Centre for Computational Geostatistics (CCG)
Guidebook Series Vol. 1

Guide to Geostatistical Grade Control and Dig Limit Determination

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Centre for Computational Geostatistics (CCG)
Guidebook Series

Volume 1. Guide to Geostatistical Grade Control and Dig Limit Determination

Volume 2. Guide to Sampling

Volume 3. Guide to SAGD (Steam Assisted Gravity Drainage) Reservoir Characterization Using Geostatistics

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Chapter 1

Introduction

Although geostatistics originated in the mining industry, most of the recent theoretical and practical advances in geostatistics have been made in the petroleum industry. This was due to significant funding made available to research and development in this area. This funding was provided because it was recognized that heterogeneity and uncertainty are important considerations in reservoir development. The last few years have seen renewed interest in the use of advanced geostatistical simulation and optimization-based tools in the mining industry. This guidebook addresses the application of some of the latest geostatistical techniques to the important problem of short-term planning and grade control.

The focus of this guidebook is twofold: (1) illustrate the geostatistical modelling steps required to quantify the spatial distribution of grades from sample data and (2) transfer of uncertainty in grade to expected profit for the purpose of selecting optimal dig limits. The focus is not on geological mapping, sampling practices, the use of dedicated grade-control sampling, or the link to advanced positioning of equipment and scheduling software.

We take a pragmatic approach to the application of geostatistics for grade control. Each geostatistical study requires a certain degree of interpretation, customization and iteration for a robust solution. This manual offers a minimal guide to the complexity of geostatistical modelling. The reader is referred to the numerous papers and books available on the subject of geostatistics and geological modelling.

Chapters 2 through 5 of this guidebook correspond to the four main steps required for geostatistical modelling and grade control. Figure 1.1 illustrates the basic steps that this guidebook will go through: data assembly, geostatistical modelling, economic calculation and dig limit determination. Chapter 1 presents an overview of the manual and illustrates the practice of grade control with a preliminary example. Chapter 2

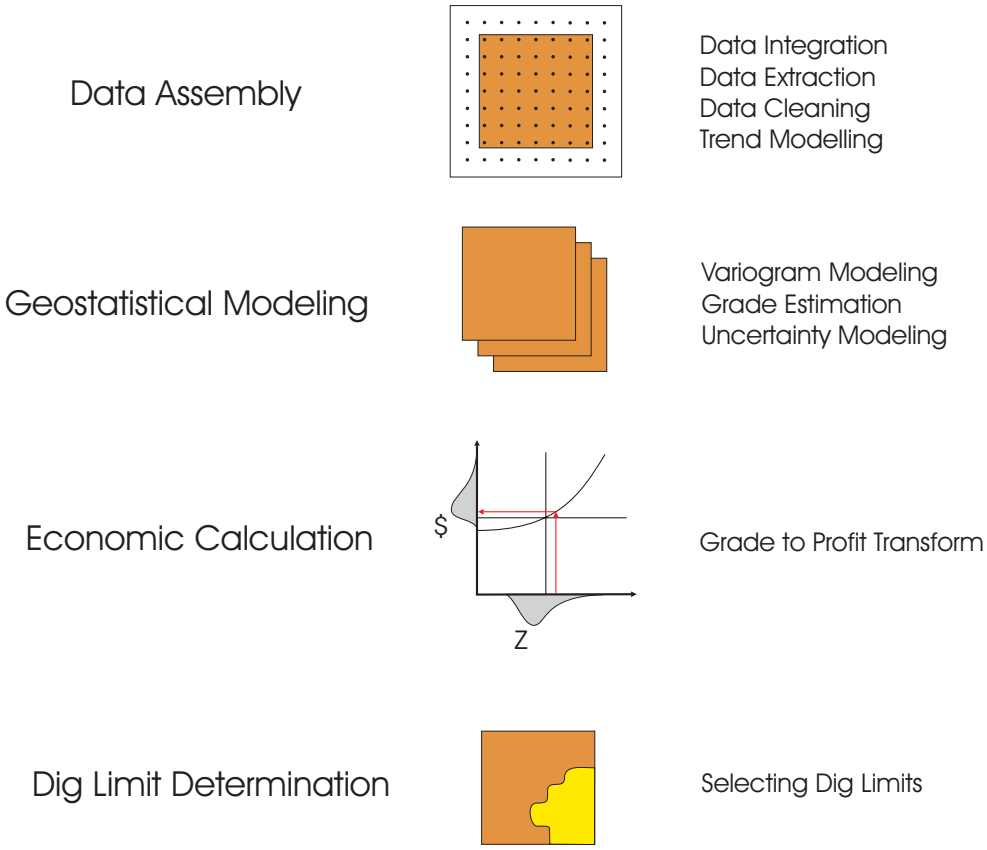

Figure 1.1: Schematic illustration of the steps explained in this guidebook for grade control and dig limit determination.

discusses data handling, grid specification, extraction of representative data and the use of rock type models. Chapter 3 tackles geostatistical modelling for grade control. Chapter 4 covers the transfer of uncertainty in grade to uncertainty in profit and the expected profit estimate for grade control. Chapter 5 describes the determination of dig limits that maximize potential profit and account for the limitation of the mining equipment. Chapter 6 presents a number of case studies and implementation details. Programs are documented in Appendix A.

1.1 Introductory Example

We present the basic steps of grade control in the form of an introductory example. The example does not discuss details of the steps as these are covered in greater detail throughout this guidebook. The details of the deposit such as the mineral type and the geologic setting of the example are not particularly important; only the steps are important.

Figure 1.2 shows a plan view of the blasthole samples with the available data. Figure 1.3 shows the model of uncertainty that consists of 100 realizations. Note that the large-scale features present in the location map are reproduced in the simulation.

The model of grade uncertainty is transformed to a model of profit uncertainty. The e-type estimate for the grade and the expected profit map are shown in Figure 1.4. The ore/waste classification map based on expected profit is shown in Figure 1.5.

The final step is to draw the optimal dig limits using the expected profit map. Optimal dig limits are shown in Figures 1.6 and 1.7 as a smooth solid line.

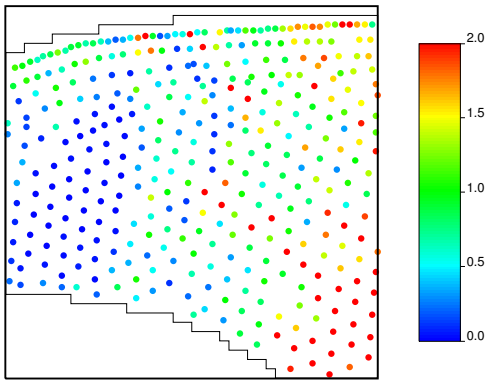

Figure 1.2: Blasthole sample locations.

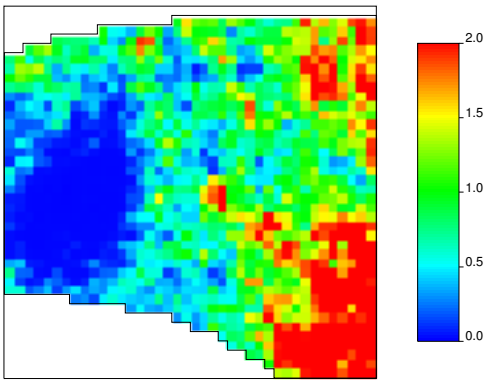

Figure 1.3: One realization from the simulation.

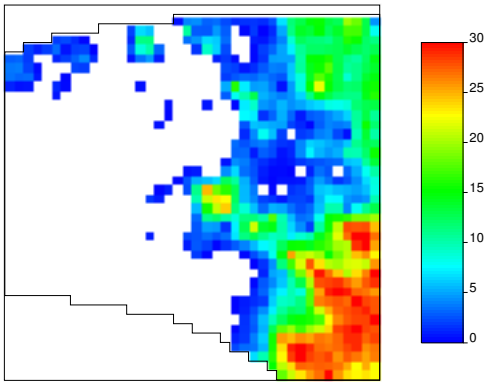

Figure 1.4: Expected profit map calculated using the simulation results.

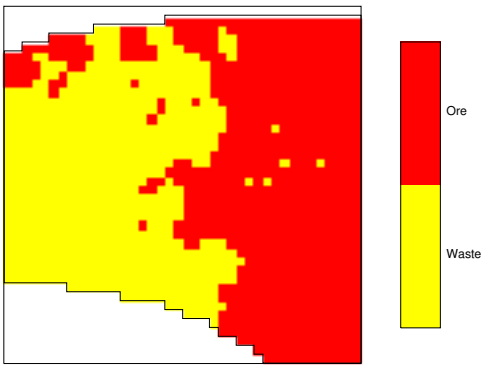

Figure 1.5: Ore/waste material classification based on the expected profit.

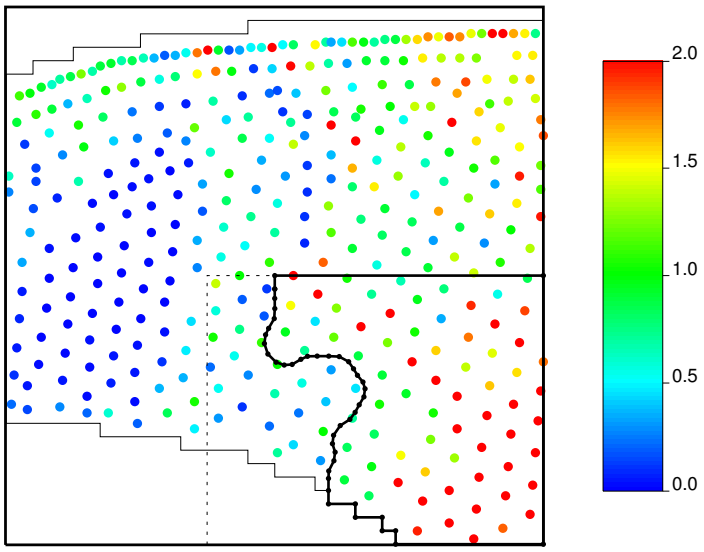

Figure 1.6: Optimal dig limits overlaid on the blasthole sample locations.

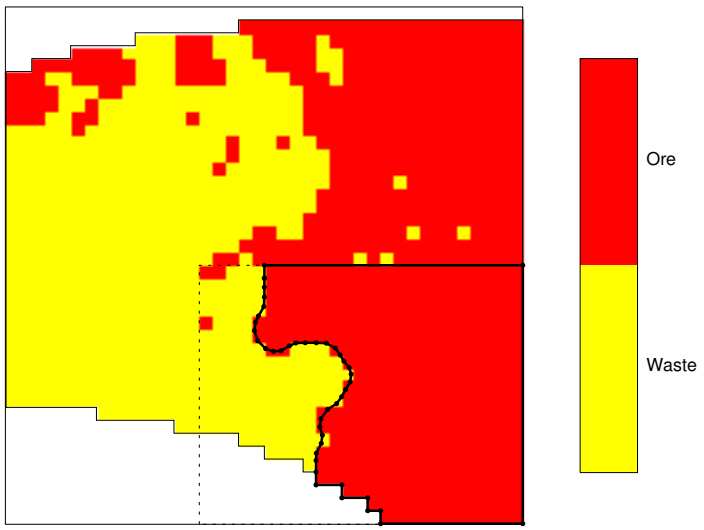

Figure 1.7: Optimal dig limits overlaid on the material classification map.

Chapter 2

Data Assembly

Inputs:

1. Area of interest.
2. Geological interpretation.
3. Raw data.
4. Trend model.

Outputs:

1. Clean data.
2. Clustered statistics.
3. Declustered statistics.
4. Residuals if the trend was modelled.

Clean data with quantified and acceptable sampling error is essential for reliable grade control. We assume that due care has been taken to review the sampling protocol and create a reliable database. The grade control program requires that data relevant to the area of interest be extracted from the database. A procedure for selecting the relevant data to be used for short term planning is discussed. Estimation should proceed using only relevant data and on a region that is judged as being stationary. Techniques for identifying outliers and pooling and splitting data are discussed. Declustering is used to account for the effect of preferential sampling. Most natural phenomena exhibit trends. Trend modelling injects deterministic knowledge into the model of uncertainty; furthermore, a trend model represents a non-stationary random function and permits stationarity techniques to be applied.

2.1 Area of Interest

The area of interest should be large enough for reliable histogram and variogram calculation. Data outside of the area of interest could be used, but caution must be exercised to ensure that only relevant data are used. Too small of an area may compromise reliable statistics, the geostatistical models, and the resulting dig limits may not be robust. Considering too large an area may compromise local accuracy by mixing disparate rock types together. A data set of 200 blastholes is considered sufficiently large.

Blasthole samples from the current bench and from the bench above should be used. Nearby exploration holes and dedicated grade-control drillholes should also be considered if available. The areal limits for data extraction should be expanded beyond the area of interest by 20–30 m to ensure that nearby data are considered. The volume support difference between the available data is of some concern; however, the difference between any of the drillhole samples and an effective SMU scale is significantly larger than the volume support difference between the available data. Pooling reliable data of different volume supports is deemed acceptable in practice when the differences are minor. A local database is thus assembled from the larger blasthole and exploration databases. Standard statistical tools should be used to display and review the available data. The data should be posted on the bench being considered and on the bench above. Figure 2.1 shows a schematic illustration of the data assembled for the area of interest.

2.2 Pooling and Splitting Data

Geostatistical modelling relies on a decision of stationarity. A decision of stationarity is not a property of the data; rather it is a choice to pool data that are deemed to belong together. The decision of stationarity permits the application of inferred statistical models such as the histogram and variogram over the area of interest. An area of interest is considered non-stationary when there are trends present or when the area of interest has different features, different geologic controls, or rock types. In the presence of these features it might be necessary to reduce the area of interest into several sub-areas. Geological maps can be used to identify areas with similar spatial and statistical features and prevent the pooling of inappropriate data. The area should not be separated into rock types when the geologic controls are not easily mapped or if they are at a suitably large scale in the pit.

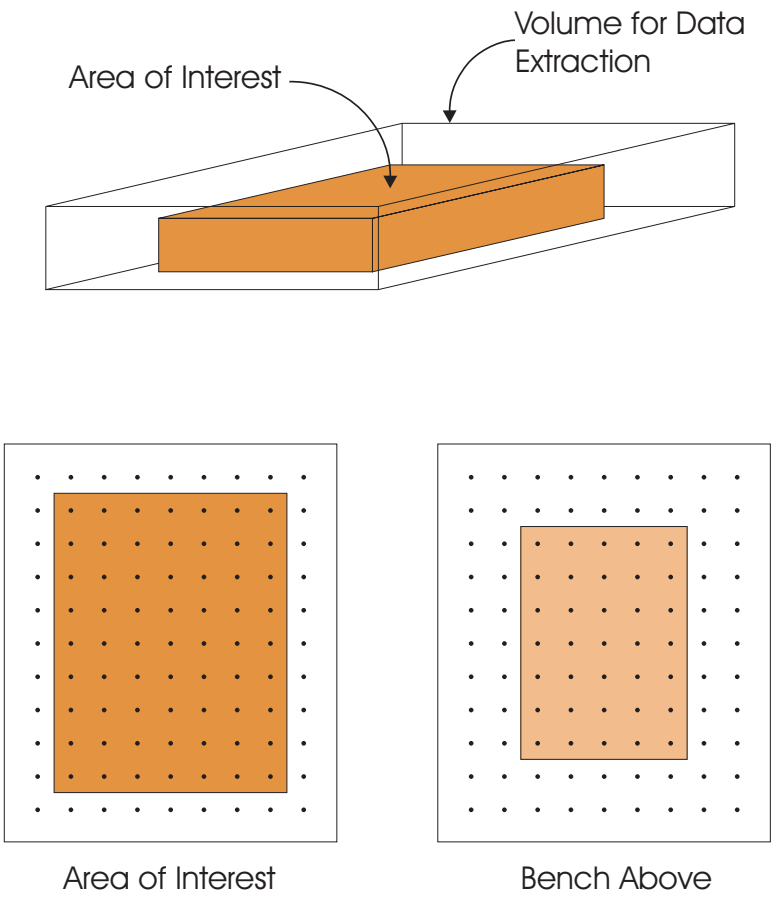

Figure 2.1: Illustration of the area of interest and the volume used for data extraction. Two plan maps are created: one for the bench being considered and one for the bench above if available. The data are considered to be within homogeneous rock types.

In the absence of a map of geology one can quickly identify potential problems by observing the histogram and looking for multiple modes or peaks. Creating a probability plot of the data and looking for kinks or breaks in the distribution is another method. Mapping using a threshold defined by a kink in a probability plot, or multiple modes may highlight areas of concern. The data plotted with different colours or symbols (above and below a threshold) might be insightful.

2.3 Outliers

Erroneous data and outliers can affect the summary statistics and subsequently the model. Data that appear unusual should be investigated and, if necessary, removed from the database. Careful examination of the data will help in identifying outliers and erroneous data; not all peculiar data are erroneous or outliers. However, it may

be necessary to remove or modify specific data. The following steps are one strategy for examining data:

- Construct a histogram of the data and focus on the extreme values.
- Calculate summary statistics with and without the extreme values.
- Construct a probability plot and examine the extreme values. Extreme values are of concern.
- Look at the probability plot for extreme values. Extreme values are of concern.
- Examine the spatial location of the suspect data. Anomalously high or low values in low or high regions respectively are of concern.
- Construct a cross plot of the local averages versus the data. Large departures from the local mean might be indicative of an outlier.

2.4 Declustering

Data are often collected preferentially from areas that are high or low in value that leads to biased data that is not volume representative. Conventional simulation routines aim to reproduce the histogram. If bias is not accounted for, the model of uncertainty will reproduce the bias. One way to account for the effect of preferential sampling is to decluster the data by weighting it according to the volume that the data represent. The most popular method is called cell declustering (the `declus` program in GSLIB). Declustering may not always be required: the regular spacing of blastholes often makes this step unnecessary.

2.5 Trend Modelling

Many variables exhibit predictable geological trends over large distances. This would normally entail some form explicit trend modelling; however, the relatively small area of interest in grade control usually makes trend modelling unnecessary. As mentioned above, deterministic rock types provide hard boundaries that must be modelled directly. The rock type model can be built using field observations, exploration data, or mapping.

The idea behind trend modelling is to define a reference datum or trend from which the data deviate in a non-systematic way, subtract the data from the trend

and proceed by modelling the residuals. A trend may be areal, vertical or both. One way to identify a trend is to draw a contour map of the data. Large-scale trends are of concern. If a trend is not obvious, or when the presence of a trend is doubted, do not attempt to model a trend. Do not over-fit a trend model; over-fitting a trend can impose too many deterministic features in the model of uncertainty. A trend may be fit as a locally varying mean or as a polynomial function. In practice an areal trend is fit independently from a vertical trend and the two are merged as a single 3-D trend model.

$$m(x, y, z) = m(z) \cdot \frac{m(x, y)}{\overline{m}} \tag{2.1}$$

Where $m(x, y, z)$ is the 3-D trend model required, $m(z)$ is the vertical trend for the location, $m(x, y)$ is the areal trend for the location, and $\overline{m}$ is the global average.

Chapter 3

Geostatistical Modelling

Inputs:

1. Clean declustered grades (or residuals plus a trend model).
2. Interpreted geological block model.

Outputs:

1. Normal scores data.
2. Normal scores variogram.
3. Kriged map.
4. Simulation results (model of uncertainty).

Geostatistical modelling requires quantitative measures of spatial correlation for estimation and simulation. The most commonly used tool for measuring spatial correlation is the variogram. Kriging is used for mapping best estimates but due to the limitations of smoothing and the lack of a joint uncertainty model, it is not recommended for decision making. Simulation overcomes the limitations of kriging and is more appropriate for decision making. Standard GSLIB notation will be used throughout this guidebook, see Appendix A.

3.1 Normal Scores Transform

Sequential Gaussian simulation is used to construct a model of uncertainty in grade. The use of Gaussian simulation requires that all data variables be transformed to a standard normal distribution prior to variogram calculation and simulation. The

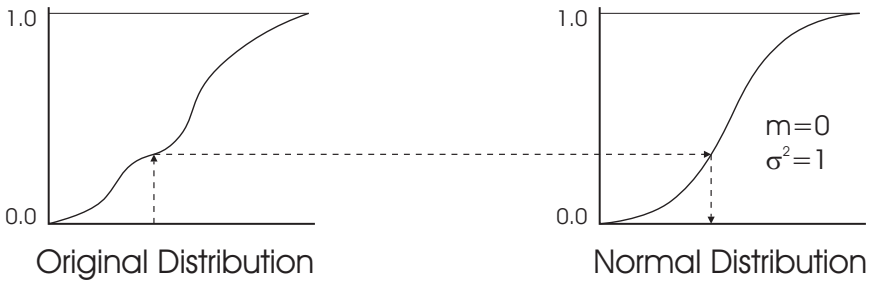

Figure 3.1: A distribution is transformed into a normal distribution evaluating the quantiles of the original distribution as the corresponding Gaussian values.

transform is shown schematically in Figure 3.1. This is accomplished by the normal scores transformation (the `nscore` program in GSLIB). There are two advantages/reasons for the transform: (1) we need to know the shape of all of the conditional distributions for simulation and (2) the variance of the transformed data ,and therefore the sill of the variogram, is equal to 1.0. Transforming the data to normal scores has the added affect of dampening outliers; variograms are less noisy because extreme values do not affect the variogram calculation as significantly.

3.2 Variogram modelling

The spatial variability of a variable is modelled by the variogram. There are three steps in variogram modelling: (1) calculation of the experimental data (the `gamv` program in GSLIB), (2) construction of a model variogram based on the experimental variogram (the `vmodel` and `varfit` programs in GSLIB) and (3) checking variogram reproduction using the model of uncertainty constructed from the modelled variogram (the `gam` program in GSLIB).

Experimental variogram calculation is done after the data have been inspected and the geological rock types and large-scale trends are accounted for. The simulation is done in Gaussian, or normal, space so the variograms must be calculated and modelled using the normal score data values that have been transformed without the declustering weights. Figures 3.2 and 3.3 show some of the key concepts related to variogram calculation: establishing a lag template to pool points for variogram calculation and scanning that template over all data points for variogram calculation. Blastholes are regularly spaced, which makes variogram calculation straightforward. It is not possible to measure spatial variability for distances less than the blasthole spacing. Determining the nugget effect, which is very important to the model, can be done using information from pre-split blastholes, exploration holes with multiple

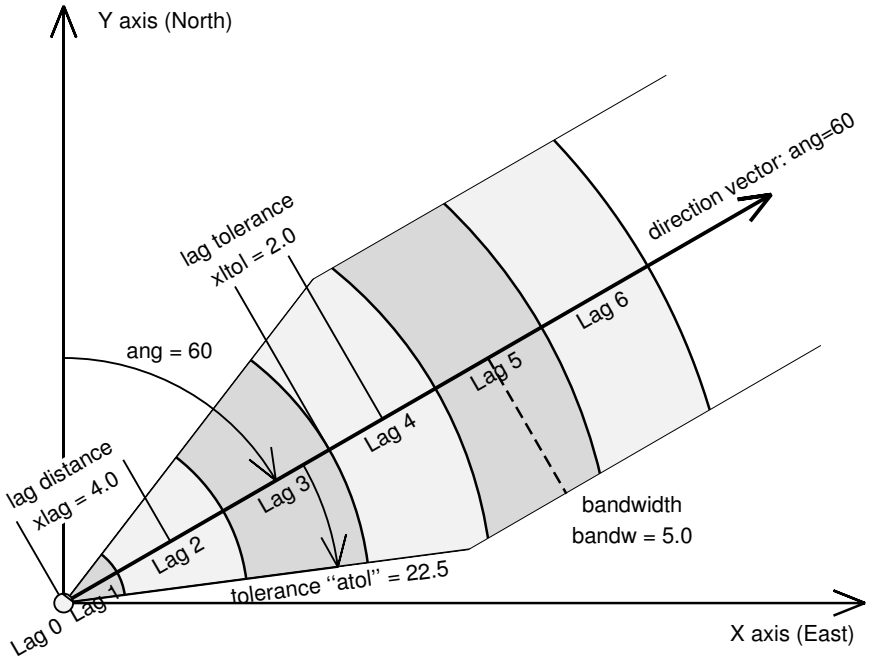

Figure 3.2: Illustration of a template used to pool data into lags for variogram calculation.

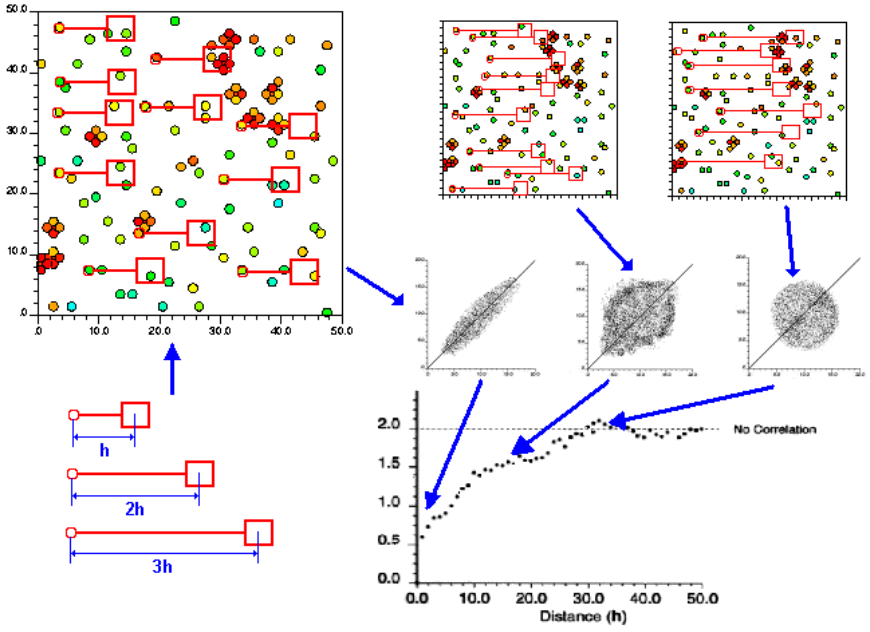

Figure 3.3: Illustration of variogram calculation for three different lag vectors. The correlation often decreases with distance and the variogram value increases.

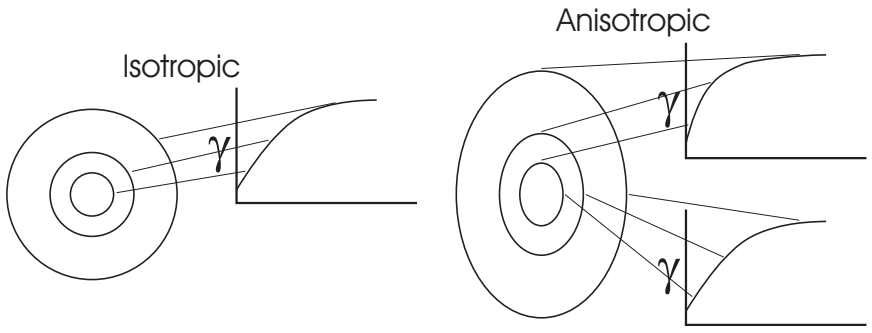

Figure 3.4: Isotropic and anisotropic contour lines and their corresponding variograms.

samples per hole, extrapolation of the variogram function or other relevant geological information.

Spatial phenomenon often exhibit anisotropic behaviour with different spatial correlations in different directions. Figure 3.4 shows contour lines for isotropic and anisotropic variables and the corresponding variogram models. Note that the isotropic variable requires only one variogram model; this is an omnidirectional variogram. Where as the anisotropic variable requires a different variogram model for each of the principle directions.

An omnidirectional experimental variogram calculates spatial variability without respect to direction. The angle tolerance is 90° and the bandwidth is larger than the area of interest. Because of the angle tolerance, the azimuth is not relevant. The lag tolerance should be approximately equal to the data spacing. Experimental omnidirectional variograms are useful for characterizing the spatial variability of isotropic phenomenon or for obtaining initial parameters for calculating the directional anisotropic variograms. Experimental directional variograms attempt to capture the anisotropic features present in the spatial phenomenon. The idea is to calculate an experimental variogram in the directions of maximal and minimal spatial continuity when 2-D spatial characterization is required and an additional experimental variogram perpendicular to the plane of maximal and minimal spatial continuity when 3-D characterization is required. A good tool for establishing the direction of spatial continuity is the location map or a contoured map of grades. In recognition of the anisotropic features of the spatial phenomenon the azimuths of the maximal and minimal directions must be specified and the lag tolerance and bandwidth are smaller to correspond to the extent of the anisotropic features: if the directional spatial variability is strong the angle tolerance and bandwidth will be very small with the opposite being true if there is only moderate anisotropic behaviour. Experimental variogram calculation is an exercise that requires iteration of parameters to achieve

good results. The experimental variograms will most likely show characteristics directly related to the geology present such as: geometric anisotropy, zonal anisotropy (stratigraphic and/or areal correlation), cyclicity and trends. These features must be understood and linked back to variations observed on the posted blasthole grades.

The experimental variogram provides information only for the specified angles and distances. The experimental variogram must be fit with a model variogram that provides a means of calculating spatial variability for any angle and distance. The model variogram is constructed as the sum of known positive definite licit variogram functions and may consist of any combination of functions. Positive definiteness guarantees a unique solution to the kriging equations that will be used in kriging and simulation. Directional experimental variograms require a model variogram for each direction. The directional variograms must not be modelled independently: there can only be an alteration of range between the angle of maximal and minimal continuity; a fundamental change in the pattern of spatial continuity does not make physical sense. The modelled directional variogram must consist of the same variogram model types with the same variance contributions in each direction, the range of correlation may be different in each direction though. Most of the spatial features can be captured in the variogram modelling; however, the sill of the stationary normal scores variograms should be locked in to the stationary variance of 1.0; otherwise, the histogram of the simulated values may not match that of the data.

3.3 Kriging

Kriging is a well-established method for estimation. The estimates are calculated as a weighted combination of the data. The weights are chosen to minimize error variance. There are some limitations to kriging. The first major consideration is that the estimates are smoother than the underlying true grades and this smoothness depends on the available data spacing. Secondly, the measure of uncertainty provided by the kriging variance does not account for data values; only the configuration of the data is accounted for. For these reasons kriging should not be used for grade control but as a confirmation tool. Kriged maps provide insight into trends. Kriging does not reproduce the histogram and does not usually assign weight to the global distribution, therefore declustering weights are not used.

3.4 Simulation

Simulation techniques overcome many of the limitations of kriging. Multiple realizations of the mineral grades are constructed to capture joint uncertainty. This uncertainty can then be transferred through economic calculations and the determination of optimal dig limits. Figures 3.5 to Figures 3.7 show a set of blasthole data, a kriged map, and a one realization from the simulation. A model of uncertainty consists of many such realizations. A model of uncertainty consisting of 20–100 realizations is required to avoid making decisions on the basis of a single "stochastic" feature. Simulation attempts to reproduce the histogram on average, that is, over many realizations. The histogram must be representative; therefore, declustering weights are used.

The goal of kriging and simulation are very different: kriging aims to provide a best estimate at an unsampled location that is as close as possible to the true unknown value, while simulation aims to construct a model of uncertainty in grades that represent the joint model of uncertainty and our state of incomplete knowledge. Retaining the kriging estimate and converting it to profit does not account for uncertainty in grade, profit, or the situation where mineral recovery depends on grade. We transform the distribution of uncertainty in grade to a distribution of profit using an economic function and thereby acount for uncertainty. Converting the sample grades to profit and simulating profit is not recommended. Profit depends on grade, recovery, selling price and the cost of producing the mineral. The only static geologic variable is grade. If the other variables change, the steps for uncertainty modelling must be repeated including the transform from grade to profit, variogram calculation and simulation.

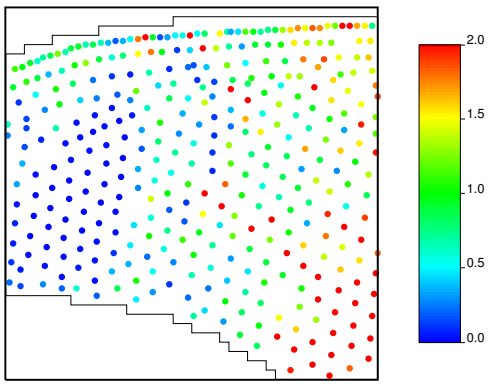

Figure 3.5: Blasthole sample locations.

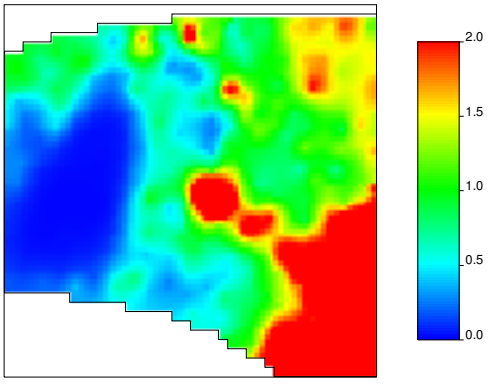

Figure 3.6: Ordinary kriging grades.

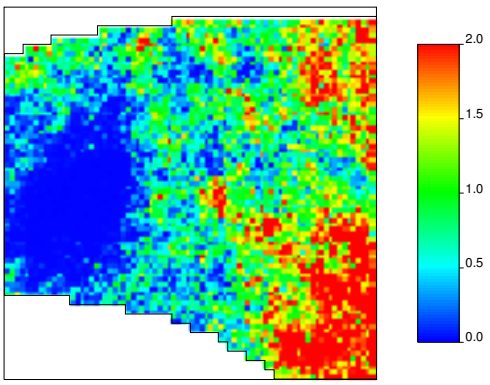

Figure 3.7: One realization from the simulation.

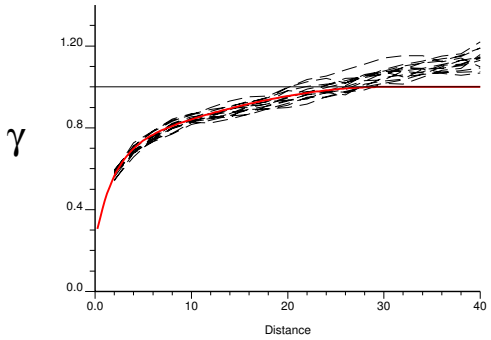

Figure 3.8: Good variogram reproduction.

We recommend that both kriging and simulation be performed. Geostatistical models of uncertainty can be difficult to visualize and interpret. The kriged map provides insight to the model of uncertainty because the average of many simulated realizations should be close to the kriged map. The non-linear transform to and from Gaussian space causes some differences. In general, the kriged map and various cross-validation tools can be used to check for gross errors in the modelling parameters such as correct orientation of coordinates and variogram modelling. Plan views of the data and simulated values should be checked for consistency.

Some important features to check include: (1) realizations should honour the data (high grade areas simulated around the high grade samples and so on), (2) the rock type interfaces should be honoured as should any "hard or soft" boundaries, and (3) the direction of continuity should be consistent with the geology. This should be done for all of the modelled variables.

The histogram of the simulated variable should be similar to the sampled histogram but with acceptable statistical fluctuations. The histogram of a single cell (block) will show the uncertainty and distribution for the variable at that point.

The variograms of the simulated realizations, should, on average, reproduce the modelled variogram. The variogram calculated from a single realization of gridded simulated values may not honour the modelled variogram due to statistical fluctuations. Figure 3.8 shows good variogram reproduction where the red line is the model and the dashed black lines are the variograms calculated from the simulation results.

It is recommended to construct the model of uncertainty at a fine scale, 1/2 to 1/3 the blasthole spacing, and then block average the simulation up to the SMU size. This accounts for volume variance that the simulation algorithms do not account for.

Chapter 4

Expected Profit Conversion

Inputs:

1. Simulation results (model of uncertainty).
2. Operating costs or a cutoff grade.
3. Selling price for each mineral.
4. Processing recovery curve for each mineral.

Outputs:

1. Model of the total expected profit.
2. Model of the expected grade(s).
3. Model of the ore, marginal ore, and waste classification.

One goal of a mining company is to maximize profit. However, there is unavoidable uncertainty in the spatial distributions of the variables that contribute to the classification of material as ore, marginal ore, or waste.

Traditionally, mines use a cutoff grade or a probability of ore threshold for their grade control. These are simple and easy to use systems but only decisions based on profit can maximize profit. A conversion from the uncertainty in grade to an uncertainty in profit is required to facilitate a grade control program using profit. The methods presented in this Chapter are guidelines only. Each operation must ensure that the conversion they use is appropriate for local conditions.

4.1 Mining for Profit

Grade control programs aim to minimize the misclassification of material. Typically grade control is done using information obtained from blasthole drilling or dedicated grade control drilling; however, there is always uncertainty in the grade information as consequence of the spacing between the data. Uncertainty in the grade makes grade control difficult. Figure 4.1 shows distributions for three different models of uncertainty. The material on the left is clearly waste, the material on the right is clearly ore, but the material in the middle near the cutoff grade, z_c, is difficult to classify. The practice of classifying material as ore or waste using a cutoff grade would be appropriate if the grades were known with certainty, but in practice this is not the case. A profit based classification scheme is the preferred method for grade control.

Material should not be classified using a "probability of ore" threshold. Figure 4.2 shows two models of uncertainty that have the same probability of being above the cutoff grade. and being classified as ore. The material characterized by the distribution on the right should be classified as ore because of the potential to make a large profit, whereas the material on the left should be classified as waste; even though they both have the same probability of being above the cutoff grade.

Material can be misclassified when a cutoff grade is used for the classification

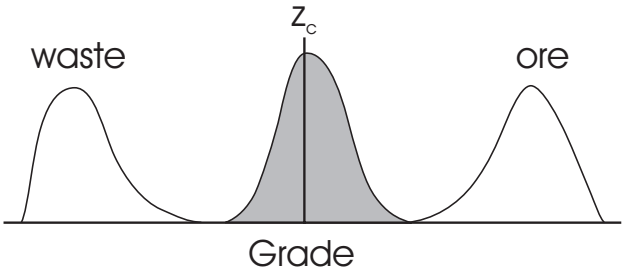

Figure 4.1: Three different distributions of uncertainty: on the left is waste, on the right is ore, and in the middle is material that is not as easy to classify.

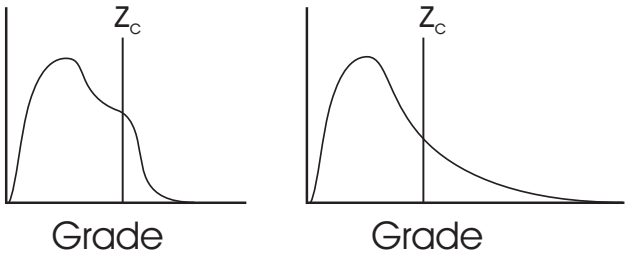

Figure 4.2: Distributions where the probability of exceeding cutoff is equal. However, the potential to make a large profit is higher for the distribution on the right.

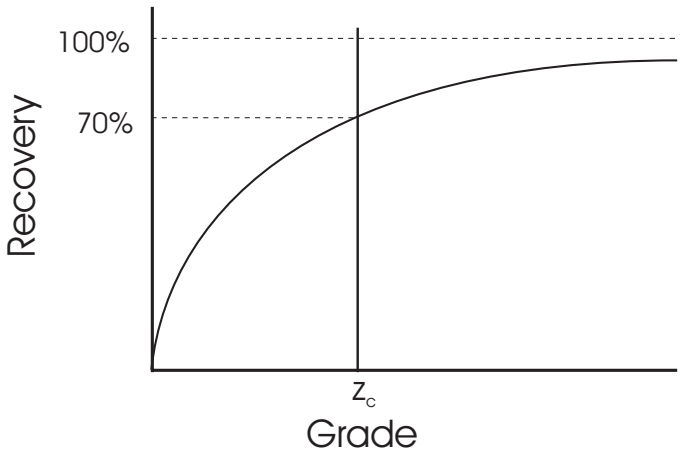

Figure 4.3: A typical recovery curve.

Table 4.1: Cutoff grade versus expected profit classification.

Realization	Grade (%)	Recovery (%)	Price ($)	Profit ($)
1	0.80	0.70	1500	0.00
2	0.60	0.59	1500	-306.65
3	0.71	0.66	1500	-140.97
4	0.67	0.63	1500	-202.24
5	1.30	0.93	1500	973.50
6	0.59	0.59	1500	-321.22
7	1.20	0.90	1500	780.00
8	0.59	0.59	1500	-321.22
9	0.60	0.59	1500	-306.65
10	0.75	0.68	1500	-78.78
Average	0.78			7.58

process. There is no economic criteria used in classifying the material as ore or waste during the decision making process.

Using the expected grade from simulation does not account for the conversion of grade to profit. For example, the material can have a grade less than the cutoff but when the economics are considered the material may be profitable. Consider an example scenario for a copper mine: 10 realizations of the grade for one tonne of material, a cutoff grade = 0.8%, a selling price of $0.75 per pound of copper, and the hypothetical recovery curve shown in Figure 4.3. Table 4.1 shows the results of the classification procedure.

The results show that the expected grade is less than the cutoff grade, but the expected profit is greater than zero. This is because of the small, yet significant, probability that the grade of the block is high ($\geq$1.2%). Therefore, the correct classification of the material is ore, not waste. Counter examples could also be shown. The

best decision depends on the full distribution of uncertainty in the grade and all of the economic parameters that affect the profitability of the material. Desired metal minerals increase the profit, while contaminants decrease it.

Two different methods for calculating the expected profit will be presented. The first is a "cutoff grade" based method and the second is "cost" based. The cutoff based method is quick, easy, and works well with single mineral deposits. If more information is required from the classification, or when in the presence of multiple minerals, the cost based approach is advocated. Using the cost based classification marginal can be identified. In general, each mine will have their own profit calculation routine accounting for local conditions.

4.2 Profit Conversion Using a Cutoff Grade

Classification should account for the risk of misclassifying the material. Transforming the distribution of uncertainty in grade to a distribution of uncertainty in profit accounts for this risk. The cutoff based grade to profit conversion requires four parameters:

1. Grade information ($z^l(\mathbf{u}),\ l = 1, \ldots, L,\ \mathbf{u} \in A$): the grades at the unsampled locations represented by a model of uncertainty.

2. Cutoff grade (z_c): in general the cutoff grade is defined as the grade at which the cost/profit is equal for the mine to process the material as ore or mine it as waste. Market conditions, the plant recovery, mining costs, administration costs, and contract obligations dictate the cutoff grade. It is generally specified by management.

3. Recovery curve ($r(z)$): in most mines the recovery varies with grade. Often, the recovery increases with increasing grade.

4. Selling price ($price$): the selling price per unit of mineral or metal produced.

5. Cost of processing waste ratio ($cpwr$): a ratio for the cost of processing material that has zero grade. In most cases the cost of processing waste is equivalent to the product of the cutoff grade, the recovery at the cutoff grade, and the price minus the cost of mining the ore plus the cost of mining the waste:

$$cpw = z_c \cdot r(z_c) \cdot price - c_o + c_w \tag{4.1}$$

In special cases the cost of processing waste is different. There may be a greater cost associated with classifying low grade material as ore for different types of processing facilities. For example, a heap leach operation would have a much lower cost of classifying low grade material as ore versus a grinding and flotation plant. We can account for these special cases by scaling the calculated profit when $z < z_c$:

$$cpwr = \frac{cpw}{z_c \cdot r(z_c) \cdot price} \tag{4.2}$$

As mentioned above, the profit calculation is very important and very mine-specific. This should be considered carefully and updated often.

Using the above parameters the grade to profit conversion for realization l at location $\mathbf{u}$ is calculated as:

$$Profit^l(\mathbf{u}) = \begin{cases} \left(z^l(\mathbf{u}) \cdot r\left(z^l(\mathbf{u})\right) - z_c \cdot r(z_c)\right) \cdot price, & \text{if } z^l(\mathbf{u}) \geq z_c \\ \left(z^l(\mathbf{u}) \cdot r\left(z^l(\mathbf{u})\right) - z_c \cdot r(z_c)\right) \cdot price \cdot cpwr, & \text{if } z^l(\mathbf{u}) < z_c \end{cases} \tag{4.3}$$

The expected profit for location $\mathbf{u}$ is calculated as:

$$E\left\{Profit(\mathbf{u})\right\} = \frac{1}{L}\sum_{i=1}^{L} Profit^l(\mathbf{u}) \tag{4.4}$$

Figure 4.4 shows the grade to profit conversion. The model of uncertainty for the grade is on abscissa axis and the transformed model of uncertainty in the profit is on the ordinate axis. The conversion function is not linear, therefore material with an expected grade less than the cutoff grade may have an expected profit greater

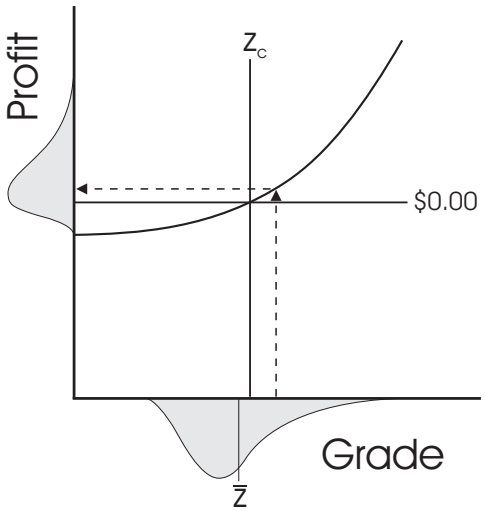

Figure 4.4: The non-linear conversion from grade to profit.

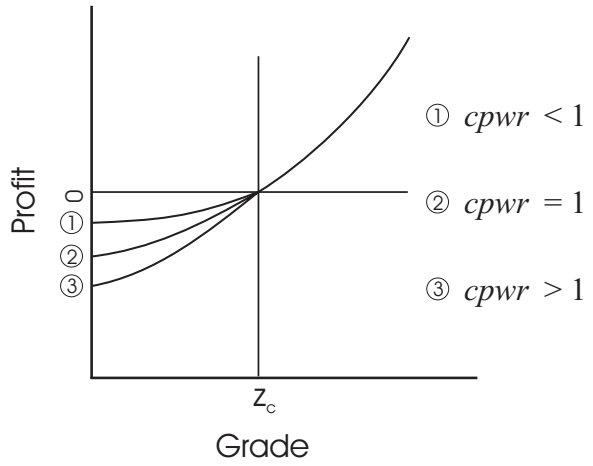

Figure 4.5: Accounting for different waste processing costs and their effect on the grade to profit conversion.

than zero and should be mined as ore. Alternatively, material with an expected grade greater than zero could have an expected profit less than zero depending on the processing method used, the distribution of uncertainty in the grade estimate, etc.

When the cost of processing waste plays a role in the expected profit transformation there are three different cases that need to be considered. The different *cpwr* ratios are:

- $cpwr < 1$: low grade blocks will have a tendency to be classified as ore, even though the cutoff grade is respected.
- $cpwr = 1$: behaves as if *cpwr* was not used.
- $cpwr > 1$: marginal blocks near the cutoff grade will have a tendency to be classified as waste.

Figure 4.5 shows the effects that the *cpwr* ratio has on the conversion function.

After the grade to profit conversion is done the material is classified as ore if the expected profit is greater than zero and waste otherwise:

$$classification = \begin{cases} ore, & \text{if } E\{Profit(\mathbf{u})\} > 0 \\ waste, & \text{otherwise} \end{cases} \tag{4.5}$$

This approach considers two aspects of accounting for costs in the classification: (1) the uncertainty in the estimate and (2) the best possible classification. Transforming the distribution of uncertainty in grade to a distribution of uncertainty in profit accounts for all potential outcomes for each block.

4.3 Profit Conversion Using Operating Costs

The cost based approach provides more information about the classification and can be used in the presence of multiple minerals. In addition to ore and waste, marginal ore can be identified. Marginal ore is material that is cheaper to put through the plant than send it to the waste dump, but it still costs the mine money. Some of the required inputs are similar to the cutoff based approach:

1. Grade information ($z^l(\mathbf{u}),\ l = 1, \ldots, L,\ \mathbf{u} \in A$): the grades at the unsampled locations represented by a model of uncertainty.
2. Milling cost (C_t): The cost per tonne of treating feed to the plant.
3. Ore mining cost (C_o): Cost of mining the ore.
4. Waste mining cost (C_t): Cost of mining the waste.
5. Recovery curve ($r(z)$): in most mines the recovery varies with grade. Often, the recovery increases with increasing grade until an optimum recovery is reached.
6. Selling price ($price$): the selling price per unit of mineral or metal produced.

Using the above parameters the grade to profit transform for realization l at location $\mathbf{u}$ is calculated as:

$$Profit^l(\mathbf{u}) = z^l(\mathbf{u}) \cdot r\left(z^l(\mathbf{u})\right) \cdot price - c_o - c_t \tag{4.6}$$

The expected profit for location $\mathbf{u}$ is calculated as:

$$E\left\{Profit(\mathbf{u})\right\} = \frac{1}{L}\sum_{i=1}^{L} Profit^l(\mathbf{u}) \tag{4.7}$$

The cost grade to profit conversion function is not linear, just like the cutoff based conversion, therefore, material with an expected grade less than the cutoff grade may have an expected profit greater than zero and should be mined as ore. Alternatively, material with an expected grade greater than zero could have an expected profit less than zero depending on the processing method used, the distribution of uncertainty in the grade estimate, etc.

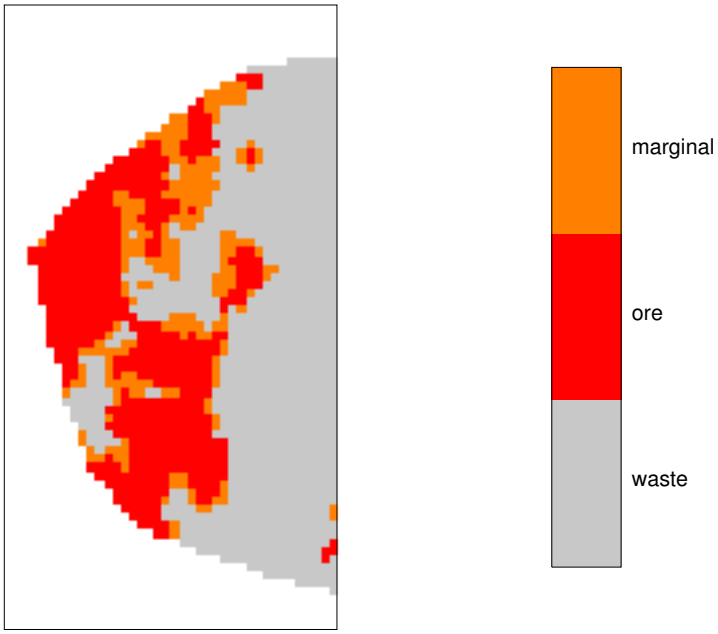

Figure 4.6: Categorical material classification map showing ore, marginal ore, and waste.

Material is classified as ore if the expected profit is greater than zero, as marginal ore if the expected profit is between zero and the cost of waste and waste otherwise:

$$classification = \begin{cases} ore, & \text{if } E\left\{Profit(\mathbf{u})\right\} > 0 \\ marginal\ ore, & \text{if } 0 \geq E\left\{Profit(\mathbf{u})\right\} \geq -c_w \\ waste, & \text{otherwise} \end{cases} \tag{4.8}$$

Marginal ore is defined as material that if mined it is cheaper to classify as ore and process even though the mine will still lose money. If marginal ore does not have to be mined it should be left in place. Figure 4.6 shows the classification results for one area using the cost based expected profit calculation. If an equivalent cutoff grade had been used to calculate the expected profit all of the material classified as marginal ore would be classified as ore with no way to decipher the marginal ore from the profitable ore.

4.4 Accounting for Multiple Minerals

In the presence of multiple mineralization the profit calculation is more complicated. There may be some minerals that contribute to profit and some that reduce profit. Moreover, different ratios of the different minerals can affect plant recoveries and the quality of the final product.

The interactions between all of the potential minerals that occur simultaneously is mine-specific. Fortunately, most operations that mine poly-metallic reserves have an understanding of these complex relationships. As mentioned previously it is up to the user to provide an appropriate grade to profit conversion function for each specific deposit.

4.5 Lost Opportunity Cost

Lost opportunity is an additional tool that can be used in the classification of material. The lost opportunity cost is defined as the cost of placing ore on the waste dump. Essentially, this is the profit that the mine would have made if the material were not sent to the dump.

Some practitioners consider the lost opportunity cost to be redundant. Nevertheless, it does cost the mine money when ore is wasted. Shown below is a methodology for using the lost opportunity cost and a cutoff grade for classifying material [*put MPS paper here*].

$$\begin{aligned} Profit^{l}_{ore}(\mathbf{u}) &= z^{l}(\mathbf{u}) \cdot r\left(z^{l}(\mathbf{u})\right) \cdot price - c_o - c_t && (4.9) \\ Profit^{l}_{waste}(\mathbf{u}) &= \begin{cases} -c_w, & \text{if } z^{l}(\mathbf{u}) \leq z_c \\ -c_w - c_{lo}(\mathbf{u}), & \text{if } z^{l}(\mathbf{u}) > z_c \end{cases} && (4.10) \\ \text{where : } c_{lo}(\mathbf{u}) &= Profit^{l}_{ore}(\mathbf{u}) \end{aligned}$$

$$\begin{aligned} E\left\{Profit(\mathbf{u})\right\} &= \frac{1}{L}\sum_{i=1}^{L} Profit^{l}(\mathbf{u}) && (4.11) \\ classification &= \begin{cases} ore, & \text{if } E\left\{Profit_{ore}(\mathbf{u})\right\} > E\left\{Profit_{waste}(\mathbf{u})\right\} \\ waste, & \text{otherwise} \end{cases} && (4.12) \end{aligned}$$

When using lost opportunity in the classification it is important to remember that it is a cost that should only be used to decide between ore and waste material. If a ore gets sent to the waste dump the mine only pays for the cost of mining the ore as waste. The lost revenue from wasting ore is never recorded.

4.6 Grade Control Using Expected Profit

The expected profit will be used for drawing dig limits. For optimization in the `diglim` program any material that has an expected profit greater that zero is treated as ore and any material below zero is considered waste. Using the cutoff based expected profit approach is simpler but not as powerful as the cost based approach.

With the cutoff based calculation any material that should be classified as ore has a positive profit and any material that is waste has a negative profit. Submitting these numbers to `diglim` will include all of the material classified as ore.

With the cost based classification there are two different sets of numbers that can be used to determine the dig limits. The first includes the marginal ore and the second one does not. This is useful when the marginal material does not have to be mined and can be left in place.

Chapter 5

Dig Limit Determination

Inputs:

1. 2-D expected profit map.
2. Initial dig limit.
3. Dig limit boundary.
4. Equipment factor or digability.

Outputs:

1. Optimal dig limit polygon.
2. Block fractions that fall in the dig limit polygon.

The classification of material as ore if the expected profit is greater than zero does not account for the limitations of the mining equipment. Dig limits that are identified by the interface between ore and waste blocks, block-wise dig limits, are almost always impossible to mine. This section discusses an automatic dig limit selection algorithm that accounts for the profitability of the material and the limitations of the mining equipment simultaneously.

5.1 Dig Limits and Simulated Annealing

The algorithm used to automatically select dig limits that simultaneously and systematically account for the profitability of blocks and the limitations of the mining equipment relies on simulated annealing. The general idea is to propose an initial dig limit, supplied by the user, and perform small random changes until the optimal

dig limits are converged upon. To find the optimal dig limits an objective function is used. The objective function measures the profit of the blocks contained within the dig limit against a penalty based on the limitations of the mining equipment. When the objective function has converged to a maximum value the optimal solution has been found.

Some of the random changes or perturbations result in an increased objective function while others do not. Simulated annealing always accepts changes that increase the objective function and sometimes accepts changes that do not. The simulated annealing schedule dictates how the perturbations that decrease the objective function are accepted or rejected. The program `diglim`, used for determining the optimal dig limits, sets the simulated annealing schedule automatically. Users that want additional control can use the manual simulated annealing schedule explained in Section 5.7.

5.2 Initial Dig Limit and Boundary

The initial dig limit must be supplied by the user. Typically a geologist or engineer in charge of the grade control will digitize the initial polygon. Figure 5.1 shows an initial dig limit. Dig limits may contain either ore or waste and the distance between the vertices is not important for the initial dig limit. New vertices are added if the distance between the vertices is too large and vertices are deleted if a segment is too short.

The dig limit boundary is used to constrain the limit to a certain section of the map. It is specified as a rectangular box and the dig limit is not allowed to cross the boundary even if it would result in a larger profit.

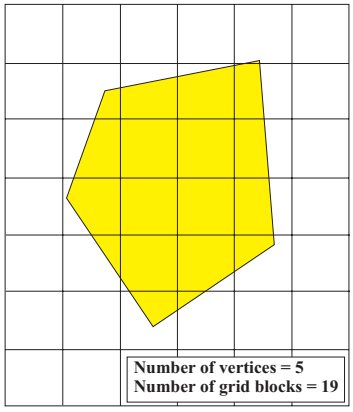

Figure 5.1: A digitized initial dig limit.

5.3 Equipment Factor or Digability

A penalty function is introduced as a method to quantify digability. One important aspect of the penalty function is calibrating it to the mining equipment. Small equipment can mine more tortuous limits than large equipment. An equipment factor or digability is used to perform this calibration. The best dig limits maximize profit by striking an optimal balance between the cost of mining and digability. Figure 5.2 shows two different dig limits that have low and high digability. Selecting a high equipment factor or digability parameter trades digability for profit because some ore is lost and some waste is included in the dig limit. Selecting low values for the digability factor yields dig limits that cost more to mine but are more selective.

An example penalty function is shown in Figure 5.3. The abscissa axis represents the angle defined by three consecutive vertices on the dig limit polygon. The sum of all these angles is the measure of digability for the dig limit polygon. A highly tortuous polygon would have many small angles resulting in a low digability, while a smooth polygon has large angles and a high digability. In the example penalty

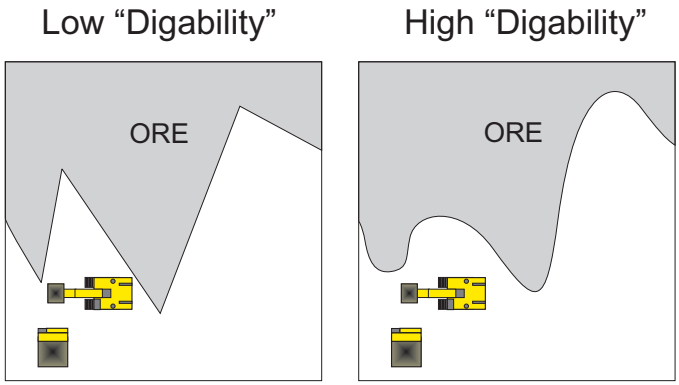

Figure 5.2: Dig limits with low digability versus dig limits with high digability.

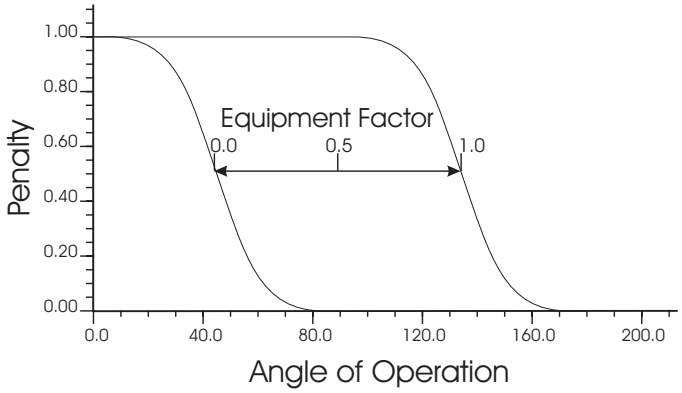

Figure 5.3: Equipment penalty curve.

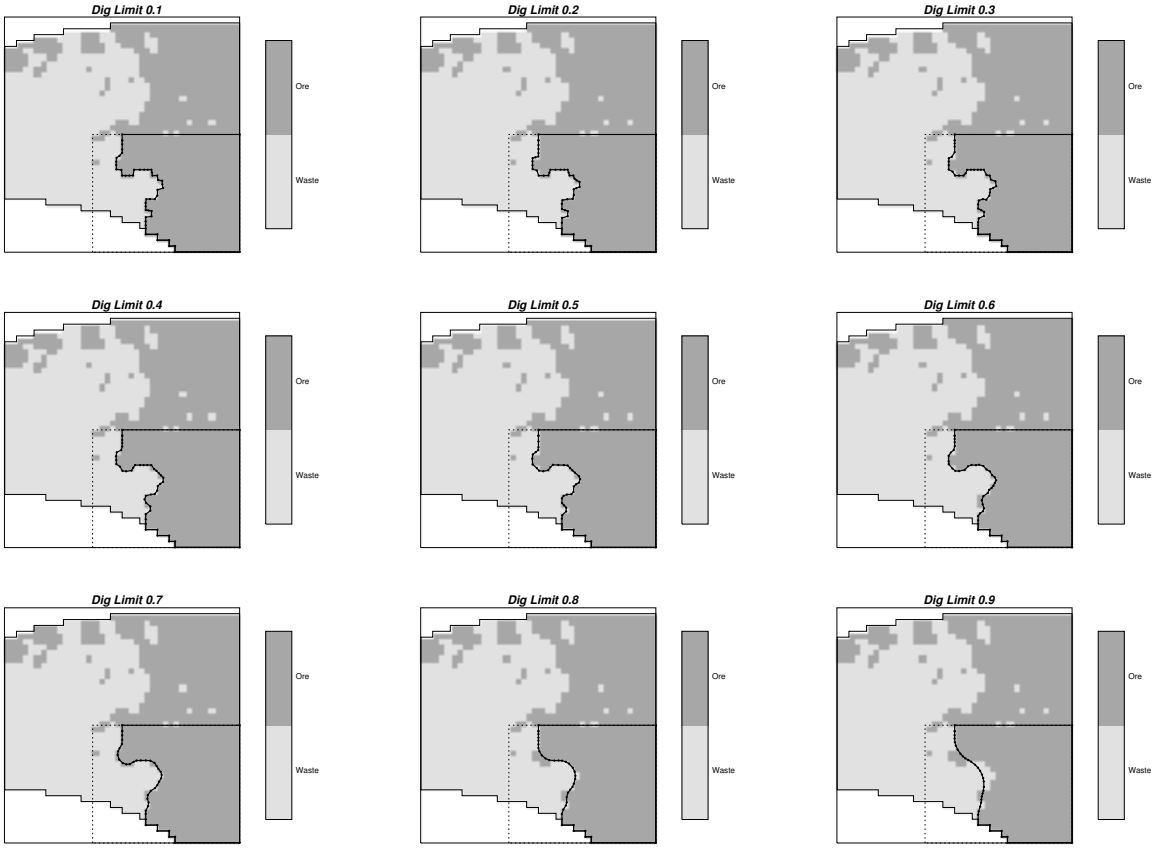

Figure 5.4: A sample catalogue from which to select an equipment or digability factor.

function angles less than 40° are penalized significantly. The ordinate axis issues a penalty, in profit, according to the measured angle for each vertex in the polygon. Low angles receive high penalties and impact the objective function more than high angles. The ordinate axis is scaled by the average profit of all the blocks within the boundary having a positive profit. The penalty function converts the digability at a vertex to a reduction in profit. The digability objective function is defined as the sum of the penalties over all the vertices.

There is no set theory on the selection of an equipment or digability factor because it represents the performance characteristics of mining equipment, the operating strategy of the mine, the skill level of the operators, and many other subjective considerations. The solution is to construct a "catalogue" of dig limits corresponding to a range of equipment factors. A mining professional can then select an appropriate equipment factor that results in a dig limit polygon that has the best balance maximum profit and digability. Figure 5.4 shows an example catalogue.

5.4 Simulated Annealing Optimization Procedure

To determine the optimal dig limit using the initial polygon a simulated annealing algorithm is used. The basis of the algorithm is to make small random changes, accept good changes, and reject most bad changes.

Objective Functions

A global objective function is required to utilize a simulated annealing optimization algorithm. Equation (5.1) is the global objective function used for the optimization.

$$O_{\text{global}} = O_{\text{profit}} - O_{\text{digability}} \tag{5.1}$$

The profit objective function is defined as the sum of all the fractional blocks contained within the dig limit, Equation (5.2).

$$O_{\text{profit}} = \sum_{ix=1}^{nx} \sum_{iy=1}^{ny} frac_{(ix,iy)} \cdot E\left\{Profit_{(ix,iy)}\right\} \tag{5.2}$$

The digability objective function is defined as the sum of the penalties over all the vertices, Equation (5.3). In order to reduce the profit objective function the penalty needs to be converted to a profit equivalent. This is done by scaling the penalty by the average profit of all the block that have a profit greater than zero.

$$O_{\text{digability}} = \sum_{iv=1}^{nv} pen_{iv} \cdot averageglobalpenalty \tag{5.3}$$

Annealing Process

During the simulated annealing process small random changes are made to the dig limit, Figure 5.5. After each change, new objective functions are calculated and the resulting change in the global objective is evaluated, Figure 5.6. If the change is positive, the change to the dig limit is accepted. If the change is negative, the change to the dig limit may or may not be accepted.

At the start of the annealing process the temperature will be high and most of the bad changes will be accepted. As the temperature cools fewer bad changes are accepted and the dig limit will converge to an optimal solution. The time that it takes for the dig limit to reach optimum is primarily a function of the number of nodes in the dig limit.

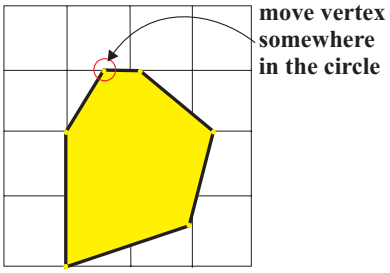

Figure 5.5: Make a random change to one of the vertices in the dig limit.

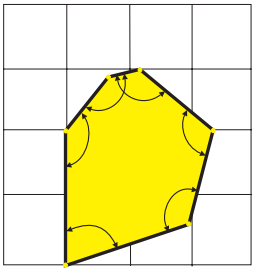

Figure 5.6: Calculate new profit and digability objective functions on the changed dig limit.

5.5 Handling Multiple Polygons

The dig limit algorithm can solve for a single ore or waste dig limit at a time. Many benches require multiple ore and waste dig limits. This is not a limitation though; multiple dig limits can be drawn using a decomposition approach. In the decomposition approach dig limits are drawn individually and the composite of individual dig limits represent the multiple dig limits required for the entire bench.

Figure 5.7 shows a map of a waste pod inside of a larger ore body and the decomposition approach for drawing multiple dig limits. The first step shown is to determine the optimal dig limit for the large ore pocket. After this is done the dig limit for the small pod of waste must be determined. Note that the steps are interchangeable. The total profit of the bench is defined as the sum of profit over all polygons.

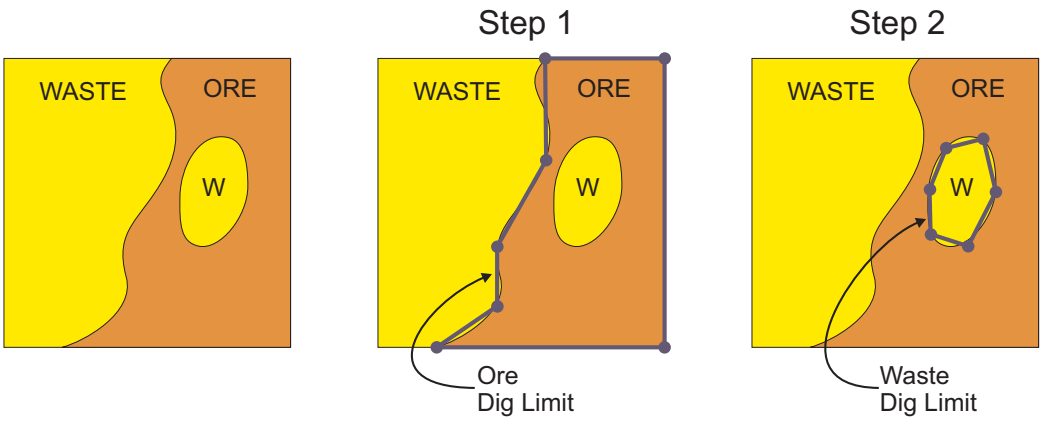

Figure 5.7: The decomposition approach breaks the dig limit problem into individual dig limit polygons. The assemblage of the polygons classifies all of the material on the bench.

5.6 Including or Excluding Marginal Ore

When using the cost based approach for classification it is possible to define ore and marginal ore and then use `diglim` to either include or exclude the marginal ore. If the marginal ore must be mined, it is cheaper to treat it as ore but if it does not need to be mined it should be left in place. Figure 5.8 shows a classification map with the black dig limit where the marginal ore has to be mined and is included in the limit and the green dig limit where the marginal ore can be left in place and is excluded.

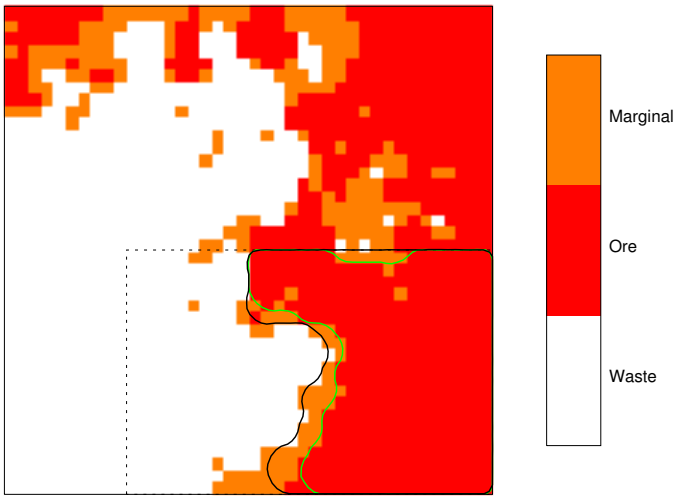

Figure 5.8: Classification map with the black dig limit including the marginal ore and the green dig limit excluding the marginal ore.

5.7 Simulated Annealing Schedule

The automatically determined simulated annealing schedule will be sufficient for most users . For added flexibility `diglim` allows for a user specified simulated annealing schedule. The advantage of a user specified annealing schedule is to reduce processing time. The parameters required for the annealing schedule are described below:

1. Initial temperature, t_0: the initial temperature specifies the decision criteria starting point for accepting perturbations. Selecting a high t_0 allows almost all perturbations to be accepted. The consequence of selecting a high value for t_0 is a greater CPU time requirement for convergence. Selecting low values for t_0 will result in too few non-optimal perturbations being accepted and the possibility that sub-optimal dig limits will be found. A value of $t_0 = 1$ is very high since it is scaled to the maximum.

2. Reduction factor, λ: the reduction factor is the multiplicative factor for reducing t. A λ between 0.75 and 0.5 is recommended.

3. Total perturbation attempts, k_A: after k_A perturbations have been attempted at one temperature t is multiplied by λ. A k_A of 20 times the initial number of dig limit polygon vertices is recommended.

4. Target perturbations k: after k perturbations have been accepted at one temperature t is multiplied by λ. A k of 10 times the initial number of dig limit polygon vertices is recommended.

5. Num: as the dig limit approaches optimal fewer perturbations will be accepted at one temperature. Num is the number of times that k_A is reached until the program will exit. This is used to prevent unnecessary processing time at the end of the optimization schedule. Normally set at 50.

6. Maximum perturbations $Maxp$: the maximum number of perturbations to be carried out. Around 100 000 perturbations are suggested.

The suggested parameters are good starting points. Experience plays a significant role in fine tuning them. For example $Maxp$ could be reduced according to the rate of convergence: if a large number perturbations are performed with no change in profit $Maxp$ could be reduced. In practice, the program runs so fast that there is no need to change these parameters,

Chapter 6

Case Studies

The case studies that were done to illustrate the steps required for dig limit determination are documented in this chapter. Some practical suggestions are mentioned but practice and experience are invaluable.

6.1 First Example — Small Mining Bench

This example shows the step-by-step procedure to determine an optimal dig limit. The data available is a set of blasthole assays from one bench of a copper mine. An area of interest was chosen, the data was extracted, analyzed and cleaned, the variogram was modelled, the grades were simulated, an expected profit map was made and the dig limits were determined.

Data Analysis

Understanding the data and the general statistical and geological characteristics is important. This understanding will help ensure that good results are obtained. Any "blind faith" in the programs must be avoided. An area of interest was used to extract a set of data from a larger set of blasthole samples; shown in Figure 6.1. The area of interest is 200 by 200 metres square with 372 blastholes. The data shows a high grade area to the east and good continuity north-south. This should result in a final dig limit that runs north-south through the blastholes and then follows along the edge of the pattern.

The data must be extracted and validated/checked for accuracy. Outliers and erroneous data must be isolated and either corrected or removed from the data set. A histogram of the sample grades showing the cutoff grade of 0.8% is shown in Figure 6.2.

This shows several samples that do not fit the upper end of the distribution. These high valued outliers were plotted on a location map, Figure 6.3, that shows the outliers have a definite north-west/south-east trend meaning they are most likely part of the same geological structure. The probability plot, Figure 6.4, confirmed this conclusion as the high valued samples fit the upper tail of the distribution. Both of these checks show that the data appears valid. The kink in the probability plot shows that there are most likely two different distributions of grades and therefore, two different rock types. This is also evident in the location map as the majority of the low grade samples are in the west and most of the high grade samples are in the east. No rock type information was used directly in the estimation though.

Proper representation of the true grade distribution is important for the simulation results to be accurate. Data that is clustered in certain areas and sparse in others must be corrected by declustering. The location map of the blasthole samples shows a regular spacing and the need for declustering will be minimal, if at all. Declustering was done for multiple cell sizes and the declustered mean was plotted against the cell size, Figure 6.5. This showed that the declustered mean hardly changes from the clustered mean and that declustering is not needed.

There is a slight west-to-east increasing trend apparent in the location map. Due to the small area the impact of the trend will be minimal and no effort was made to model or remove it.

Grade Estimation

A good normal scores variogram model is essential for reliable results from Gaussian simulation. The data was normal scored and the histogram of the transformed data is shown in Figure 6.6. To determine the major and minor directions of continuity a variogram map was made, Figure 6.7. From this map the major direction of continuity had an azimuth of 25° and the minor direction is perpendicular at an azimuth of 115°. Experimental variograms were calculated in the major direction, minor direction, and omni-directionally; shown in Figure 6.8. The experimental variogram was modelled with a nugget effect and two nested structures, Equation 6.1. Figure 6.9 shows the experimental variogram as points and the modelled variogram as a solid line overlaid on the experimental points.

$$\gamma(\mathbf{h}) = 0.15 + 0.15 \cdot Exp_{\substack{\mathbf{aa}_{hmax}=30 \\ \mathbf{aa}_{hmin}=30}} + 0.7 \cdot Sph_{\substack{\mathbf{aa}_{hmax}=130 \\ \mathbf{aa}_{hmin}=200}} \tag{6.1}$$

Kriging was done to establish a baseline for comparing the simulation results. The kriged map, Figure 6.10, is inherently too smooth, but it will only be used to verify the simulation results. To quantify the uncertainty in the grade estimation 100 realizations were generated. The simulation was done using 2.5 m square blocks with the data values being assigned to the closest node. Most of the volume variance issues are mitigated by block averaging the results up to 5 m square blocks. One of the fine scale realizations is shown in Figure 6.11 and the block averaged realization is shown in Figure 6.12.

Expected Profit

To determine the optimal dig limits the grade uncertainty model was converted to a map of expected profit. The model of grade uncertainty was converted to a model of profit uncertainty and the realizations were averaged together. The profit was calculated using the grade of the block multiplied by the recovery at that grade minus the cutoff grade times the recovery at the cutoff grade all multiplied by the selling price of the mineral of interest, Equation 6.2.

$$Profit^l(\mathbf{u}) = price \cdot \left(z^l(\mathbf{u}) \cdot r\left(z^l(\mathbf{u})\right) - z_c \cdot r\left(z_c\right)\right) \tag{6.2}$$

$$E\left\{Profit(\mathbf{u})\right\} = \frac{1}{L}\sum_{i=1}^{L} Profit^l(\mathbf{u}) \tag{6.3}$$

Using the cutoff grade in the profit calculation will give blocks above the cutoff a positive profit and blocks below the cutoff a negative profit. The expected profit map with the positive profits is shown in Figure 6.13. If the expected profit is positive, the block will be called ore otherwise; it is considered waste. When compared to the classification based on the cutoff grade, there are four possible scenarios. In the first two the expected profit and expected grade classification are the same; ore as ore and waste as waste. For the other two cases the classification does not agree. First the expected profit is positive, and the block is classified ore, while the expected grade is less than the cutoff grade, and would have been called waste. Secondly the expected profit is negative, and classified waste, while the expected grade is greater than cutoff and would have been called ore.

The ore/waste map in Figure 6.14 shows how the blocks were classified. The red blocks are ore where the expected profit and grade classification were the same, the yellow blocks are ore where the expected profit and grade classification were the same, and the blue blocks are where the expected profit classified blocks as ore but

the expected grade was less than the cutoff. In this case there were no instances where the expected profit of a block was less than zero but the grade was above cutoff. The summary in Figure 6.15 shows that the two different classification methods were the same for 97.4% of the blocks and there was a difference in only 2.6% of the blocks. To show how this can occur one block was investigated.

The effect that a few high valued realizations have is quite easy to see in a scatterplot of the profit versus the grade; shown in Figure 6.16. The expected grade of the 100 realizations is 0.7318, below the cutoff of 0.8, while the expected profit is greater than zero at 83.58. This is because of several high valued realizations that have the potential to make a huge profit. This potential outweighs the losses if 100 blocks having this distribution were to be mined and processed. The histogram of the grades for the 100 realizations are shown in Figure 6.17 and the histogram of the profits are shown in Figure 6.18.

Automatic Dig Limit Determination

The final step is to determine the optimal dig limits subject to the equipment constraints. A small area in the lower right hand corner of the map is where the first dig limit will be determined. A good practice to develop is to work step-by-step through the area. This means that there will be several non-overlapping dig limits. Although there could be one large dig limit in hindsight, the blasthole data becomes available incrementally and the limits are determined that way in practice.

After the area has been chosen, the initial polygon, the maximum boundary window, the type of dig limit, and the equipment factor must be chosen. Shown in Figure 6.20 is the initial dig limit polygon, a solid black line, and the boundary window, shown as a dashed line. The boundary window defines the maximum and minimum in the x and y directions. Ore will be contained within the dig limit, so the type of limit is ore. The equipment factor is based on the type of equipment used. It was chosen both low, to simulate a small loader, and high, to simulate a large shovel. An example parameter file for `diglim` that was used is shown in Figure 6.19. With these parameters, the program `diglim` can now be run.

Figures 6.21 and 6.23 show two different dig limits with different equipment factors. The higher equipment factor resulted in a smoother dig limit that included waste and lost ore. The optimal dig limit for the mine would have the lowest equipment factor possible and still be easy to mine. One way to compare the effectiveness of the different limits is to calculate the amount of ore and waste within the limits and

the amount of ore lost. Figures 6.24 and 6.24 show sample tables for the dig limits. The program `postdig` was used to generate the tables. It calculates the amount and value of the material if free and perfect selection were possible and then the amount of ore and waste inside the dig limit, the dollar value of the material inside the limit and the amount of ore and waste outside the limit. To choose the best equipment factor several different dig limits should be done and compared.

The next step is to move the maximum boundary window to the north and determine the next dig limit. This is repeated until all of the dig limits for this map have been determined.

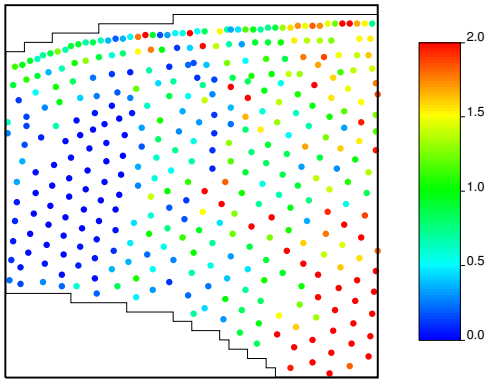

Figure 6.1: Blasthole sample locations.

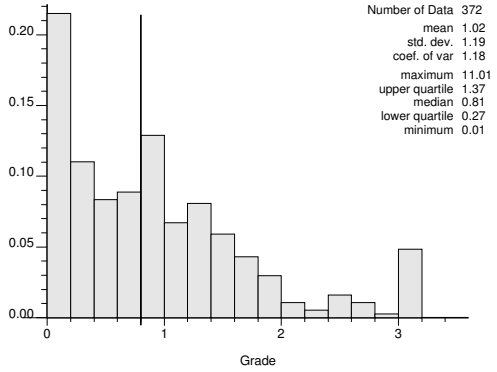

Figure 6.2: Histogram of the blasthole assays.

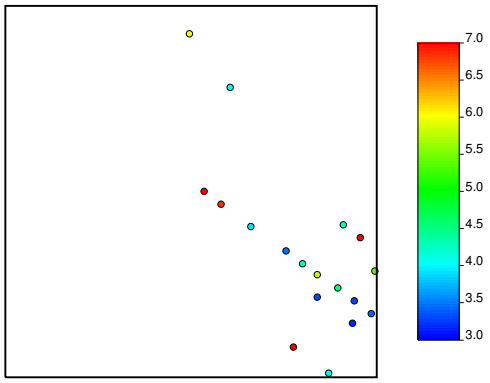

Figure 6.3: Location map showing the outliers.

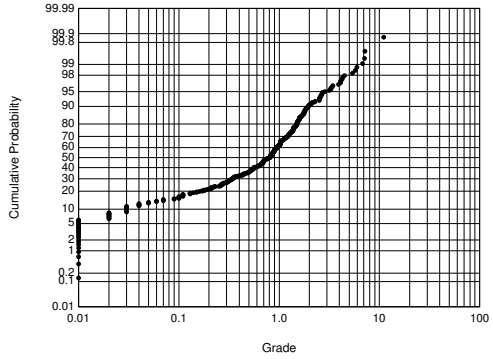

Figure 6.4: Probability plot.

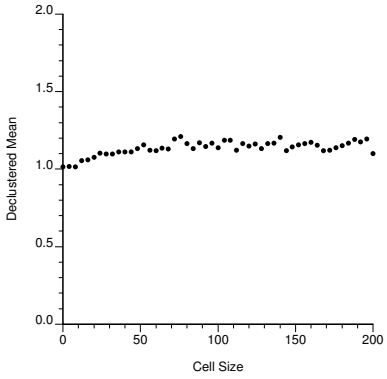

Figure 6.5: Declustered mean versus cell size.

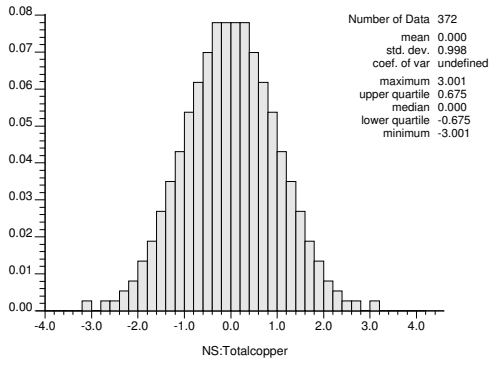

Figure 6.6: Data transformed to normal scores.

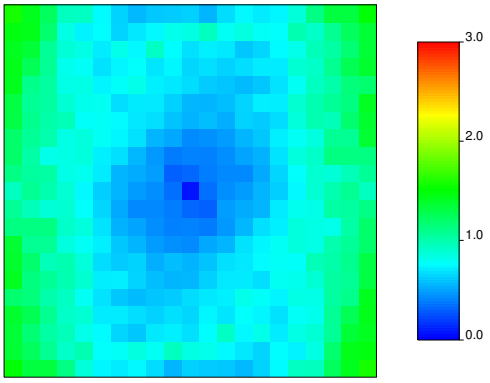

Figure 6.7: Variogram map.

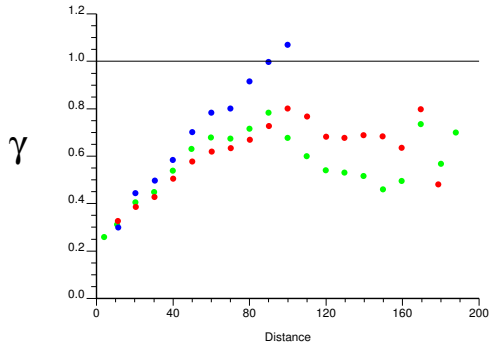

Figure 6.8: Experimental variograms (omni-directional — green, minor — blue, major — red).

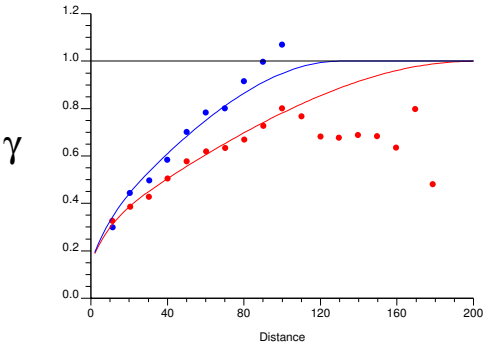

Figure 6.9: Modelled variogram (minor — blue, major — red).

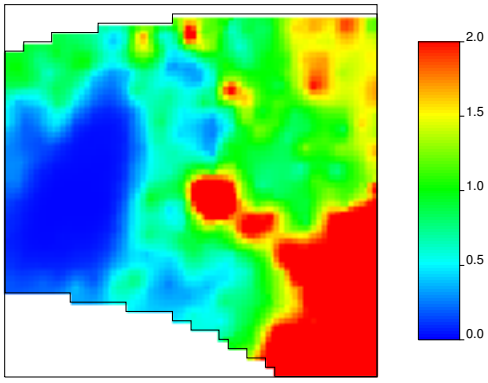

Figure 6.10: Kriged map.

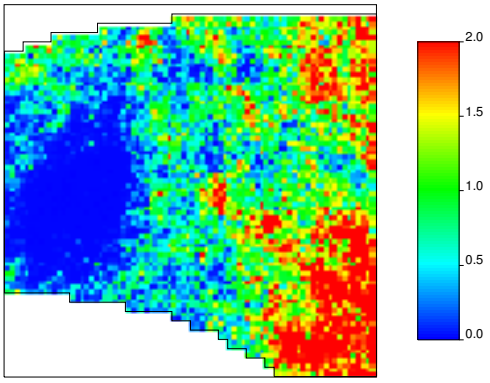

Figure 6.11: One realization of the simulated grades.

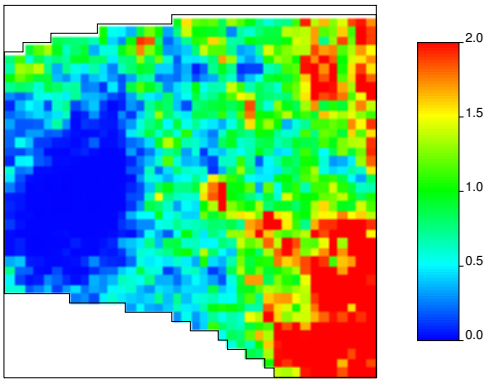

Figure 6.12: One realization of the simulated grades averaged to a larger block size.

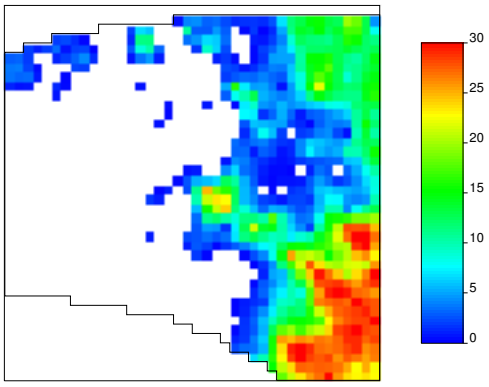

Figure 6.13: Expected profit map.

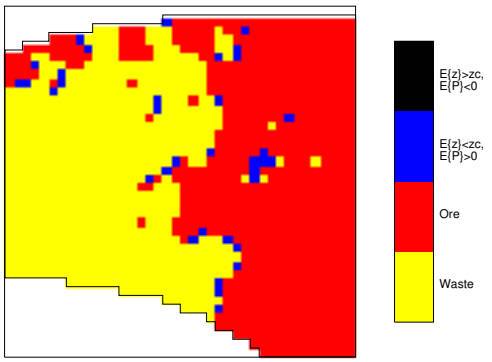

Figure 6.14: Ore/Waste classification map.

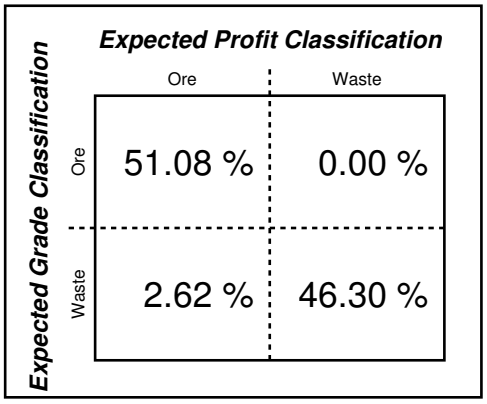

Figure 6.15: Summary of the classification.

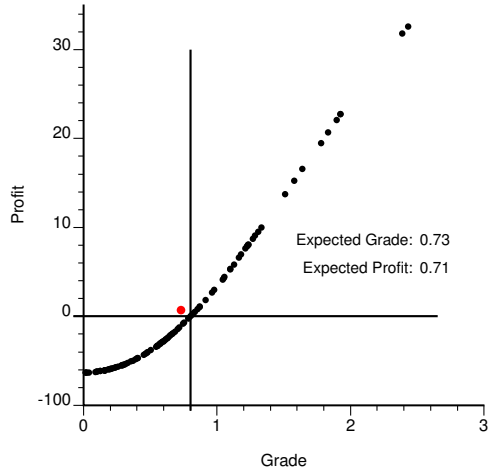

Figure 6.16: Scatterplot of profit versus grade for one location.

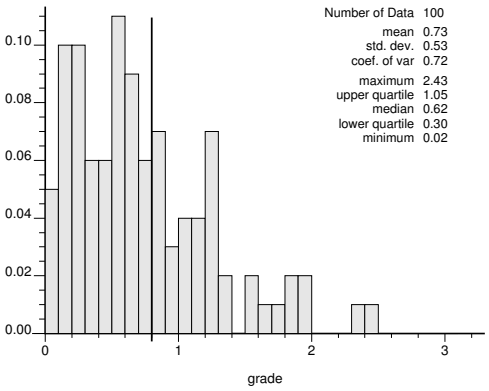

Figure 6.17: Histogram of the grade realizations for one location.

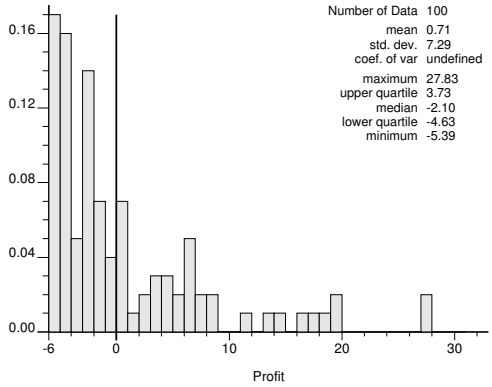

Figure 6.18: Histogram of the transformed profit realizations for one location.

```
            Parameters for DIGLIM
            *********************

START OF PARAMETERS:
eprofit.out          -file with the input data
2                    -   expected profit column
initial_limit.dat    -file with seed polygon
diglim_03.out        -file for dig limit output
diglim_03.dbg        -file for debugging output
gr_03.out            -file for gridded output
40   652.5   5.0     -nx, xmn, xsiz
40   152.5   5.0     -ny, ymn, ysiz
725   850 150   250  -window: xmn,xmx,ymn,ymx
1                    -dig limit type(1=o, 0=w)
0.3                  -equipment factor
1                    -fix vertices? (1=y, 0=n)
750 245   770 170    -fixed vertices xs,ys,xe,ye
1                    -automatic=1, user=0
```

Figure 6.19: Example parameter file for diglim.

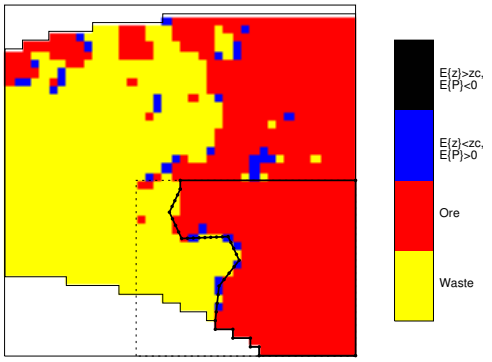

Figure 6.20: Initial dig limit polygon for `diglim`.

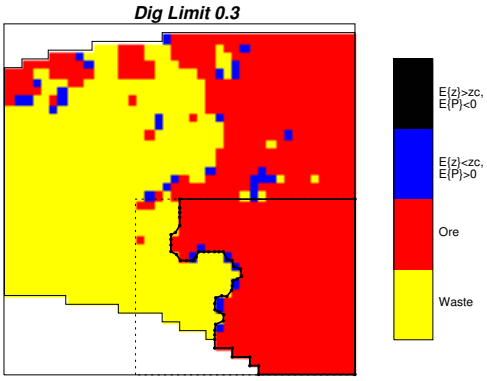

Figure 6.21: Optimal dig limit with an equipment factor of 0.3.

Perfect and Free Selection

	Tonnes	Grade	Profit / Cost
Ore	296100	1.720	$ 4617719
Waste	84240	0.544	$ -421200
Total	380340	1.459	**$ 4196519**

Inside Diglimit

	Tonnes	Grade	Profit
Ore	292206	1.731	$ 4614593
Waste	738	0.720	$ -336
Total	292944	1.729	$ 4614257

Outside Diglimit

	Tonnes	Grade	Cost
Ore	3894	0.826	$ -19469
Waste	83502	0.542	$ -417511
Total	87396	0.555	$ -436980

Total Potential Profit **$ 4177277**

Figure 6.22: Summary table for the dig limit performance.

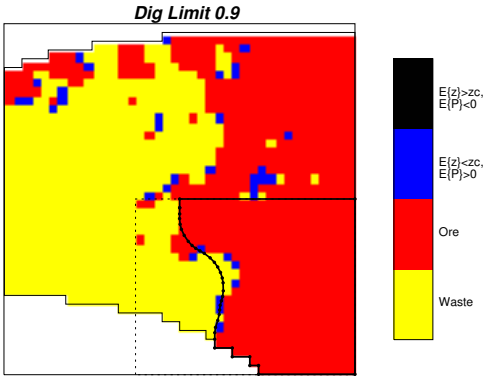

Figure 6.23: Optimal dig limits with an equipment factor of 0.9.

Perfect and Free Selection

	Tonnes	Grade	Profit / Cost
Ore	296100	1.720	$ 4617719
Waste	84240	0.544	$ -421200
Total	380340	1.459	**$ 4196519**

Inside Diglimit

	Tonnes	Grade	Profit
Ore	285966	1.750	$ 4604665
Waste	6793	0.648	$ -8944
Total	292758	1.724	$ 4595721

Outside Diglimit

	Tonnes	Grade	Cost
Ore	10134	0.857	$ -50672
Waste	77447	0.535	$ -387236
Total	87582	0.572	$ -437908

Total Potential Profit **$ 4157813**

Figure 6.24: Summary table for the dig limit performance.

Appendix A

Program Documentation

A.1 Expected Profit Transform Based on a Cutoff Grade `eprofit`

The expected profit transformation program, `eprofit`, converts a grade uncertainty model to an expected profit map. The transformation is based on a cutoff grade, a variable plant recovery, and the economic value of the mineral of interest. To show the differences between the material classification based on grade and the material classification based on expected profit a summary PostScript table can be made.

The output file contains four columns: (1) the expected profit, (2) the expected grade, (3) an ore/waste indicator(0=waste, 1=ore) and an ore/waste indicator where the expected profit and the expected grade classification differ (0=waste from both, 1=ore from both, 2=ore from expected profit and waste from expected grade), and (4) 3=waste from expected profit and ore from expected grade). The parameters are listed below and shown in Figure A.1:

- **datafl:** the input data file with the grade simulation.
- **icol:** column number for the grade.
- **nx, ny,** and **nz:** the number of nodes in the x, y, and z directions.
- **nreal:** the number of realizations.
- **igrd:** the units that the grade is in (0=fraction, 1=percent, 2=parts per million or grams per tonne).
- **outfl:** the output file.

```
                  Parameters for EPROFIT
                  **********************

START OF PARAMETERS:
sgsim.out               -file with realizations
1                       -   column number for grade
80  80  1               -   nx, ny, nz
51                      -   number of realizations
1                       -   grade: 0=fraction, 1=percent, 2=ppm
eprofit.out             -file for output
0                       -write a postscript summary table (1=yes)
epsum.ps                -file for postscript table
0.6                     -cutoff grade
6000.0                  -metal price ($/tonne)
0     15.0              -(1=use) cost of processing waste ratio ($/tonne)
6                       -number of points in the recovery curve
0.0  0.0                -   grade, fraction recovered
0.2  0.50
0.4  0.70
0.6  0.77
0.8  0.80
1.0  0.82
```

Figure A.1: An example parameter file for `eprofit`.

- **isum:** indicator to write a PostScript summary classification table (0=no, 1=yes).
- **sumfl:** file for the classification summary table if requested.
- $\mathbf{z_c}$**:** cutoff grade.
- **price:** the selling price for the mineral of interest.
- **icpw** and **cpwr:** indicator to use the cost of processing waste ratio (0=no, 1=yes) and the cost of processing waste ratio.
- **irec:** number of points in the recovery curve.
- **grd(i)** and **recov(i):** the grade and corresponding recovery for constructing the recovery curve.

A.2 Expected Profit Transform Based on Costs `eprofitcost`

The expected profit transformation program, `eprofitcost`, converts a grade uncertainty model to an expected profit map. The transform is based on the operating costs of the mine, a variable plant recovery, and the economic value of the mineral of

interest. To show the differences between the material classification based on grade and the material classification based on expected profit a summary PostScript table can be made.

The output file contains six columns: (1) the expected grade, (2) the shifted expected profit (used for including the marginal ore during dig limit determination with `diglim`), (3) the expected profit if ore (correct expected profit value used for calculating the summary tables and excluding the marginal ore during dig limit determination), (4) the expected profit if waste, (5) an ore/waste indicator (0=waste, 1=ore), and (6) an ore/marginal ore/waste indicator (0=waste, 1=ore, 2=marginal ore). Listed below, and shown in Figure A.2, are the parameters for the program:

- **datafl:** the input data file with the grade simulation.
- **icol:** column number for the grade.
- **nx, ny,** and **nz:** the number of nodes in the x, y, and z directions.
- **nreal:** the number of realizations.
- **igrd:** the units that the grade is in (0=fraction, 1=percent, 2=parts per million or grams per tonne).
- **outfl:** the output file.
- **isum:** indicator to write a PostScript summary classification table (0=no, 1=yes).
- **sumfl:** file for the classification summary table if requested.
- **costmill:** cost of milling or treating the ore.
- **costore:** cost of mining the ore.
- **costwaste:** cost of mining the waste.
- **price:** the selling price for the mineral of interest.
- **irec:** number of points in the recovery curve.
- **grd(i)** and **recov(i):** the grade and corresponding recovery for constructing the recovery curve.

```
                Parameters for EPROFITCOST
                **************************

START OF PARAMETERS:
sgsim.out                         -file with realizations
1                                 -   column number for grade
80  80  1                         -   nx, ny, nz
51                                -   number of realizations
1                                 -   grade: 0=fraction, 1=percent, 2=ppm
eprofitcost.out                   -file for output
0                                 -write a postscript summary table (1=yes)
epcsum.ps                         -file for postscript table
13.0                              -unit milling cost ($/tonne)
 1.0                              -unit ore mining cost ($/tonne)
 1.0                              -unit waste mining cost ($/tonne)
5000.0                            -metal price ($/tonne)
6                                 -number of points in the recovery curve
0.0  0.0                          -   grade, fraction recovered
0.2  0.50
0.4  0.70
0.6  0.77
0.8  0.80
1.0  0.82
```

Figure A.2: `eprofitcost` parameter file.

A.3 Dig Limits `diglim`

The automatic dig limit selection program, `diglim`, aims to select dig limits that account for uncertainty, profitability, and the mining equipment.

There are two output files from the program. One contains the final dig limit in Geo-EAS format listing the x and y coordinates of the polygon vertices. The second file is the input file with the block fractions appended in an additional column. The block fractions are the fraction of the block that is contained within the dig limit. The blocks are assigned a value of -1 if the block is outside of the dig limit boundary window, a value of 0 if the block is within the maximum window but outside the dig limit, and a value greater than 0 and up to 1 if the block is inside the dig limit partially or completely. The final dig limit is an optimal solution that will maximize the potential profit given the uncertainty in the estimation, the expected profit, and the limitations of the mining equipment. Listed below, and shown in Figure A.3, are the parameters for the program:

- **datafl:** the input data file with the expected profit.
- **icol:** column number for the expected profit.
- **polyfl:** file with the initial polygon (the x coordinate must be in column one and the y coordinate in column two).

- **outfl:** the output file for the dig limit polygon (the x coordinate will be in column one and the y coordinate in column two).
- **dbgfl:** the debugging file. The information written to the debugging file is the same as what appears on the screen during the program execution.
- **grdfl:** the gridded output file that contains the input file and the appended block fractions.
- **nx, xmn,** and **xsiz:** definition of the grid system (x-axis).
- **ny, ymn,** and **ysiz:** definition of the grid system (y-axis).
- **xmin, xmax, ymin** and **ymax:** definition of the dig limit boundary window. Minimum and maximum x and y values.
- **iorelim:** indicator for the material type contained in the dig limit (0=waste, 1=ore).
- **freezeflag:** indicator for freezing a portion of the dig limit during the optimization (0=no, 1=yes).
- **xs, ys, xe,** and **ye:** start and end x and y coordinates of the vertices to freeze. The two vertices closest to the points are frozen and all the vertices between are frozen in a clockwise direction.
- **eqfac:** the equipment factor, or digability (0 to 1).
- **schedflag:** indicator for an automatic or user specified simulated annealing schedule (0=automatic, 1=user specified). With an automatic schedule the remaining lines in the parameter file are ignored.
- **seed:** random number seed (a large odd integer).
- **t_0, redfac, ka, k,** and **num:** the initial temperature, reduction factor, number of perturbation attempts at one temperature, target number of perturbations at one temperature, and the number of times that **ka** can be reached before the program exits. These parameters set the simulated annealing schedule and are described in detail in Section 5.7.
- **maxpert** and **nrepo:** total number of perturbations to be carried out and the interval for reporting.

```
                  Parameters for DIGLIM
                  *********************

START OF PARAMETERS:
eprofit.out                       -file with the input data
2                                 -   column for expected profit
initial_polygon.dat               -file with initial polygon
diglim.out                        -file for dig limit output
diglim.dbg                        -file for debugging output
grid.out                          -file for gridded output
50    0.5    1.0                  -nx, xmn, xsiz
50    0.5    1.0                  -ny, ymn, ysiz
20   50    5   35                 -dig limit window: xmin, xmax, ymin, ymax
1                                 -type of dig limit (1=ore, 0=waste)
0.3                               -equipment factor
0                                 -use fixed vertices? (1=yes, 0=no)
0.0 0.0 10.0 10.0                 -fixed vertices xs,ys(start),xe,ye(end)
1                                 -automatic schedule=1, user schedule=0
69069                             -     random number seed
 2.0  0.5  1000  500  100         -     SA schedule: t0, redfac, ka, k, num
50000  100                        -     maximum perturbations, reporting
0.5  1.0                          -     minimum and maximum interval length
0.4                               -     maximum perturbation distance
```

Figure A.3: `diglim` parameter file.

- **dismin** and **dismax:** the minimum and maximum distance between nodes. If the distance is smaller than **dismin** a node is removed and if the distance is greater than **dismax** a node is added.

- **dmax:** the maximum distance that a node can be moved during a single perturbation. The amount a node moves is drawn randomly from 0 to the maximum.

A.4 Post Processing of Dig Limits `postdig`

Once a dig limit has been determined a summary table can be made using `postdig`. The table is a summary for tonnes, expected profit, and expected grade. There are two cases show in the table: (1) perfect and free selection where each block can be extracted with no losses or dilution and (2) the imperfect selection that results with the dig limit.

The tonnes are calculated using the specific gravities of the different materials, read in from the input file or default values, and the block dimensions. The profit (for ore) and cost (for waste) are calculated by multiplying the expected profit per tonne of the block by the tonnage in the block. The grades are calculated using a tonnage weighted average. If no marginal ore is present in the area it is excluded. Listed below, and shown in Figure A.4, are the parameters for the program:

```
              Parameters for POSTDIG
              **********************

START OF PARAMETERS:
grid.out              -input data file
 1                    -   grade column
 -1.0e21   1.0e21     -   trimming limits for grade
 3                    -   expected profit if ore column
 0                    -   cost if waste column or cost of waste (<=0)
 -1.0e21   1.0e21     -   trimming limits for profit(ore or waste)
 6    1    2    0     -   ore/marg/waste indicator column and indicators
 9  3.2  3.0  2.7     -   spec. grav. column(0=default), ore, marg, waste
 7                    -   fraction within block column
80  80                -nx, ny
 5   5  10            -block dimensions (x, y, z)
36     36      48     -column widths (72's of an inch)
Grade                 -column 2 header
0       3       0     -decimal places for columns
postdig.ps            -postscript file for output
```

Figure A.4: `postdig` parameter file.

- **datafl:** the input data file with the expected profit, expected grade, material classification, and block fractions.
- **ig:** column number for the grade.
- **trimgl** and **trimgh:** lower and upper trimming limits for the grade.
- **iep:** column number for expected profit (make sure not to use the shifted expected profit column if a cost based profit transformation was used).
- **icw:** column for the cost if waste or the default cost of waste if **icw**$\leq$0. When using the cutoff based expected profit transformation there is no cost of mining waste and **icw**=0.
- **trimepl** and **trimeph:** lower and upper trimming limits for the expected profit.
- **iow, iore, imarg,** and **iwaste:** column number for the material classification indicators and the ore, marginal ore, and waste indicators.
- **isg, oresg, margsg,** and **wastesg:** column for the specific gravity or if isg=0 the default values **oresg**, **margsg**, and **wastesg** will be used.
- **ifwb:** column number for the block fractions.
- **nx** and **ny:** the number of blocks in the x and y directions.
- **dx, dy,** and **dz:** dimensions of the blocks in the x, y, and z directions.

- **tbcw2, tbcw3,** and **tbcw4:** width of the second, third, and fourth column in the summary table (72's of an inch).
- **tbhd2:** header for the second column (up to 30 characters).
- **ndec1, ndec2,** and **ndec3:** number of decimal places for the second, third, and fourth columns.
- **outfl:** output file for the postscript summary table.

A.5 Location Map with Dig Limits `locmappoly`

`Locmappoly` is the same as the `locmap` program that comes with the GSLIB library but with the added feature of plotting polygons over the sample locations. One input file is required for each polygon to be plotted. Listed below, and shown in Figure A.5, are the parameters for the program:

- **datafl:** the data file with the sample locations and grades.
- **ixl, iyl,** and **ivrl:** columns for the x-coordinate, y-coordinate and the variable.
- **tmin** and **tmax:** lower and upper trimming limits.
- **outfl:** file for PostScript output.
- **xmn** and **xmx:** the limits in the x direction.
- **ymn** and **ymx:** the limits in the y direction.
- **npoly** and **bulsiz:** number of polygons to plot and the bullet size. If plotting multiple polygons the next three lines get repeated for each additional polygon.
- **polyfl:** the data file with the polygon.
- **ivrx** and **ivry:** columns for the x and y coordinates of the polygon vertices.
- **linew, idsh, ibul,** and **lcol:** line width, dashing (same as `vargplt`), plot bullets at the vertices (1=yes), and line colour (same as categorical colours).
- **ibw:** plot boundary window as a thin dashed line (1=yes).
- **bminx** and **bminy:** minimum x and y of the boundary.

```
                  Parameters for LOCMAPPOLY
                  *************************

START OF PARAMETERS:
../data/cluster.dat             -file with data
1   2   3                       -   columns for X, Y, variable
-1.0    1.0e21                  -   trimming limits
locmappoly.ps                   -file for PostScript output
0.0     50.                     -xmn,xmx
0.0     50.                     -ymn,ymx
2   0.5                         -number of polygons, bullet size
polyfile_1.dat                  -1 file with polygon
1   2                           -   columns for x and y
0.5   0   0   10                -   line width, dashing, points?, colour
polyfile_2.dat                  -2 file with polygon
1   2                           -   columns for x and y
0.5   0   0   10                -   line width, dashing, points?, colour
0                               -plot boundary window (0=no, 1=yes)
   0.0   0.0                    -   minimum x and y
 100.0 100.0                    -   maximum x and y
0                               -0=data values, 1=cross validation
1                               -0=arithmetic,  1=log scaling
1                               -0=gray scale,  1=color scale
0                               -0=no labels,   1=label each location
0.01   10.0    1.               -gray/color scale: min, max, increm
0.5                             -label size: 0.1(sml)-1(reg)-10(big)
Locations of Clustered Data     -Title


Color Codes for Categorical Variable Plotting:
      1=red, 2=orange, 3=yellow, 4=light green, 5=green, 6=light blue,
      7=dark blue, 8=violet, 9=white, 10=black, 11=purple, 12=brown,
      13=pink, 14=intermediate green, 15=gray
```

Figure A.5: `locmappoly` parameter file.

- **bmaxx** and **bmaxy:** maximum x and y of the boundary.
- **idata:** when plotting data values (**idata**=0) circular symbols are used, when cross validation results are being plotted (**idata**=1) plus or minus signs are used and filled in by the magnitude of the error.
- **ilog:** =0 for arithmetic grey/colour scaling, =1 for base 10 logarithmic scaling.
- **icolor:** =0 for grey scale, =1 for colour scale.
- **ilabel:** =0 no labels, =1 each location will be labelled with the attribute value.
- **gmin, gmax,** and **ginc:** minimum and maximum limits for the grey/colour scale plotting and increment for the legend bar. Values less than **gmin** or above **gmax** appear at either end of the spectrum.
- **sizfac:** size of the labels if used (0.1=small, 1=default and 10=large).
- **title:** a 40-character title for the top of the plot.

A.6 Gridded Map with Dig Limits `pixelpltpoly`

`Pixelpltpoly` is the same as the `pixelplt` program that comes with GSLIB but with the added feature of plotting polygons over the gridded data. The slice of data can be oriented in any fashion but the plane that the polygon is in must match the plane of the slice. One input file is required for each polygon to be plotted. The parameters are listed below and shown in Figure A.6:

- **datafl:** the data file with the 2-D grid.
- **icol:** column number for the variable.
- **tmin** and **tmax:** lower and upper trimming limits.
- **outfl:** file for the PostScript output.
- **ireal:** realization number.
- **nx, xmn,** and **xsiz:** definition of the grid system (x-axis).
- **ny, ymn,** and **ysiz:** definition of the grid system (y-axis).
- **nz, zmn,** and **zsiz:** definition of the grid system (z-axis).
- **iview:** slice orientation(1=horizontal XY, 2=vertical XZ, and 3=vertical YZ).
- **islice:** slice number.
- **title:** 40-character title for the plot.
- **xlabel:** 40-character label for the x-axis.
- **ylabel:** 40-character label for the y-axis.
- **npoly** and **bulsiz:** number of polygons to plot and the bullet size. If plotting multiple polygons the next three lines get repeated for each additional polygon.
- **polyfl:** the data file with the polygon.
- **ivrx** and **ivry:** columns for the x and y coordinates of the polygon vertices.
- **linew, idsh, ibul,** and **lcol:** line width, dashing (same as `vargplt`), plot bullets at the vertices (1=yes), and line colour (same a categorical colours).
- **ibw:** plot boundary window as a thin dashed line (1=yes).

```
                Parameters for PIXELPLTPOLY
                ***************************

START OF PARAMETERS:
../data/true.dat                  -file with gridded data
1                                 -  column number for variable
-1.0e21  1.0e21                   -  data trimming limits
pixelpltpoly.ps                   -file with PostScript output
1                                 -realization number
50   0.5     1.0                  -nx,xmn,xsiz
50   0.5     1.0                  -ny,ymn,ysiz
1    0.0     1.0                  -nz,zmn,zsiz
1                                 -slice orientation: 1=XY, 2=XZ, 3=YZ
1                                 -slice number
2-D Reference Data                -Title
East                              -X label
North                             -Y label
2   0.5                           -number of polygons, bullet size
polyfile_1.dat                   -1 file with polygon
1   2                            -   columns for x and y
0.5   0   0   10                 -   line width, dashing, points?, colour
polyfile_2.dat                   -2 file with polygon
1   2                            -   columns for x and y
0.5   0   0   10                 -   line width, dashing, points?, colour
0                                 -plot boundary window (0=no, 1=yes)
   0.0   0.0                      -   minimum x and y
 100.0 100.0                      -   maximum x and y
0                                 -0=arithmetic, 1=log scaling
1                                 -0=gray scale, 1=color scale
0                                 -0=continuous, 1=categorical
0.0  20.0  1.0                    -continuous:  min, max, increm.
4                                 -categorical: number of categories
1     3    Code_One               -category(), code(), name()
2     1    Code_Two
3     6    Code_Three
4    10    Code_Four


Color Codes for Categorical Variable Plotting:
      1=red, 2=orange, 3=yellow, 4=light green, 5=green, 6=light blue,
      7=dark blue, 8=violet, 9=white, 10=black, 11=purple, 12=brown,
      13=pink, 14=intermediate green, 15=gray
```

Figure A.6: `pixelpltpoly` parameter file.

- **bminx** and **bminy:** minimum x and y of the boundary.
- **bmaxx** and **bmaxy:** maximum x and y of the boundary.
- **ilog:** =0 for arithmetic grey/colour scaling, =1 for base 10 logarithmic scaling.
- **icolor:** =0 for grey scale, =1 for colour scale.
- **icat:** =0 for a continuous variable, =1 for a categorical variable.
- **cmin, cmax,** and **cinc:** minimum, maximum and labelling increment for a continuous variable. Values less than cmin or above cmax will appear just beyond either end of the grey/colour spectrum.

- **ncat:** number of categories for the categorical variable.
- **icode, ccode,** and **cname:** the integer code of the category, the colour code for the category and the name for the legend for each category. The colour codes are listed in the parameter file.

A.7 Digitizing the Initial Dig Limit `DigXY`

One of the simplest methods for digitizing the initial dig limit required by `diglim` is to use `DigXY`. `DigXY` is a simple program designed to digitize data from bitmaps. A 30 day free trial is available from Thunderhead Engineering Consultants (`http://www.thunderheadeng.com/DigXY`).

The first step is to take the location map of the sample data, or the gridded map of the simulation, and convert it to a bitmap. `GSview` is an easy to use program that can quickly convert images from PostScript to bitmaps (`http://www.cs.wisc.edu/~ghost`). After the bitmap has been generated load it into `DigXY`. Once the image has been loaded the x and y axis need to be defined; they can be rotated and/or log-scale. After defining the axis the polygon points for the initial dig limit can be digitized; remember to do so in a clockwise direction. Once the digitizing is complete save the points; the sort data check box needs to be unchecked.

Before the file with the polygon points can be used a header needs to be added. An example polygon file is shown in Figure A.7. The final file with the header can now be used as an initial dig limit for `diglim`.

```
example polygon file
2
x-coordinate for the vertices
y-coordinate for the vertices
850   250
850   150
795   150
790   155
780   160
780   165
783   204
777   217
750   216
743   231
750   245
750   250
```

Figure A.7: Example polygon file digitized with `DigXY`.

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