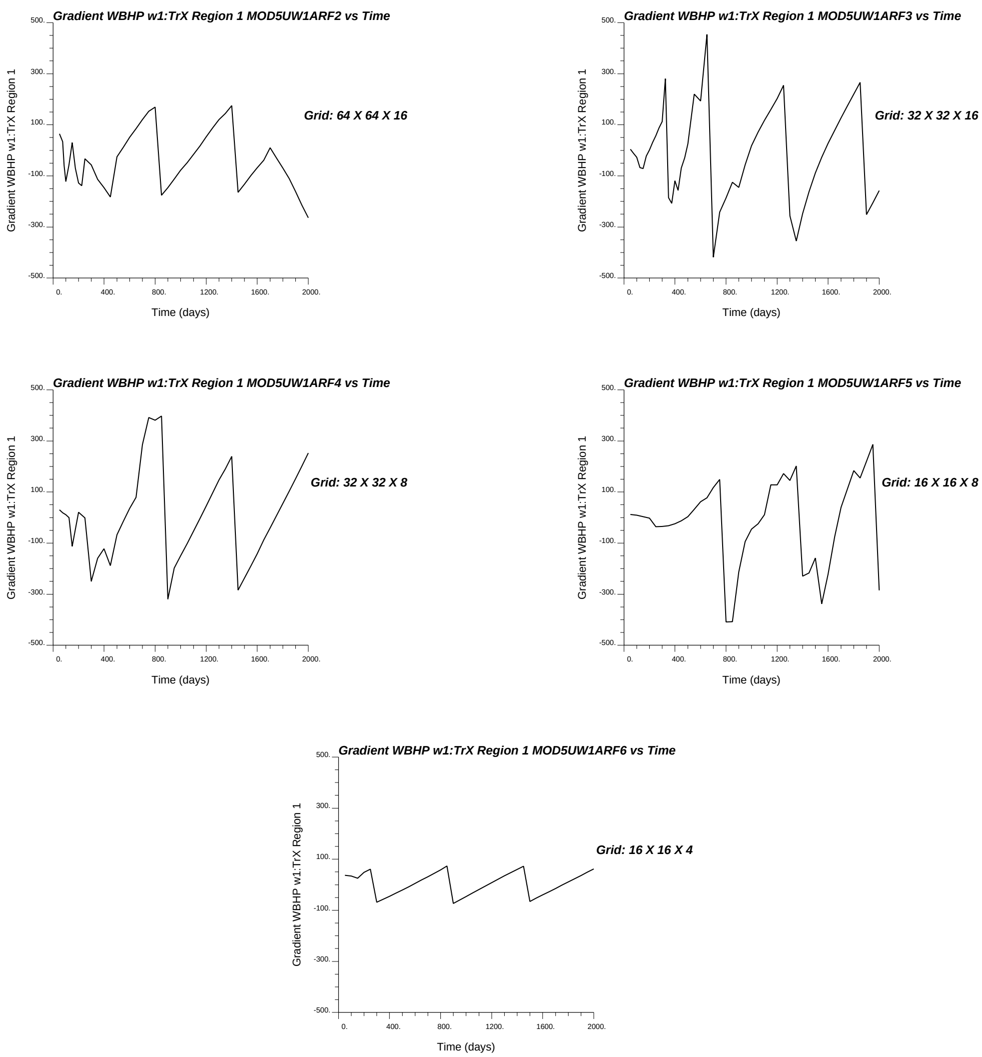

Figure 27: Effect of grid coarsening on the sensitivity coefficients of the Well 1 borehole pressure gradients with respect to transmissibility of Region 1.

References

[1] C. V. Deutsch and A. G. Journel. *GSLIB: Geostatistical Software Library and User's Guide*. Oxford University Press, New York, 2nd edition, 1998.

[2] Schlumberger Geoquest, Schlumberger Technology Corporation, UK. *ECLIPSE 100 Reference Manual*, 1997.

Appendix

```
                   Parameters for NORMSIM
                   **********************

START OF PARAMETERS:
sgsimpor2.out              !file with Normal score data
1                          !column number of the variable
por2.out                   !file for output
1                          !realization number
64   64   16               !nx, ny, nz
10.0                       !mean of the Normal Distribution
25.0                       !variance of the Normal Distribution
```

Figure 28: Parameter file for program `normsim`.

```
                   Parameters for LOGNSIM
                   **********************

START OF PARAMETERS:
sgsimperm2.out         !file with Normal score data
1                      !column number of the variable
per2.out               !file for output
1                      !realization number
64   64   16           !nx, ny, nz
100.0                  !mean of the Normal Distribution
1000.0                 !variance of the Normal Distribution
```

Figure 29: Parameter file for program `lognsim`.

```
-- ===================================================================
-- BIG MODEL WITH 64*64*16 RESERVOIR GRID BLOCKS FOR SIMOPT
-- ===================================================================
RUNSPEC
TITLE
   BIG MODEL WITH 64*64*16 RESERVOIR GRID BLOCKS FOR SIMOPT
DIMENS
   64 64 18  /
OIL
WATER
FIELD
EQLDIMS
    1  100   20    1   20 /
TABDIMS
    1    1   18   12    1   12 /
WELLDIMS
   10    13    2    6 /
NUPCOL
    4 /
START
  1 'MAR' 2000   /
NSTACK
    25 /
UNIFOUT
FMTOUT

GRID    ==============================================================
EQUALS
      DX    500      /
      DY    500      /
      TOPS  8000      1  64  1  64  1  1  /
      DZ    7.5       1  64  1  64  1  16  /
      DZ    25        1  64  1  64  17 18  /
/       EQUALS IS TERMINATED BY A NULL RECORD
INCLUDE
'fltpp2.data'  /
COPY
      PERMX    PERMY   1  64  1  64  1 18  /
      PERMX    PERMZ   /
/
MULTIPLY
    PERMZ 0.1  1  64  1  64  1  16   /
    PERMZ 0.1  1  64  1  64  17 18   /
/
INIT
DEBUG
6*0 0 /
RPTGRID
  'TRANX' 'TRANY' 'TRANZ' 'PORO'  /

PROPS    =============================================================
SWFN
0.210   0.000    1.200
0.248   0.001    1.125
0.289   0.003    1.058
0.327   0.007    0.993
0.355   0.010    0.912
0.394   0.020    0.837
0.423   0.034    0.745
0.452   0.056    0.658
0.485   0.085    0.558
0.529   0.103    0.414
0.578   0.127    0.306
0.628   0.182    0.157
0.705   0.356    0.029
1.000   1.000    0.000  /
SOF2
0.285  0.000
0.322  0.005
0.365  0.015
0.401  0.035
0.450  0.072
```

Figure 30: ECLIPSE data file to for "base case" reservoir simulation run.

```
0.498  0.142
0.543  0.249
0.586  0.360
0.615  0.545
0.653  0.755
0.696  0.821
0.743  0.910
0.790  1.000  /
PVTW
       5514.7     1.029        5.0D-6           0.55            0 /
ROCK
    5514.7   3.8E-6           /
DENSITY
         42.1   60.79  0.06054  /
PVDO
     14.7  1.0255   1.06
    264.7  1.0172   1.064
    514.7  1.0091   1.067
   1014.7  1.0011   1.074
   2014.7  0.9931   1.078
   2514.7  0.9852   1.082
   3014.7  0.9774   1.085
   4014.7  0.9607   1.09
   5014.7  0.9410   1.105
   9014.7  0.9192   1.124
/

SOLUTION =============================================================
EQUIL
       8000   5500   8125    0     7900    0      0      0       0  /

SUMMARY ==============================================================
SEPARATE
RUNSUM
WOPR
/
WWCT
/
WBHP
    /
RPTONLY
RPTSMRY
1                /

SCHEDULE =============================================================
TUNING
50. 150. 1. 1.5 5.0 0.25 0.2  /
/
/
RPTSCHED                                         FIELD   10:29 13 JUN 85
    'PRES' 'SWAT'  'WELLS' 'SUMMARY=2' 'CPU=2' /
WELSPECS
    'PRODUCR1' 'G'   16  17  8090 'OIL'  /
    'PRODUCR2' 'G'   45  15  8090 'OIL'  /
    'PRODUCR3' 'G'   14  40  8090 'OIL'  /
    'PRODUCR4' 'G'   39  47  8090 'OIL'  /
/
COMPDAT
    'PRODUCR1'  16 17 1 12  'OPEN' 0   -1   0.335  /
    'PRODUCR2'  45 15 1 12  'OPEN' 0   -1   0.335  /
    'PRODUCR3'  14 40 1 12  'OPEN' 0   -1   0.335  /
    'PRODUCR4'  39 47 1 12  'OPEN' 0   -1   0.335  /
  /
WCONPROD
     'PRODUCR1' 'OPEN' 'ORAT' 5000  4*                      1000 /
     'PRODUCR2' 'OPEN' 'ORAT' 5000  4*                      1000 /
     'PRODUCR3' 'OPEN' 'ORAT' 5000  4*                      1000 /
     'PRODUCR4' 'OPEN' 'ORAT' 5000  4*                      1000 /
  /
TSTEP
 5*50 10*100 7*150
  /
END     ==============================================================
```

Figure 31: ECLIPSE data file to for “base case” reservoir simulation run (continued).