Angle Rotations in GSLIB

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Abstract

Geological formations do not conform to simple Cartesian coordinate systems. It is essential to work in a coordinate system that is rotated with the geology to effectively model geological properties. The rotation convention and matrices used in GSLIB are presented, along with two programs. These programs are used for determining the azimuth and dip of the rotated principal directions and for calculating the apparent anisotropic ranges in a non-principal direction.

Introduction

Geological formations do not conform to simple coordinate systems. It is rare to find an ore body or oil reservoir that is oriented north-south with a dip of zero. We have to account for the true orientation of these deposits prior to performing any 3-D modelling. This requires some coordinate transformation.

Working with a rotated coordinate system has benefits and drawbacks. Some of the benefits are: (1) the true anisotropy of the deposit can be characterized in the modelling process, (2) it aids in understanding the geology, and (3) it reduces computational time. **Figure 1** shows the apparent anisotropy for the original coordinate system (solid lines) and the true anisotropy for the rotated coordinate system (dashed lines). Some drawbacks are: (1) it is hard to determine the orientation of the rotated principal axis as the rotations become more complex and (2) for some cases a coordinate system rotation may be too simplistic.

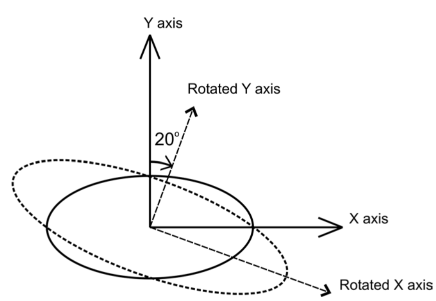

Figure 1: Rotation to capture the true anisotropy. The solid lines are the original coordinate system and the apparent anisotropy. The dashed lines are the rotated coordinate system and the true anisotropy.

This paper presents the coordinate system rotation convention used in all of the GSLIB programs. The required rotation matrices are derived as well as the back rotation matrix. Two utility programs were written to help facilitate working with rotated coordinate systems.

Coordinate Rotation Convention

The logic of all GSLIB coordinate systems is that the original X direction is in the East direction, the original Y direction is North, and the original Z direction is elevation vertically upward. The goal is to rotate the original coordinate system {X,Y,Z} to be along strike, down dip, and perpendicular to the structure coordinates $\{X_R,Y_R,Z_R\}$. All of the rotations presented here adhere to the GSLIB conventions.

Consider a clockwise rotation by angle α (ang1 in GSLIB) to orient the X axis along the strike direction and the Y axis in the dip direction. The Z axis remains unchanged while the X and Y axes are rotated in a clockwise angle α. Note that the clockwise angle is looking down along the Z axis toward the origin. This is shown as the first step in **Figure 2**. The equation for this rotation is:

$$\begin{bmatrix} X_I \\ Y_I \\ Z \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \tag{1}$$

The rotated X_I and Y_I coordinates are intermediate. The next step is to rotate Y_I and Z around the X_I axis to orient the Y_I axis down the dip direction and the Z axis perpendicular to the structure. Consider a counter clockwise rotation of the Y_I and Z axes by angle β (ang2 in GSLIB). Note that the counter clockwise angle is looking along the X_I axis toward the origin. This is shown as the second step in **Figure 2**. The equation is:

$$\begin{bmatrix} X_I \\ Y_R \\ Z_I \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & \sin\beta \\ 0 & -\sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} X_I \\ Y_I \\ Z \end{bmatrix} \tag{2}$$

The rotated Y_R coordinate is the final coordinate. The Z_I coordinate is intermediate. The next step is to rotate X_I and Z_I around the Y_R axis to orient the X_I and Z_I axes along the plunge of the structure. Consider a clockwise rotation by angle θ (ang3 in GSLIB). Note that the clockwise angle is looking along the Y_R axis away from the origin. This is shown as the third step in Figure 2. The equation is:

$$\begin{bmatrix} X_R \\ Y_R \\ Z_R \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} X_I \\ Y_R \\ Z_I \end{bmatrix} \tag{3}$$

These three rotations can be combined into a single matrix multiplication. The result is X along strike, Y down dip, and Z perpendicular to structure.

$$\begin{bmatrix} X_R \\ Y_R \\ Z_R \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & \sin\beta \\ 0 & -\sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$
$$= \begin{bmatrix} \cos\alpha\cos\theta + \sin\alpha\sin\beta\sin\theta & -\sin\alpha\cos\theta + \cos\alpha\sin\beta\sin\theta & -\cos\beta\sin\theta \\ \sin\alpha\cos\beta & \cos\alpha\cos\beta & \sin\beta \\ \cos\alpha\sin\theta - \sin\alpha\sin\beta\cos\theta & -\sin\alpha\sin\theta - \cos\alpha\sin\beta\cos\theta & \cos\beta\cos\theta \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \quad (4)$$

The rotation can be reversed at any time by inverting this matrix or by reversing the transformations. Inverting the matrix leads to the following (note that the matrix inverse and the matrix transpose of Equation (4) happen to be identical in this case):

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \cos\alpha\cos\theta + \sin\alpha\sin\beta\sin\theta & \sin\alpha\cos\beta & \cos\alpha\sin\theta - \sin\alpha\sin\beta\cos\theta \\ -\sin\alpha\cos\theta + \cos\alpha\sin\beta\sin\theta & \cos\alpha\cos\beta & -\sin\alpha\sin\theta - \cos\alpha\sin\beta\cos\theta \\ -\cos\beta\sin\theta & \sin\beta & \cos\beta\cos\theta \end{bmatrix} \begin{bmatrix} X_R \\ Y_R \\ Z_R \end{bmatrix} \quad (5)$$

The matrix equations (4) and (5) are useful for converting UTM or local coordinates to a specific coordinate system that is aligned along strike, down dip, and perpendicular to structure. Geostatistical models are constructed in the rotated coordinate system $\{X_R, Y_R, Z_R\}$ and all values are rotated back to original coordinates {X,Y,Z} afterwards.

Working in the Rotated Coordinate System

The three angle rotations can result in a complex orientation of the rotated coordinate axis. This leads to two application problems: (1) how do we calculate an experimental variogram, and (2) how do we check variogram reproduction after modelling? Two utility programs were written to address these problems. The first program, `azmdip`, is used to calculate the orientation of the three principal directions after the rotations. The second program, `app_range`, calculates an apparent range in a non-principal direction.

The orientation of the three principal directions is required for calculating experimental variograms. Determining the azimuth and dip of the principal directions is relatively easy when working with one or two angle rotations. When using all three rotations this becomes much more difficult. A simple program called `azmdip` takes the three rotation angles specified in GSLIB and calculates the azimuth and dip of the three principal directions in the rotated coordinate system. The input parameters are the three rotation angles:

```
                  Parameters for AZMDIP
                  *********************

START OF PARAMETERS:
 0.0   0.0   0.0             -ang1, ang2, ang3
```

and the program outputs the azimuth and dip of the three principal directions to the screen:

```
hmax azimuth and dip =       0.000      0.000
hmin azimuth and dip =     -90.000      0.000
vert azimuth and dip =       0.000     90.000
```

The azimuth and dip angles from `azmdip` are then used to calculate the experimental variograms for the three principal directions.

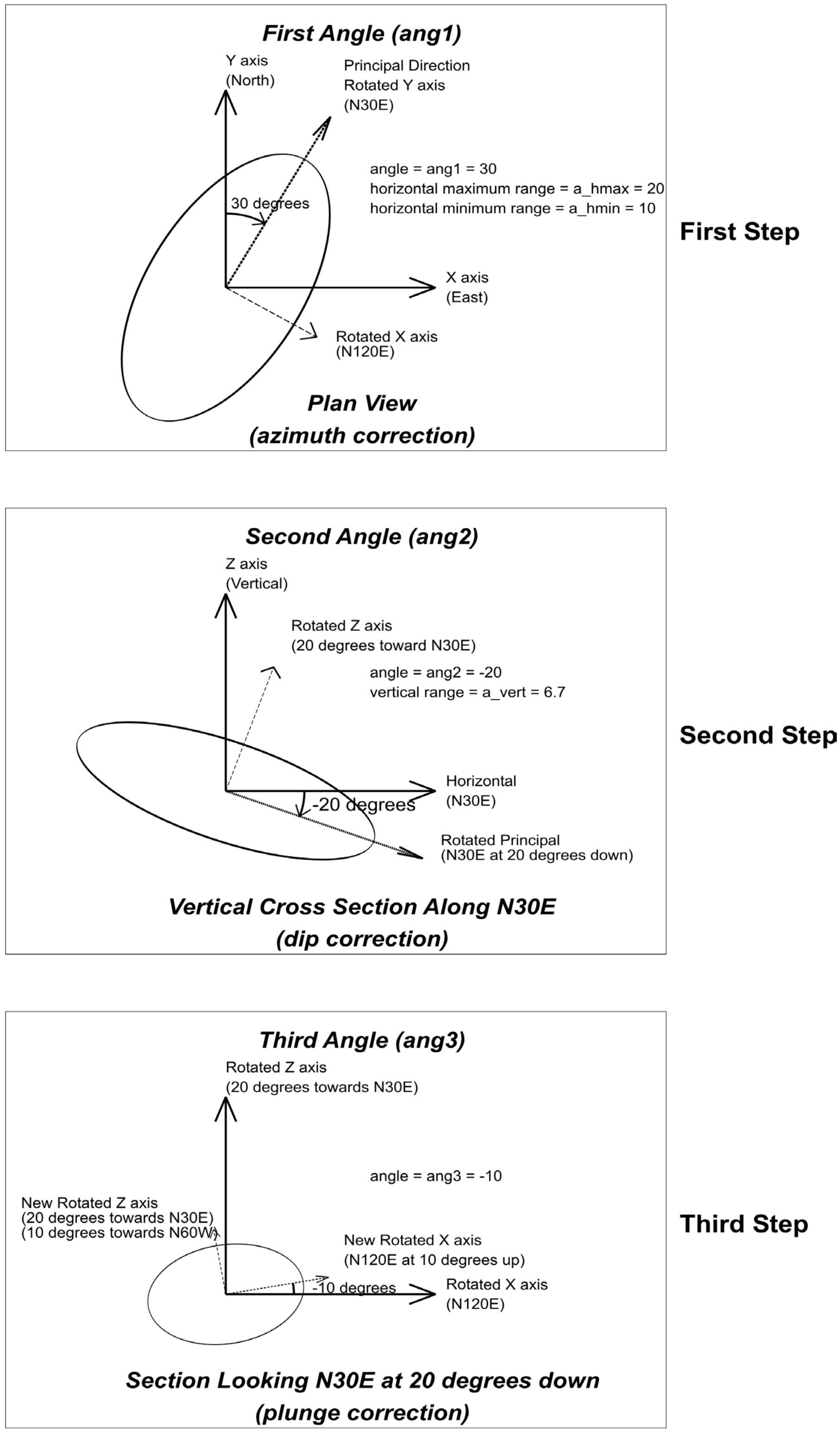

Figure 2: GSLIB convention for axis rotations (Source: Deutsch & Journel, 1996, pg. 28). Note that the third step has been corrected.

It is good practice to check the output model for variogram reproduction. The GSLIB programs `gam` provides a fast algorithm to check variogram reproduction. The limitation of `gam` is that it cannot deal with complex angles. This is because it works with gridded data and cell offsets are used to specify directions. A more accurate check is to use `gamv` for this task. However, the large amount of time that `gamv` requires make this infeasible.

The approach presented here is to calculate the apparent range of the anisotropy from the variogram model in directions that are easy to specify in `gam`. Then variogram reproduction can be checked in these non-principal directions using `gam` and the apparent ranges. The input parameters are the true anisotropy and the directions for calculating the apparent ranges:

```
                        Parameters for APP_RANGE
                        ************************

START OF PARAMETERS:
 70.0  -20.0  0.0              -ang1, ang2, ang3
 10.0   5.0   1.0              -a_hmax, a_hmin, a_vert
  3                            -number of directions
  0.0   0.0                    -azm and dip for apparent range
 90.0   0.0                    -azm and dip for apparent range
  0.0  90.0                    -azm and dip for apparent range
```

Example

Consider the deposit shown in **Figure 3**. We want to align the X, Y, and Z axis to be along strike, down dip, and perpendicular to the structure respectively. This allows us to better characterize the anisotropy of the deposit.

The azimuth correction is the first rotation. Setting ang1 = 25° aligns the Y axis with the dip and the X axis with the strike. The dip correction is the second rotation. Setting ang2 = -40° aligns the Y axis down dip and the Z axis perpendicular to the dip. The plunge correction is the third rotation. Setting ang3 = -20° aligns the X axis along strike and the Z axis perpendicular to the structure.

The next step is to calculate the experimental variograms for the principal directions. The azimuth and dip for the three principal directions can be calculated using the three rotation angles as input to `azmdip`. The output from `azmdip` is:

```
hmax azimuth and dip =      25.000    -40.000
hmin azimuth and dip =     101.832     15.189
vert azimuth and dip =      -4.520     46.042
```

It would be difficult to specify the above angles in `gam` to check the variogram reproduction. Calculating the apparent ranges in three directions that are easy to specify in gam is the preferred method. Consider using the original X, Y, and Z directions and checking the variogram reproduction along those axis. Suppose the anisotropy is represented by the following ranges:

```
hmax = 100, hmin = 30, and vert = 5
```

Using `app_range`, the apparent ranges in the original X, Y, and Z directions are:

```
azm =   90.000, dip =    0.000, apparent range =    29.86
azm =    0.000, dip =    0.000, apparent range =     7.21
azm =    0.000, dip =   90.000, apparent range =     6.93
```

Conclusions

Rotating the coordinate system to capture the true anisotropy is a common practice. Understanding the results of the rotations can be difficult though. The program `azmdip` is used for determining the orientation of the rotated principal directions, and the program `app_range` is used for calculating the apparent anisotropic range in a non-principal direction. These two utility programs will facilitate variogram calculations prior tom modelling and variogram reproduction checks.

References

[1] Deutsch, C.V. and Journel, A.G., 1998. *GSLIB: Geostatistical Software Library and User's Guide*, 2nd edn. Oxford University Press, New York, 369 pp.

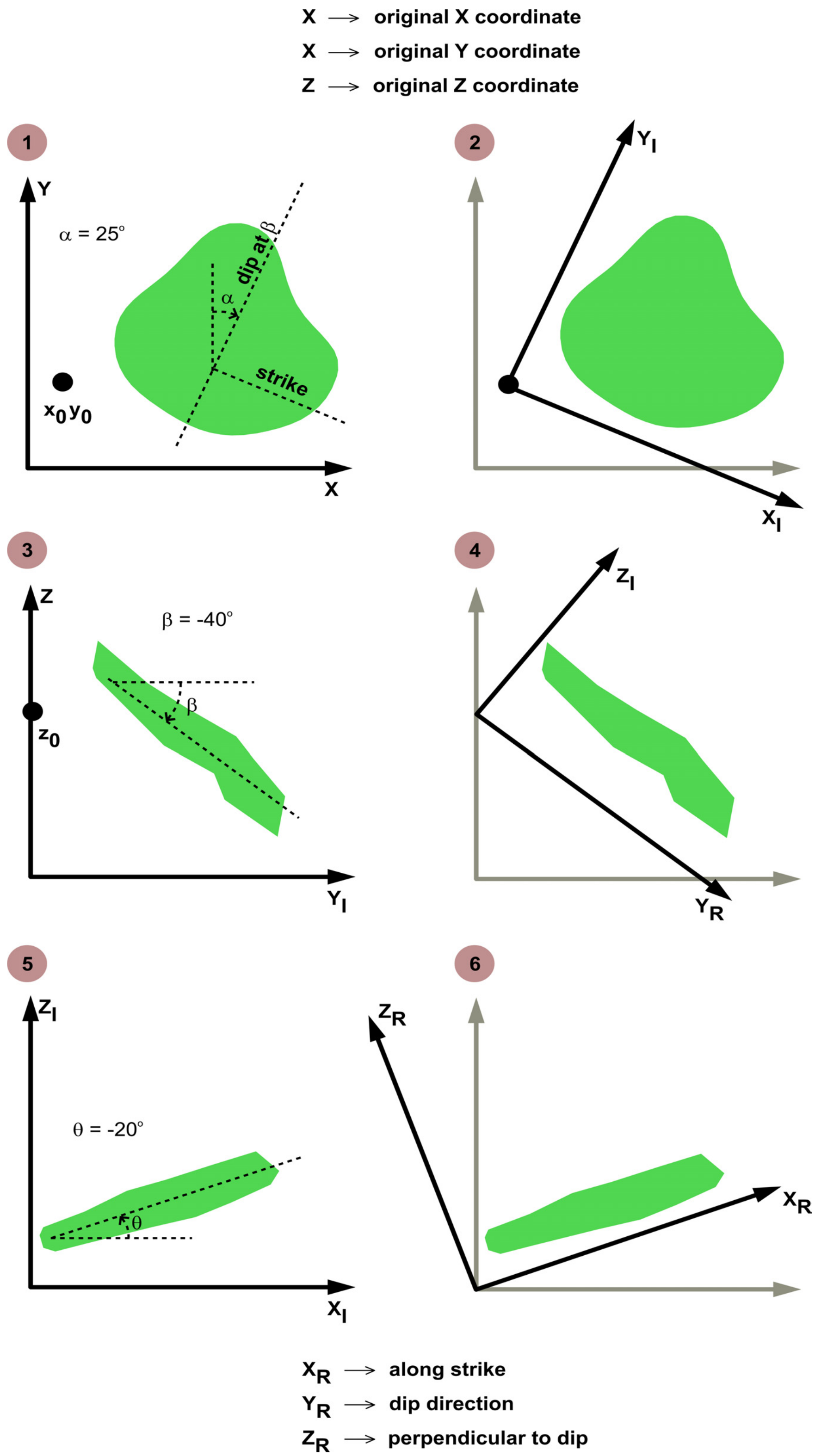

Figure 3: Example rotated coordinate system.