Optimal grade control using geostatistics and economics: Methodology and examples

C.V. Deutsch, E. Magri V and K. Norrena

Abstract

An important problem in mine operations is the classification of material as waste, low-grade stockpile and ore. This classification must often be made with blasthole data that are widely spaced and that have sampling errors. Geostatistical-simulation techniques combined with basic economic principles allow a procedure for classification that maximizes the expected profit. Geostatistical-simulation methods (Gaussian, indicator or annealing-based) allow the integration of hard and soft data in the creation of alternative, equally probable realizations of the mineral grades. At each location, for each realization one calculates the "profit" if the block were to be classified as ore or waste. The optimal classification is the one that maximizes expected profit (a maximum profit selection or MPS procedure). In this paper, the authors discuss the theoretical justification of the method and implementation details. The use of blasthole data from different types of mineral deposits, that is, different levels of continuity, are considered. The authors also show the efficacy of the procedure with different levels of sampling error. The increased revenue due to the MPS procedure and improved sampling is shown. The paper shows the geostatistical-simulation procedure and the uncertainty in block grades that result from incomplete and imperfect sampling. The optimal classification is presented. Optimal block classifications are transferred to realizable dig limits by hand-drawing polygonal boundaries. The results of the proposed method are compared to classification based on kriging. Limitations and areas of future work are identified.

Introduction

Some orebodies are mined by visible differences to delineate between ore and waste. Other orebodies are very continuous and grade control amounts to delineating the ore/waste contact and providing a reasonable estimate of the ore grade for accounting purposes. Such cases are not considered in this paper. This paper is concerned with erratic orebodies that must be mined to a grade cutoff. There are concerns with blasthole sampling, reliable prediction of block grades, practical dig limits and internal dilution.

An important decision at the time of mining is to determine what materials to take to the mill, waste dump and low-grade stockpiles. This decision must be made with no access to the true grades; widely spaced and notoriously imprecise blasthole samples must be used. Moreover, the ultimate classification must be made for reasonably sized mining blocks that must ultimately fall within contiguous dig limits.

The classical approach to grade control is to hand contour the blasthole grades and smooth the limits, accounting for practical constraints imposed by the mining equipment. Although very attractive due to simplicity, there are often a number of problems with this classical approach, such as:

- errors in individual blasthole samples are not accounted for, that is, the limits "too closely" follow high-grade samples;
- it is difficult to account for exploration drilling and blastholes from the bench above; and
- there is neither an objective measure of optimality nor a repeatable procedure.

As alluded to above, geological rock types and visible ore/waste contacts must be taken into account. This should not be forgotten in the following "geostatistical" discussion. Most orebodies permit some grade control from geological interpretation. In practice, the geostatistical grade prediction should consider the geological rock types and *all* available information derived from actual grade assays of different types and interpretive geological information.

The first, most obvious extension to the classical hand-contouring approach to grade control is to consider some form of smooth interpolation, such as a moving window average, inverse distance or kriging. The estimates are then mapped and practical dig limits are established, usually by hand. These estimation methods have the advantage of providing a single "optimal" prediction that is easy to use with a predefined cutoff grade for ore/waste classification. There are limitations with such estimation methods:

- the "smoothing" effect of kriging and linear interpolators in general lead to conditional bias, that is, overestimation of low grades and underestimation of high grades; and
- there is no ability to account for uncertainty in the block estimates, in particular, the risk of discarding ore or processing waste.

C.V. Deutsch and **K. Norrena** are professor and Ph.D. candidate, respectively, with the University of Alberta, Edmonton, Alberta, Canada; **E. Magri V.** is professor with the University of Chile, Santiago, Chile. Preprint 99-018, presented at the SME Annual Meeting, March 1-3, 1999, Denver, CO. Manuscript accepted for publication March 2000. Discussion of this peer-reviewed and approved paper is invited and must be submitted to SME prior to Sept. 30, 2001.

Notwithstanding these limitations, kriging followed by hand contouring for the dig limits is common practice in grade control.

The classic references by David (1977) and Journel and Huijbregts (1978) discuss the application of kriging in mining engineering. There are many other case studies that use kriging for grade control and ore reserves, e.g., the papers by Raymond (1979), Westley (1986) and Davis et al. (1989). As described in Snowden et al. (1994), indicator kriging can be used to better quantify the continuity of extreme values.

The major concern with hand contouring and smooth mapping, however, is that such maps do not account for uncertainty in the grade estimates and the economic consequences of wasting ore or processing waste. A smooth map is optimal only when these consequences are the same (and increase with the magnitude of the error squared), which is never the case. The idea of using economic loss functions from decision analysis in the context of mineral-resources estimation overcomes this major concern (see Srivastava, 1987, and Glacken, 1996, for more details). This paper advocates a similar procedure with, perhaps, a more straightforward implementation.

The essential idea of decision analysis is to make decisions based on minimum loss or maximum profit. These decisions must account for the uncertainty of all variables and the consequences of making the wrong decision. For grade control:

- geostatistical simulation is used to quantify uncertainty in block grades;
- the consequences, or reduced profit, of wasting ore is the lost opportunity cost offset by less milling cost; and
- the consequences of processing waste are the increased milling cost offset, perhaps, by some recovered metal.

The optimal ore/waste classification is made by calculating the profit P_{ore} of each block if it were treated as ore and the profit P_{waste} of each block if it were treated as waste. The optimal classification is the one that maximizes profit. This procedure can be automated so that the mining engineer/geologist gets a plot of the blasthole grades with the optimal ore/waste classification. Of course, the detailed calculations could be examined to ensure data integrity and the correctness of the results.

There are numerous advantages to basing the ore/waste classification on maximum profitability rather than on an estimated grade map. An evident advantage is improved profitability by explicitly accounting for uncertainty in the grades and the consequences of misclassification. It is also remarkably easy to consider spatial/temporal variations in milling costs, mining costs, recovery and metal price. Multiple metals may be simply added to the profit calculation. One can also increase ore treatment costs due to spatially variable hardness, sulfur or other contaminants, and so on. The increased CPU cost of this method is not an issue because it takes only minutes on a modest Pentium PC to process many days of production.

The methodology will be presented with all necessary details. A number of examples will be presented to illustrate the procedure and quantify the economic benefit. These examples will consider different levels of spatial continuity and different levels of sampling error. A small 30- by 30-m example will also be considered to gain a deeper understanding of why the proposed methodology works. Outstanding issues, future work and limitations will also be documented.

Methodology

Definitions/cutoff grade. There are six essential geological and economic parameters needed to calculate a break-even cutoff grade for grade control:

- c_t = unit treatment (milling) cost (\$/t). This cost includes all operating costs related to processing a ton of rock sent to the mill. This often depends on the rock hardness (increased energy and materials consumption for hard ore), the ore grade and certain gangue minerals that are considered contaminants and may increase treatment cost.
- c_o = unit ore mining cost (\$/t). The total mining cost (minus stripping) for excavating a ton of rock and transporting to the mill. Again, this cost is variable depending on the location in the pit and differences in loading/hauling equipment.
- c_w = unit waste mining cost (\$/t). Mining cost to take a ton of rock and transport it to the waste dump. This cost may also vary depending on location and equipment.
- r = metal recovery factor (fraction). The fraction of metal in the ore feed retained in the final product in pure metal or concentrate form. The recovery could depend on the grade (perhaps higher recovery with higher grades) and on other metallurgical considerations.
- z = metal grade (fraction or other units). The metal grade is usually unknown and will be predicted with geostatistical estimation/simulation methods.
- p = unit metal price (\$/t or units consistent with units used for metal grade z).

There may be multiple metals/minerals of interest, in which case there will be multiple recovery (r), grade (z) and price (p) values. One should point out that the proposed methodology could straightforwardly handle multiple metals and variations in all six parameters noted above. Although an "equivalent grade" can be determined for multiple metals, it is difficult to handle cost/recovery variations in conventional grade-control practices.

Common practice consists of calculating a cutoff grade from the six parameters defined above. The simplest definition of cutoff grade is the grade at which the profit generated by processing as ore is equal to that of treating it as waste, i.e.

$$p \cdot r \cdot z_c - c_o - c_t = -c_w$$
$$z_c = \frac{c_t + (c_o - c_w)}{p \cdot r} \tag{1}$$

Any material at or above the cutoff grade will be cheaper to put in the mill than to place on the waste dump. Such a simple definition does not consider the time value of money, the capacity of the plant, fluctuations in strip ratio and other mining considerations. There are good reasons to consider an elevated cutoff grade in early years and to stockpile low-grade ore for supplementing mill feed in later years when the strip ratio increases. These issues must be handled on a case-by-case basis by the mining engineers. All grade-control methods, including our proposed simulation approach, allow flexibility to handle these issues.

Proposed approach. A block of material will be called "ore" if the expected profit (-cost) of processing the block as ore exceeds the expected profit (-cost) of considering the block to be waste. Maximizing profit is a well-established principle in mining and economics. Low-grade material is identified by establishing a threshold on the expected profit. The profit if a block is called ore P_{ore} is calculated as:

$$P_{ore} = E\{p \cdot r \cdot z - c_o - c_t\} \quad (2)$$

where
p is the metal price,
r is recovery,
z is the grade,
c_o is ore mining cost and
c_t is treatment cost.

P_{ore} will be less than $-c_w$ (the waste mining cost) when the grade is below cutoff.

The profit if a block is called waste (P_{waste}) is calculated as

$$P_{waste} = E\{-c_w - c_{lo}\} \quad (3)$$

where
$-c_w$ is the waste mining cost; and
c_{lo} is the lost opportunity cost, which only applies if the grade is greater than cutoff grade.

$$C_{lo} = E\{i(z;z_c) \cdot (-p \cdot r \cdot z + c_o + c_t - c_w)\} \quad (4)$$

where
the indicator $i(z;z_c)$ is defined as

$$i(z;z_c) = \begin{cases} 1, & \text{if } z > z_c \\ 0, & \text{otherwise} \end{cases} \quad (5)$$

The mine will only see $p \cdot r \cdot z - c_o - c_t$ if the material is called ore and $-c_w$ if the material is called waste. Some consider the inclusion of lost opportunity in Eq. (3) to be redundant; the argument is that this concept is included in the expected profit if ore. Nevertheless, the notion of lost opportunity is very real; it *does* cost money to mistakenly put high-grade ore on the waste dump. A practitioner would be advised to carefully construct their own profit functions; then, maximize the expected value of the profit over uncertainty in the grades.

In summary, the grade-control decision is written

$$\text{decision} = \begin{cases} \text{ore} & \text{if } E\{P_{ore}\} > E\{P_{waste}\} \\ \text{waste} & \text{otherwise} \end{cases} \quad (6)$$

A "maximum profit selection" criterion is advocated.

This numerical approach is very flexible. It is straightforward to consider complicating factors in the calculation of P_{ore} and P_{waste}.

Numerical laboratory. The concept of calculating P_{ore} and P_{waste} and then choosing the maximum profit selection (MPS) is simple and consistent with engineering and economic principles; however, one has to check that the results are significantly better than conventional practice (kriging). Checking with real data is notoriously difficult because the underlying true grades are never known and the grade of the material placed on the waste dump is poorly known. Here, Monte Carlo simulation will be used as a *numerical laboratory* for testing different methods.

The experimental steps in our numerical laboratory consist of:

- build a fine-scale true-grade model (together will all coregionalized variables);
- sample blasthole grades from the truth model at some realistic spacing;
- add sampling (and perhaps location) error to the "true" blasthole grades;
- apply methods $M_1, M_2, ..., M_m$ to establish different ore/waste classification;
- calculate the revenue $R_1, R_2, ..., R_m$ generated by each method; and then
- repeat for different truth realizations, models of spatial correlation and sampling practices to establish the domain of applicability of each method, that is, where $R_j \geq R_i, \ \forall i \neq j$.

A two-dimentional truth model representing a single bench has been considered. There is no limitation to consider a single bench. In fact, all methods should perform somewhat better having data from the bench above and exploration drill-hole data.

Although well established in engineering studies, there are a number of concerns with such a Monte Carlo procedure:

- the truth models are often too simplistic,
- any method M_j that makes use of the underlying random function model used to generate the truth model could appear unrealistically good (R_j too high) and
- there may be practical considerations that make it impossible to achieve any modeled revenue gains.

Awareness of these concerns help design the experimental procedure used in the numerical laboratory. A range of truth models have been considered with spatial heterogeneity that mimics real deposits. Kriging and simulation methods are given the same information to make fair comparisons. Finally, the authors attempted to enumerate all of the practical considerations (such as free selection, practical dig limits and blast movement) that would make theoretical revenue gains impracticable.

An example

For illustration, a two-dimentional example from a gold mine will be used. Although the grades and all economic parameters are fictitious, they have been chosen to mimic the features of a real mining operation. It would have been easy to show an example with real data; however, there would have been no underlying true distribution of grades to compare the results. There is nothing special about using gold in this example. In fact, more practical applications of this methodology have (to date) been made to base metal mines.

With the economic parameters: milling cost, c_t = \$12.00/t; ore mining cost, c_o = \$1.00/t; waste mining cost, c_w = \$1.00/t; recovery, r = 0.80; price, p = \$12.00/g, the breakeven cutoff grade may be calculated as 1.25 g/t. The distribution of gold grades is illustrated in Fig. 1.

An important advantage of a synthetic example is the ability to look at different types of deposits with different levels of sampling error. The first spatial distribution of grades we will consider is shown in Fig. 2. Note the N-S anisotropy and continuous regions of high and low grade. This two-dimentional example is 200 m E-W by 300 m N-S.

Blastholes were taken on a 10-m grid. Figure 3 shows a location map of the blastholes and Fig. 4 shows a histogram. In practice, each individual blast will be much smaller, and all of the blasthole grades will not be available at the same time. Also, in practice, there will be blasthole grades available from the bench above the current working bench. These issues have no affect on the "fairness" of our comparison or the illustrative nature of the example.

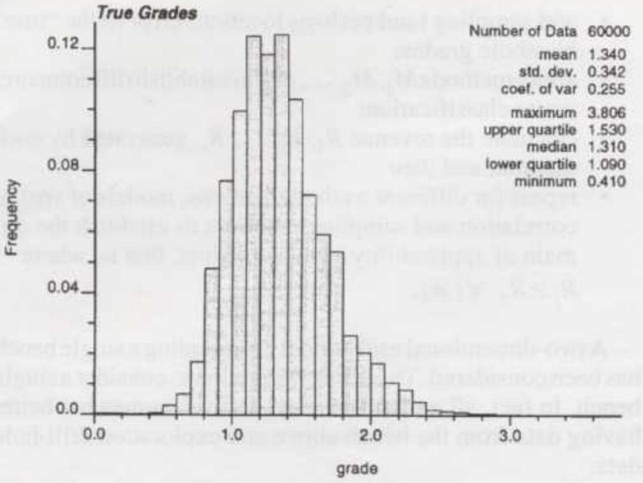

Figure 1 —Histogram of exhaustive distribution of true grade.

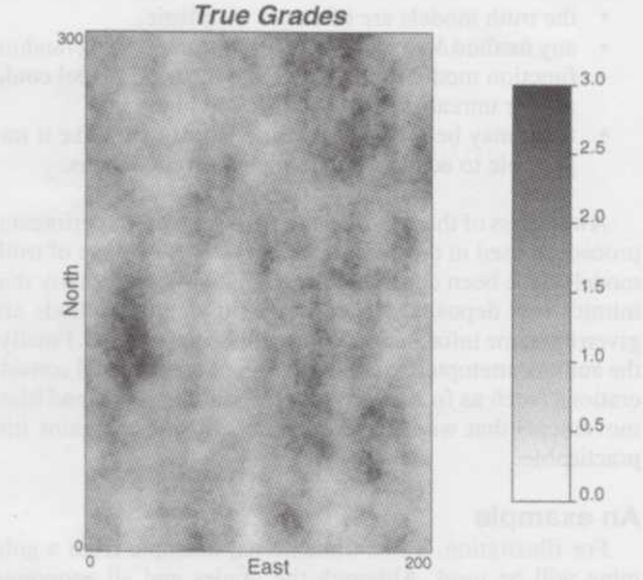

Figure 2 — Gray scale map of true grades for example application.

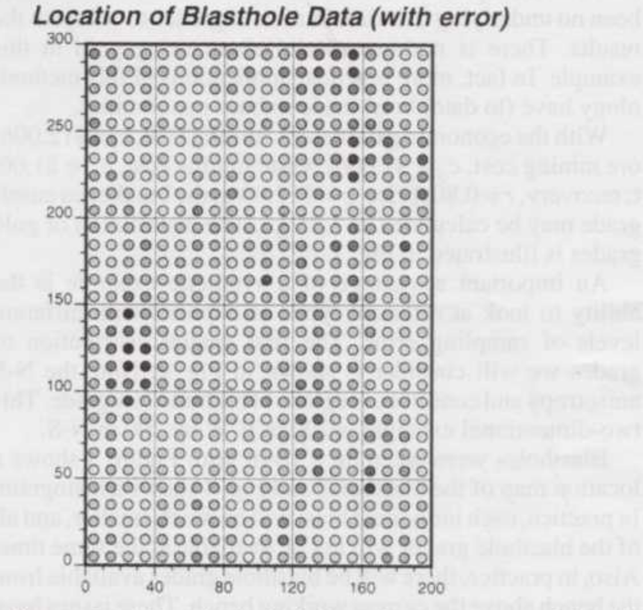

Figure 3 — Map of blasthole locations — gray scale level at each blasthole location indicates the grade.

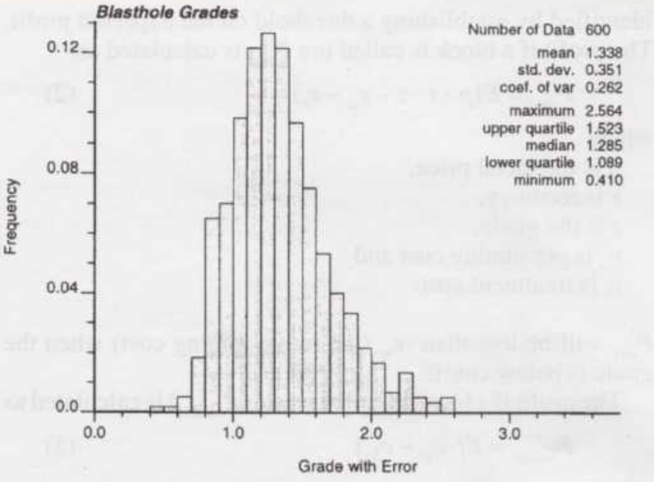

Figure 4 — Histogram of blasthole grades. Note the similarity to the exhaustive statistics shown in Fig. 1.

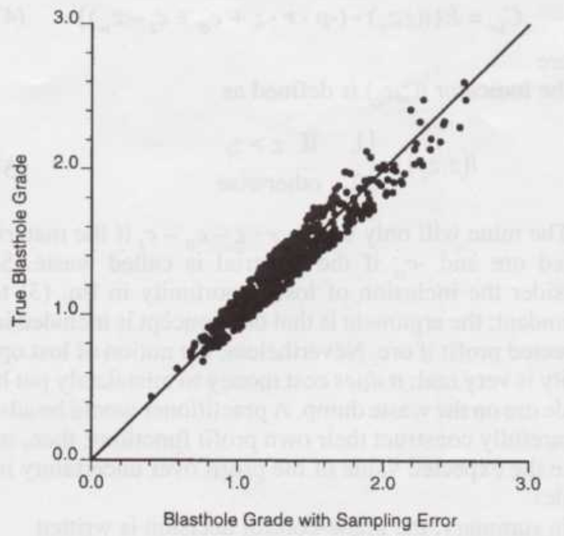

Figure 5 — Cross plot of the true grades vs. the blasthole samples with error. Note that this error level is quite small.

The blasthole grades are not perfect. The initial base case considered a 10% relative error uniformly distributed with a zero average. Figure 5 shows a cross plot of the true grades and the blasthole samples. This error level is realistic but could be considered too small in certain situations; a sensitivity will be performed later.

Kriging and simulation require a variogram model. The omnidirectional semivariogram of the normal score transform of the grades is shown on Fig. 6. The experimental points are the black dots and the solid line is the fitted model. The relative abundance of blasthole data makes it easy to infer a reliable semivariogram. The model consists of a nugget effect and a single structure spherical variogram with a range of 50 m. Additional blasthole error would increase the nugget effect and could appear to decrease the range of correlation.

Ordinary kriging was performed with the kb2d program from GSLIB, and the result is shown in Fig. 7. Note the smoothness relative to the true grades (Fig. 2). The same gray scale (Fig. 2) will be used for all gray scale images. In this base case, the kriging provides reliable estimates of the block grades because of the dense data spacing relative to the

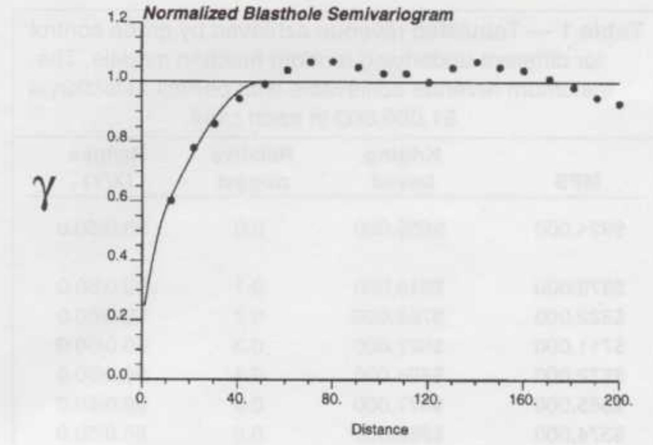

Figure 6 — Directional variograms (experimental points and modeled fit) normalized and calculated from blasthole data.

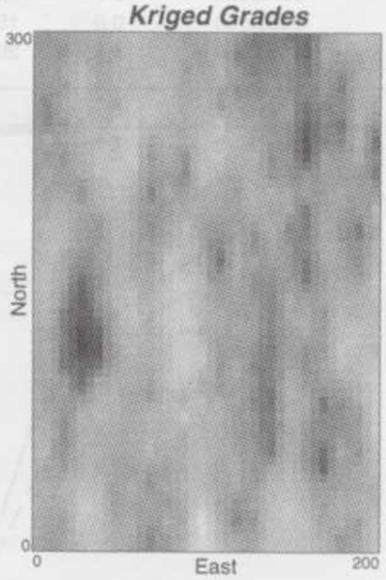

Figure 7 — Map of ordinary kriged grades using the blasthole data and variogram shown in Figs. 3 and 6. The gray scale is the same as Fig. 2

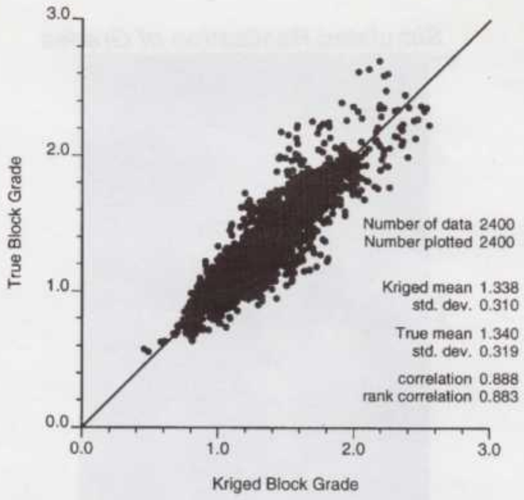

Figure 8 — Cross plot of true block grades with ordinary kriged block grades. Note the excellent correlation.

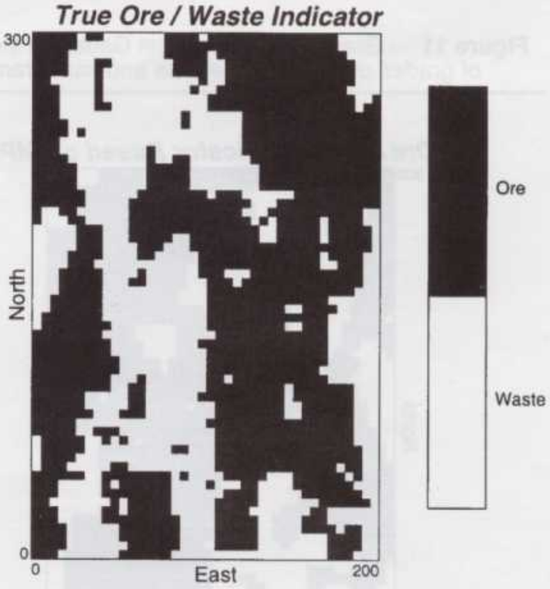

Figure 9 — Map of true ore/waste classification using true block grades and grade cutoff of 1.25.

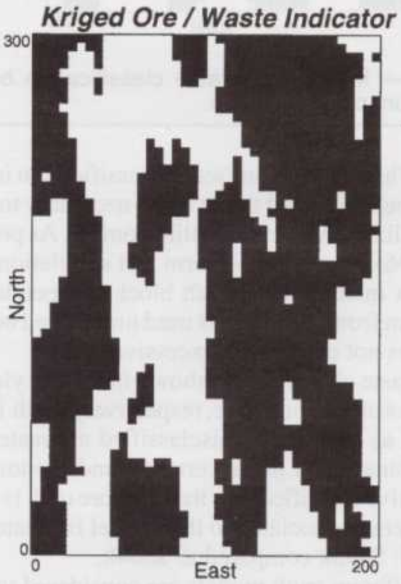

Figure 10 — Map of ore/waste classification based on kriging.

variogram range and the reliable histogram and variogram. Figure 8 shows a cross plot of the true block grades with ordinary kriged block grades; note the excellent correlation.

Figures 9 and 10 show the ore/waste classification based on the true grades and kriged estimates, respectively. The kriged limits appear somewhat smoother, which is consistent with the tendency of kriging to generate a smooth estimate. According to the kriged estimates there are 1,412 and 988 blocks of ore and waste, respectively. In truth, there are 1,358 and 1,042 blocks of ore and waste, respectively. However, the kriged model has 225 blocks misclassified as ore and 171 misclassified as waste. The maximum attainable revenue, from the true model with perfect selection, is $1,011k. The revenue from the kriged model is $864k, which must be considered quite good.

The maximum profit selection (MPS) method calls for stochastic simulation at a fine scale, block averaging to the mining scale, and then application of the "maximum profit selection" criteria. Figure 11 shows an example map of the first Gaussian realization at the fine scale. Fifty realizations were generated, block averaged, and then used for profit

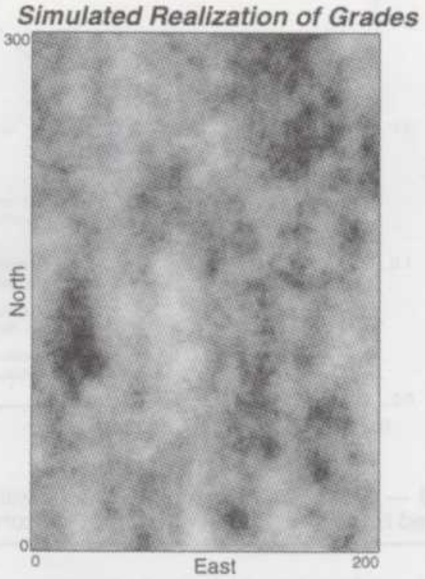

Figure 11 — Gray scale map of first Gaussian simulation of grades using blasthole data and variogram.

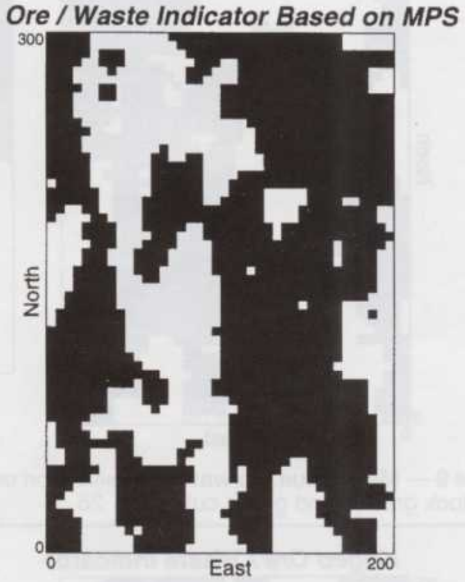

Figure 12 — Map of ore/waste classification based on maximum profit selection.

Table 1 — Tabulated revenue achieved by grade control for different underlying random function models. The maximum revenue achievable with perfect selection is $1,000,000 in each case.

MPS	Kriging based	Relative nugget	Ranges (X/Y)
$924,000	$855,000	0.0	50.0/50.0
$879,000	$810,000	0.1	50.0/50.0
$822,000	$754,000	0.2	50.0/50.0
$711,000	$621,000	0.3	50.0/50.0
$672,000	$604,000	0.4	50.0/50.0
$565,000	$477,000	0.5	50.0/50.0
$374,000	$256,000	0.6	50.0/50.0
$519,000	$498,000	0.0	10.0/10.0
$789,000	$756,000	0.0	15.0/15.0
$835,000	$808,000	0.0	20.0/20.0
$946,000	$872,000	0.0	100.0/100.0
$955,000	$880,000	0.0	150.0/150.0
$960,000	$884,000	0.0	200.0/200.0

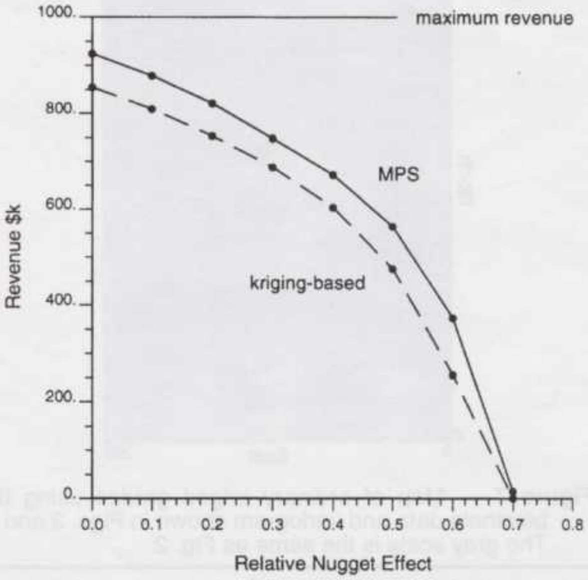

Figure 13 — Chart of revenue (1,000s $) vs. relative nugget effect with the simulation-based approach to grade control and kriging-based approach to grade control.

calculation. The resulting ore/waste classification is shown in Fig. 12. It should be noted that it is not necessary to construct fine-scale realizations over the entire domain. As proposed by Glacken (1996), one could perform fast simulation, say with an LU matrix method, over each block independently. The sgsim program from GSLIB was used unchanged because the CPU time was not considered excessive.

The ore/waste classification shown in Fig. 12 yields 1,450 and 950 blocks of ore and waste, respectively, with 157 blocks misclassified as ore and 65 misclassified as waste. It seems that the combination of the uncertainty and economics leads to a conservative classification, that is, more rock is sent to the mill. The revenue associated to this model is greater than the kriged model: $933k compared to $864k.

Twelve different truth models are considered to establish the generality of these results (Table 1). Each truth model leads to slightly different maximum revenue (recall that the maximum was $1,011k in the base case). To make the results comparable, the revenues from MPS and kriging have been normalized by the maximum truth revenue divided by $1,000k; thus, the maximum revenue is $1,000k in all cases. The same blasthole spacing and sampling error has been considered in all cases; only the nugget effect and range of correlation have been changed. MPS systematically leads to greater revenue than kriging, which is not surprising because the objective of MPS is to maximize profit, whereas the goal of kriging is to create an estimate with minimum squared error (see Srivastava, 1987).

Figure 13 shows a plot of the revenue of MPS and kriging vs. the relative nugget effect of the underlying grade models. The revenue attained by both methods decreases as the nugget effect increases. Beyond a nugget effect of 0.7, the predictability of the grades is so poor that the revenue is negative (mining the entire bench as either ore or waste would lead to

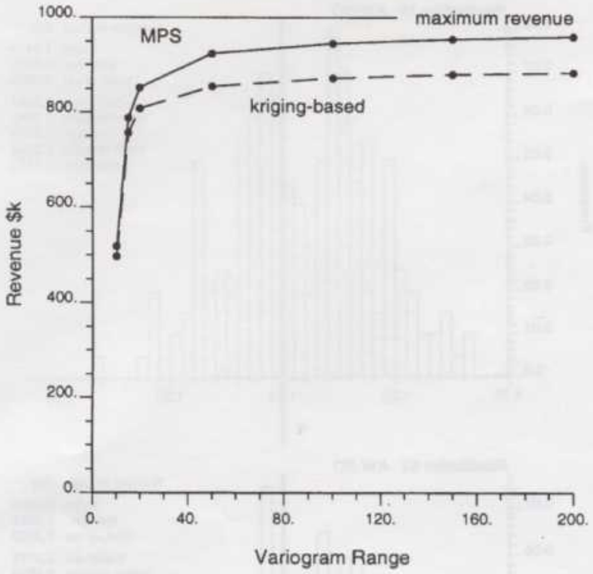

Figure 14 — Chart of revenue (1,000s $) vs.variogram range with simulation-based approach to grade control and the kriging-based approach to grade control.

a loss). The MPS procedure leads to significantly greater revenue when the nugget effect is less than 0.7.

Figure 14 shows a plot of the revenue of MPS and kriging vs. the range of correlation of the underlying grade models. The revenue attained by both methods increases as the range increases. A very low range appears like a high nugget effect, that is, negative revenue. The MPS procedure leads to significantly greater revenue when the range of correlation is greater than the twice the blasthole spacing.

The previous numerical experiments considered a small, yet realistic, sampling error. Pitard (1993) describes many sources of errors in sampling. These errors can be significant, particularly for gold deposits. Different levels of sampling error will be considered to evaluate the degradation in results with increasing error and to judge the relative performance of MPS and kriging in presence of sampling error. The sampling error will be modeled as a normally distributed residual

$$Z_{we} = z + error \rightarrow N(0, re \cdot z) \quad (7)$$

where

z_{we} is the blasthole grade with error,

z is the true blasthole grade and

error is a normally distributed error with a zero mean and variance equal to a relative error (*re*) multiplied by the grade.

The relative error will be varied from 0.0 to 0.5. Figure 15 shows a cross plot of true grades vs. blasthole grades with relative errors of 0.1 and 0.3. Note that the correlation decreases rapidly for errors in excess of 0.3.

Repeating the variogram analysis, kriging and simulation leads to the results in Table 2 and Fig. 16. The revenue decreases as the sampling error increases. It is interesting to note that the rate of decrease is less for errors less than 10% to 15% and then increases to a nearly constant rate of decrease. This coincides with conventional wisdom, which tells one that the sampling error should be less than 15% (Pitard, 1993) for industrial control. Further, it is noted that the MPS procedure systematically generates more revenue than the kriging-based procedure.

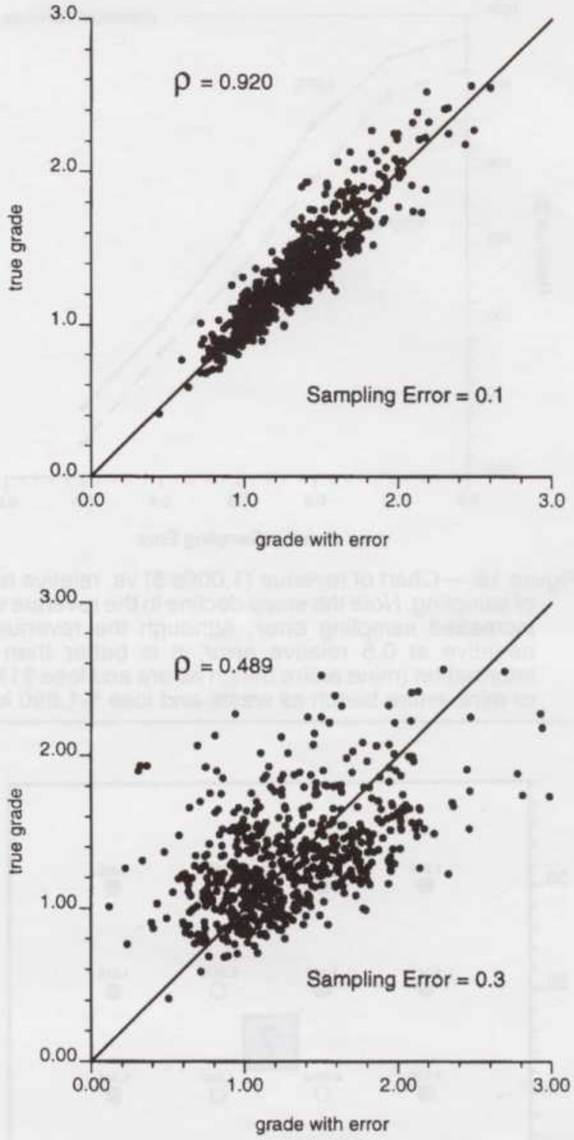

Figure 15 — Cross plot of true grades vs. blasthole grades with relative errors of 0.1 to 0.3.

Table 2 — Tabulated revenue achieved by grade control for levels of sampling error.

Sampling error	Kriging	MPS
0.0	$853,000	$945,000
0.1	$798,000	$889,000
0.2	$571,000	$713,000
0.3	$325,000	$426,000
0.4	$120,000	$188,000
0.5	$-113,000	$-19,000

Another small example

It has been shown that MPS outperforms kriging in terms of the revenue generated; however, the only explanation put forward is that MPS is directly based on profit criteria, whereas kriging is based on minimum variance. A small example will now be considered to help one see how the MPS criterion arrives at different, more optimal ore/waste classifications. A small 30- by 30-m area will be considered, see Fig. 17, with 16 blastholes on a 10- by 10-m pattern. These data follow the base-case histogram and variogram used in the

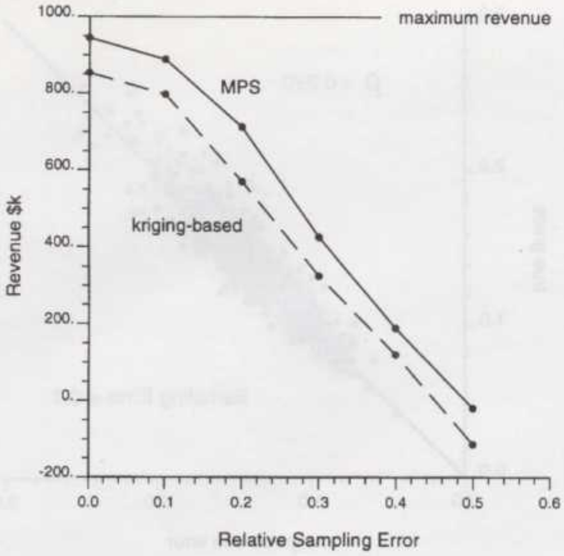

Figure 16 — Chart of revenue (1,000s $) vs. relative error of sampling. Note the steep decline in the revenue with increased sampling error. Although the revenue is negative at 0.5 relative error, it is better than no information (mine entire bench as ore and lose $170 k or mine entire bench as waste and lose %1,590 k).

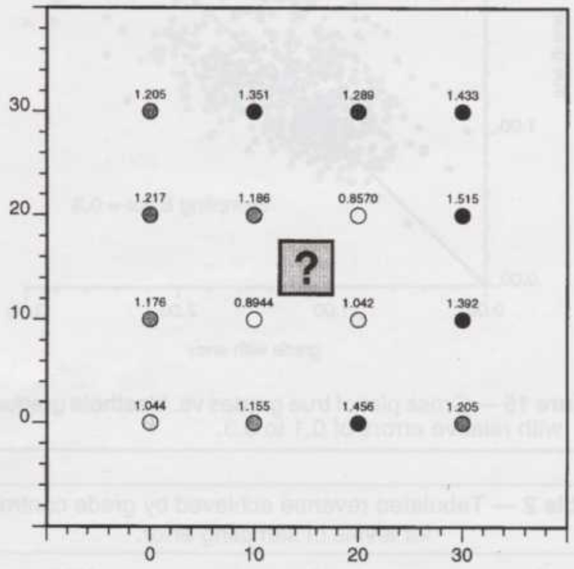

Figure 17 — Blasthole layout for small-scale example.

preceding example. Here, the interest is in classifying the 5- by 5-m block highlighted in the center of the pattern. Note that different blasthole grades will be considered, although the configuration of the blastholes and the block will remain unchanged.

One hundred different realizations of the 16 blasthole grades were constructed. The grade of the 5- by 5-m block was estimated by kriging in all cases. The uncertainty in the block grade was also assessed by performing 200 simulations of the grades (25 grades on a 1- by 1-m spacing inside the block). Thus, the central block has 100 different ore/waste classifications. There are four situations:

- kriging and MPS both classify the block as ore (40 times),
- kriging and MPS both classify the block as waste (54 times),

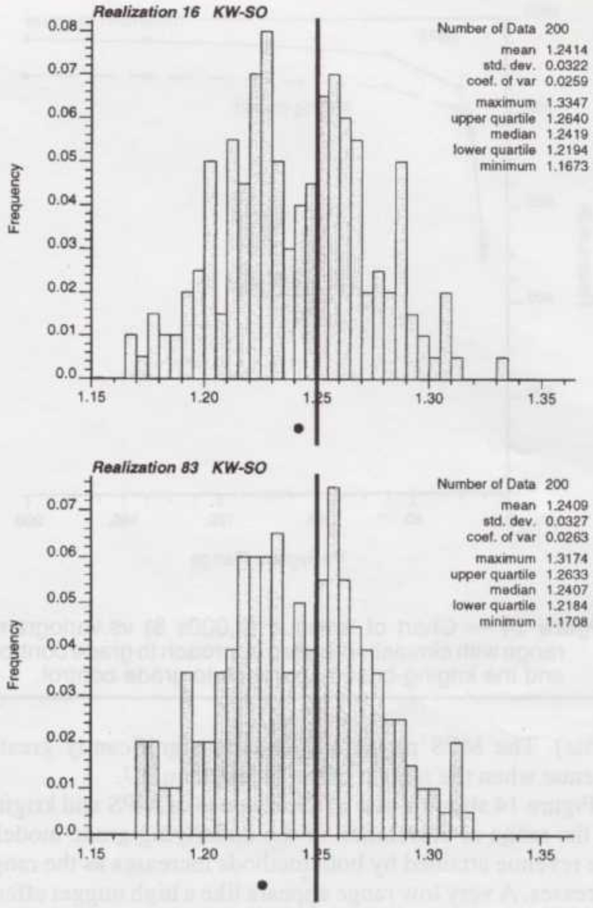

Figure 18 — Uncertainty in block grades at two locations where the kriged value would indicate waste (black dot) and simulation (MPS) would indicate ore.

- kriging classifies the block as ore but MPS classifies it as waste (four times) and
- kriging classifies the block as waste and MPS classifies it as ore (two times).

Although most classifications are the same, the six differences are interesting.

Figure 18 shows the uncertainty in block grades at the two locations where the kriged value would indicate waste (black dot) and simulation (MPS) would indicate ore. The dark black line at 1.25 indicates the ore/waste cutoff. MPS indicates ore because there is sufficient variability/probability that the block is ore, that is, the lost opportunity cost that would be incurred for all situations falling to the right of the cutoff cause the block to be classified as ore.

Figure 19 shows more startling results; the uncertainty in block grades at two locations where the kriged value (black dot) would indicate ore and the simulated grades would indicate waste. It was initially surprising to note that the simulated grades systematically are less than the kriged grades. Figure 20 shows the 16 blasthole grades used in both cases. The characteristic feature of both cases is the presence of nearby isolated high-grade samples surrounded by nearby low values. The kriged estimate is sensitive to these nearby high values whereas the simulated block grade is not.

Note that the uncertainty in a "blasthole" grade at the center of the small grid would be significantly higher than the uncertainty in a "block" grade. In fact, these two different levels of uncertainty for realization 29 are shown on Fig. 21.

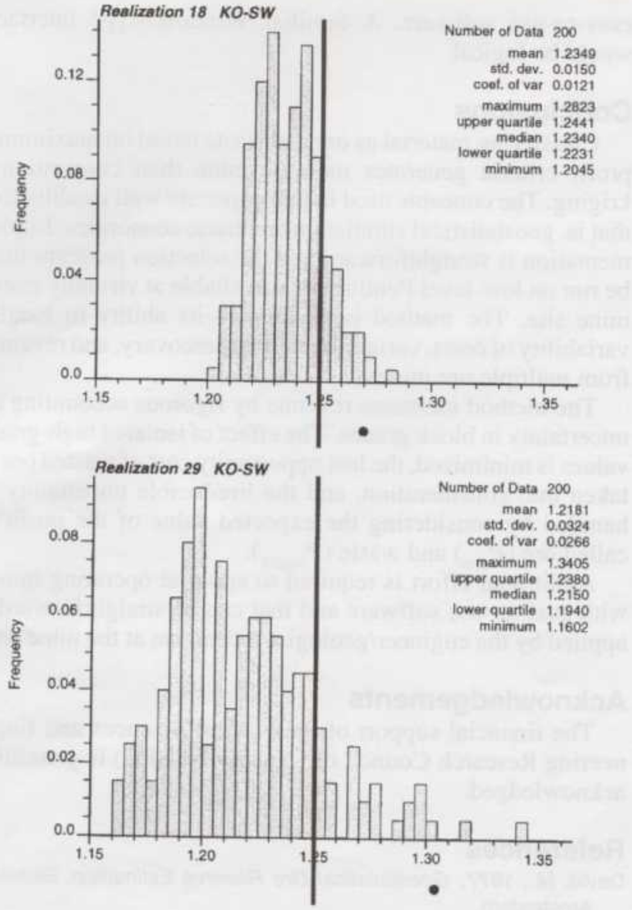

Figure 19 — Uncertainty in block grades at two locations where the kriged value would indicate ore (black dot) and simulation (MPS) would indicate waste.

The block grades have less variability, in particular, less probability of being above the cutoff grade of 1.25.

From this small example, one can draw two tentative conclusions:

- when the block is borderline ore/waste with large uncertainty, the MPS procedure would tend to classify it as ore; and
- when the block is borderline but based on an isolated high-grade sample, the MPS procedure is less sensitive to lone high-grade values and would tend to classify the block as waste.

As seen in the preceding example, these effects lead to greater revenue/profit.

Outstanding issues

Real data could have been used for the example, but it is very difficult to check the efficacy of the algorithm, because the truth is inaccessible. In other words, application of the proposed methodology to real data is straightforward, but its performance would be difficult to quantify. An important extension of this work is to present documented examples at operating mines.

A significant concern with synthetic examples is that one assumes "free selection," that is, the 5- by 5-m blocks can be selected independently and without error. The relative benefits of MPS may be less once the revenue is established based on practical dig limits. The Monte Carlo exercise could be extended to include a "hand smoothing" of the ore limits.

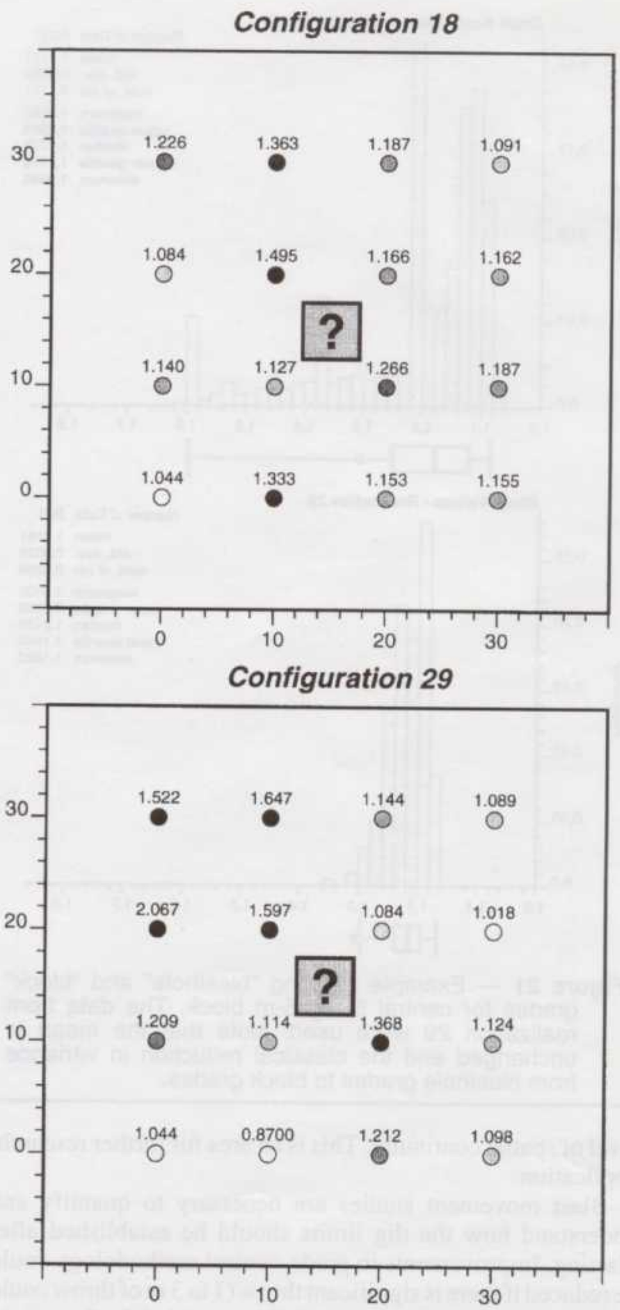

Figure 20 — Blasthole data used for small examples 18 and 29 (see distributions of uncertainty in Fig. 19).

Hand contouring of the blasthole grades can simultaneously account for the mining equipment (practical dig limits), erratic high grades and geological information. Here, kriging and simulation (MPS) were compared, but results that would have been obtained by hand calculation/contouring of the ore/waste boundary were not shown. Although such a comparison would be difficult and nonrepeatable, due to the interpretive nature of the contouring, it would be worthwhile future work. The Monte Carlo exercise could also be extended to include a "hand contouring" of the ore limits directly from the blasthole grades.

The effects of blasthole sampling errors were found to be significant; however, the sensitivity to blasthole sampling errors were only evaluated for the base case variogram (zero nugget effect and range 10 times the block size). The importance of sampling errors will likely depend on the underlying

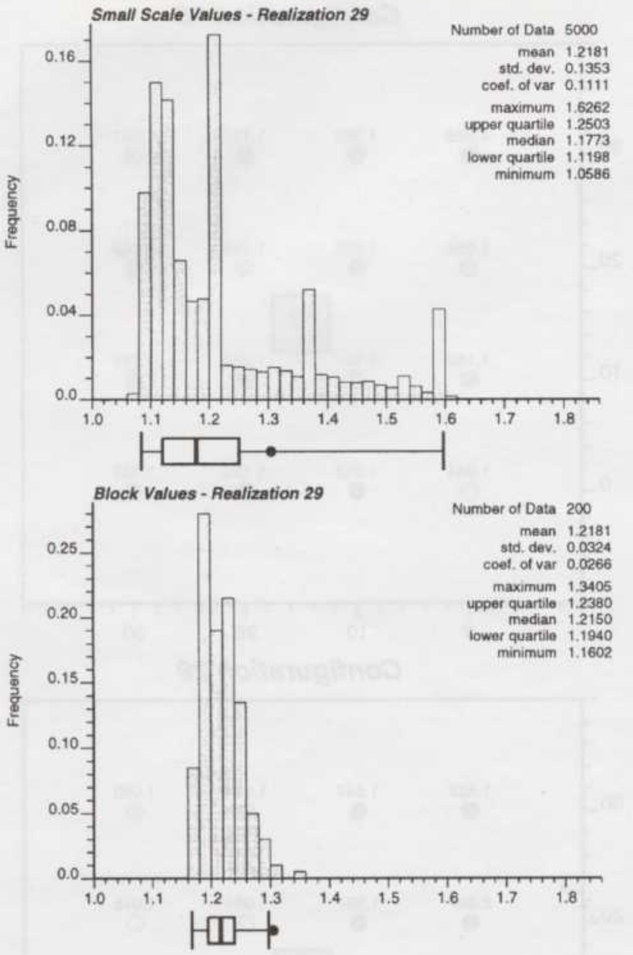

Figure 21 — Example showing "blasthole" and "block" grades for central 5- by 5-m block. The data from realization 29 were used. Note that the mean is unchanged and the classical reduction in variance from blasthole grades to block grades.

level of spatial continuity. This is an area for further research/application.

Blast movement studies are necessary to quantify and understand how the dig limits should be established after blasting. Improvements in grade-control methodology could be reduced if there is significant throw (1 to 3 m of throw could be expected). This may be acceptable provided the throw is in a single known direction; however, a general mixing would be difficult to overcome. This complicating factor would have to be handled on a case-by-case basis.

In general, there is more information for grade control than the blasthole grades on the current bench. There are geological rock types, exploration holes and blastholes from the benches above. The additional value of this information could be assessed by calculating the additional revenue using this data. Of course, it would depend on the closeness of the data.

Computer software implementing the numerical algorithms presented in this paper is not difficult. In fact, the authors will send FORTRAN code on request. There is a need, however, for the entire procedure to be automated in easy-to-use software. A familiar Windows-type interface would be logical.

Conclusions

Classifying material as ore and waste based on maximum-profit criteria generates more revenue than conventional kriging. The concepts used in this paper are well established, that is, geostatistical simulation and basic economics. Implementation is straightforward and the selection program may be run on low-level Pentium PCs available at virtually every mine site. The method is flexible in its ability to handle variability of costs, variability of metal recovery, and revenue from multiple ore minerals.

The method increases revenue by rigorous accounting of uncertainty in block grades. The effect of isolated high-grade values is minimized, the lost opportunity cost of wasted ore is taken into consideration, and the irreducible uncertainty is handled by considering the expected value of the profit if called ore (P_{ore}) and waste (P_{waste}).

Additional effort is required to apply at operating mines with easy-to-use software and that can be straightforwardly applied by the engineer/geologist/technician at the mine site.

Acknowledgements

The financial support of the Natural Sciences and Engineering Research Council of Canada (NSERC) is gratefully acknowledged.

References

David, M., 1977, *Geostatistical Ore Reserve Estimation*, Elsevier, Amsterdam.

Davis, B.M., Trimble, J., and McClure, D., 1989, "Grade control and ore selection practices at the Colosseum Gold Mine," *Mining Engineering*, August, pp. 827-830.

Deutsch, C.V., and Journel, A.G., 1992, GSLIB: *Geostatistical Software Library and User's Guide*, New York, Oxford University Press, p. 340.

Glacken, I.M., 1996, "Change of support by direct conditional block simulation," *Geostatistics Wollongong '96*, Kluwer Academic Publishers, Vol. 1, September.

Journel, A.G., and Huijbregts, Ch. J., 1978, *Mining Geostatistics*, New York, Academic Press, p. 600.

Pitard, F.F., 1993, *Pierre Gy's Sampling Theory and Sampling Practice*, New York, CRC Press, p. 488.

Raymond, G.F., 1979, "Ore estimation problems in an erratically mineralized orebody," *CIM Bulletin*, June pp. 90-98.

Snowden, V., Moore, C., and Kelly, D., 1994, "using geostatistics to assist in optimising grade control estimation at KCGM's Flimston Open Pit," *Proceedings of the 4th Large Open Pit Mining Conference*, Perth, Australia, Australasian Institute of Mining & Metallurgy, September, 1994, pp. 117-131.

Srivastava, R.M., 1987, "Minimum variance or maximum profitability?" *CIM Bulletin*, Vol. 80, No. 901, pp. 63-68.

Westley, J.F.H., 1986, "Product monitoring, grade control, and ore:waste reconciliation at Bougainville Copper Limited (BCL)," *Proceedings of the Large Open Pit Mining Conference*, Newman, WA, Australia, Australasian Institute of Mining & Metallurgy, October, pp. 277-284.