Mathematical Geology, Vol. 33, No. 4, 2001

Teacher's Aide
Variogram Interpretation and Modeling[1]

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The variogram is a critical input to geostatistical studies: (1) it is a tool to investigate and quantify the spatial variability of the phenomenon under study, and (2) most geostatistical estimation or simulation algorithms require an analytical variogram model, which they will reproduce with statistical fluctuations. In the construction of numerical models, the variogram reflects some of our understanding of the geometry and continuity of the variable, and can have a very important impact on predictions from such numerical models. The principles of variogram modeling are developed and illustrated with a number of practical examples. A three-dimensional interpretation of the variogram is necessary to fully describe geologic continuity. Directional continuity must be described simultaneously to be consistent with principles of geological deposition and for a legitimate measure of spatial variability for geostatistical modeling algorithms. Interpretation principles are discussed in detail. Variograms are modeled with particular functions for reasons of mathematical consistency. Used correctly, such variogram models account for the experimental data, geological interpretation, and analogue information. The steps in this essential data integration exercise are described in detail through the introduction of a rigorous methodology.

KEY WORDS: kriging, stochastic simulation, covariance, zonal and geometric anisotropy.

INTRODUCTION

The variogram has been used widely to quantify the spatial variability of spatial phenomena for many years; however, calculation and interpretation principles have advanced slowly. This is particularly true in the petroleum industry due to the limited number of well data. The preliminary steps of variogram calculation, interpretation, and modeling are often performed hastily or even skipped altogether. This practice should be reversed and much more attention devoted to establishing

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a robust model of spatial variability (variogram) before proceeding with building numerical geological models.

Geostatistical model-building algorithms such as sequential Gaussian simulation, sequential indicator simulation, and truncated Gaussian simulation take an input variogram model and create a three-dimensional (3D) model constrained to local data and the variogram model. The variogram has an extremely important role to play in the appearance and behavior of the resulting 3D models.

Thorough variogram interpretation and modeling are important prerequisites to 3D model building. The practice of variogram modeling and the principle of the Linear Model of Regionalization have been covered in many texts (e.g., Journel and Huijbregts, 1978; Armstrong, 1984; Cressie, 1993; Olea, 1995; Goovaerts, 1997). However, none have presented a strict and rigorous methodology to easily and systematically produce a licit and consistent 3D variogram model. We present a methodology of variogram interpretation and modeling whereby the variance is divided into a number of components and explained over different length scales in different directions.

THE VARIOGRAM

The variogram has been defined in many books and technical papers. For completeness, however, we recall the definition of the variogram and related statistics. Consider a stationary random function Y with known mean m and variance σ^2. The mean and variance are independent of location, that is, $m(\mathbf{u}) = m$ and $\sigma^2(\mathbf{u}) = \sigma^2$ for all locations $\mathbf{u}$ in the study area. Often there are areal and vertical trends in the mean m, which are handled by a deterministic modeling of the mean and working with a residual from the locally variable mean. The variogram is defined as

$$2\gamma(\mathbf{h}) = \mathrm{Var}[Y(\mathbf{u}) - Y(\mathbf{u}+\mathbf{h})] = E\left\{[Y(\mathbf{u}) - Y(\mathbf{u}+\mathbf{h})]^2\right\} \quad (1)$$

In words, the expected squared difference between two data values separated by a distance vector $\mathbf{h}$ is the variogram. The *semi*variogram $\gamma(\mathbf{h})$ is one half of the variogram $2\gamma(\mathbf{h})$. To avoid excessive jargon we simply refer to the variogram, except where mathematical rigor requires a precise definition. The variogram is a measure of variability; it increases as samples become more dissimilar. The covariance is a statistical measure that is used to measure correlation (it is a measure of similarity):

$$C(\mathbf{h}) = E\{[Y(\mathbf{u}) \cdot Y(\mathbf{u}+\mathbf{h})]\} - m^2 \quad (2)$$

By definition, the covariance at $\mathbf{h} = \mathbf{0}$, $C(\mathbf{0})$, is the variance σ^2. The covariance $C(\mathbf{h})$ is 0.0 when the values $\mathbf{h}$-apart are not linearly correlated.

Expanding the square in Equation (1) leads to the following relation between the semivariogram and covariance:

$$\gamma(\mathbf{h}) = C(\mathbf{0}) - C(\mathbf{h}) \quad \text{or} \quad C(\mathbf{h}) = C(\mathbf{0}) - \gamma(\mathbf{h}) \tag{3}$$

This relation depends on the model decision that the mean and variance are constant and independent of location. These relations are the foundation for variogram interpretation. That is, (1) the "sill" of the variogram is the variance, which is the variogram value that corresponds to zero correlation; (2) the correlation between $Y(\mathbf{u})$ and $Y(\mathbf{u}+\mathbf{h})$ is positive when the variogram value is less than the sill; and (3) the correlation between $Y(\mathbf{u})$ and $Y(\mathbf{u}+\mathbf{h})$ is negative when the variogram exceeds the sill. This is illustrated by Figure 1, which shows three **h**-scatterplots corresponding to three lags on a typical semivariogram. Geostatistical modeling generally uses the variogram instead of the covariance for mainly historical reasons. The flexibility of the variogram to handle cases of infinite variance is of little practical consequence.

A single variogram point $\gamma(\mathbf{h})$ for a particular distance and direction **h** is straightforward to interpret and understand. Practical difficulties arise from the fact that we must simultaneously consider many lag vectors **h**—that is, many distances and directions. The variogram is a measure of "geological variability" vs. distance. The "geologic variability" is quite different in different directions; for example, in sedimentary formations there is typically much greater spatial correlation in the horizontal plane.

Understanding Variogram Behavior

The link between geological variations and observed variogram behavior must be understood for reliable variogram interpretation and modeling. Figure 2(A–C) shows three geologic images and corresponding semivariograms in the vertical and horizontal directions for each image. In practice, we do not have an exhaustive image of the variable and the variogram behavior must be interpreted and related to geological principals from directional variograms. The primary variogram behaviors are as follows:

1. *Randomness or lack of spatial correlation*: Certain geological variations appear to have no spatial correlation. These random variations are the result of deterministic geological processes. At some scales, however, the processes are highly nonlinear and chaotic leading to variations that have no spatial correlation structure. Typically, only a small portion of the variability is explained by random behavior. For historical reasons, this type of variogram behavior is called the *nugget effect*. In early mining geostatistics, the presence of gold nuggets in drillhole samples would lead to apparently random variations—hence, nugget effect.

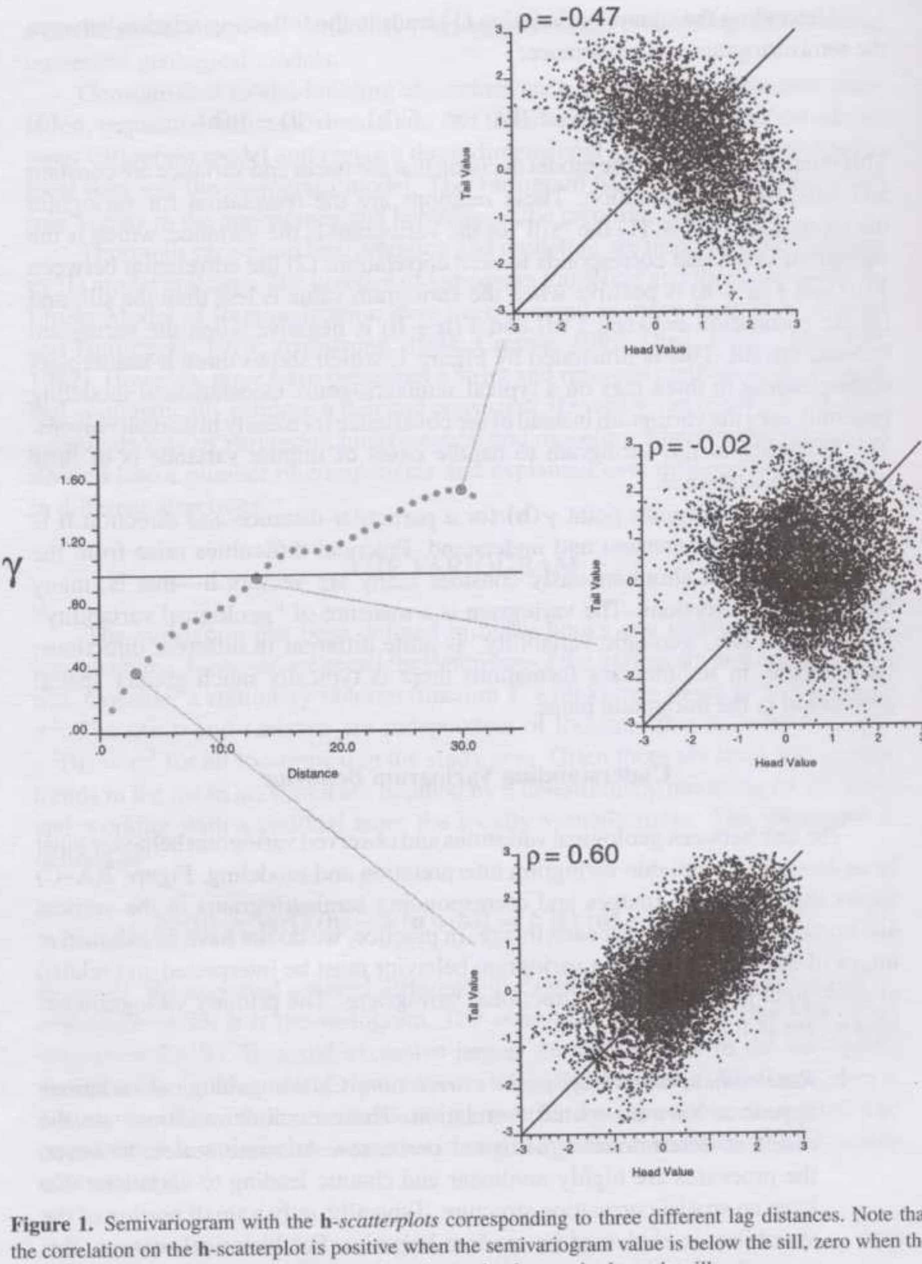
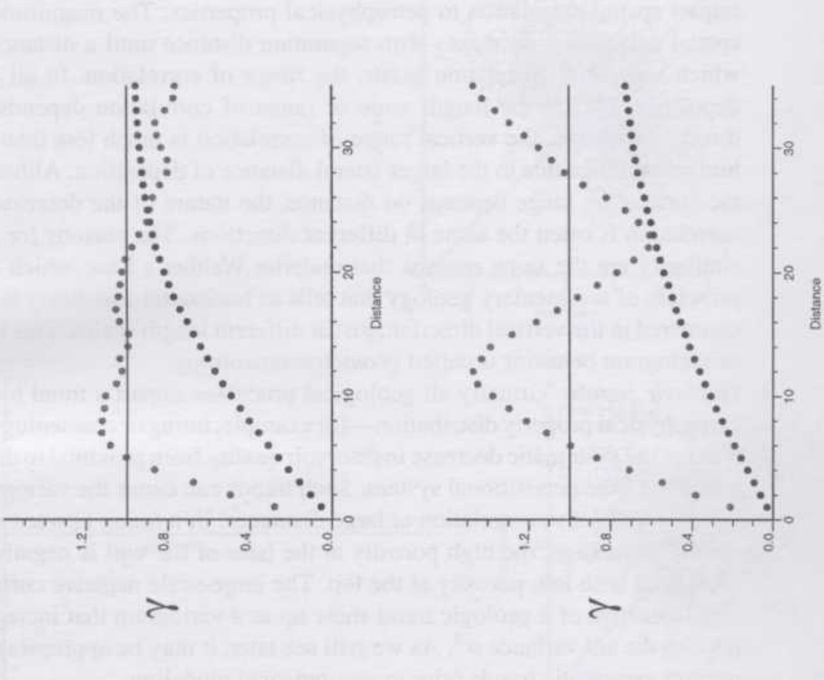


Figure 1. Semivariogram with the **h**-*scatterplots* corresponding to three different lag distances. Note that the correlation on the **h**-scatterplot is positive when the semivariogram value is below the sill, zero when the semivariogram is at the sill, and negative when the semivariogram is above the sill.

2. *Decreasing spatial correlation with distance*: Most depositional processes impart spatial correlation to petrophysical properties. The magnitude of spatial correlation decreases with separation distance until a distance at which no spatial correlation exists, the *range* of correlation. In all real depositional cases the length scale or range of correlation depends on direction—that is, the vertical range of correlation is much less than the horizontal range due to the larger lateral distance of deposition. Although the correlation range depends on distance, the nature of the decrease in correlation is often the same in different directions. The reasons for this similarity are the same reasons that underlie Walther's Law, which is a principle of sedimentary geology that tells us horizontal variability is encountered in the vertical direction; just at different length scales. This type of variogram behavior is called *geometric anisotropy*.
3. *Geologic trends*: Virtually all geological processes impart a trend in the petrophysical property distribution—for example, fining or coarsening upward or the systematic decrease in reservoir quality from proximal to distal portions of the depositional system. Such trends can cause the variogram to show a negative correlation at large distances. In a fining upward sedimentary package, the high porosity at the base of the unit is negatively correlated with low porosity at the top. The large-scale negative correlation indicative of a geologic trend show up as a variogram that increases beyond the sill variance σ^2. As we will see later, it may be appropriate to remove systematic trends prior to geostatistical modeling.
4. *Areal trends*: These have an influence on the vertical variogram—that is, the vertical variogram will not encounter the full variability of the petrophysical property. There will be positive correlation (variogram $\gamma(\mathbf{h})$ below the sill variance σ^2) for large distances in the vertical direction. This type of behavior is called *zonal anisotropy*. A schematic illustration of this is given in Figure 3.
5. *Stratigraphic layering*: There are often stratigraphic layer-like features or vertical trends that persist over the entire areal extent of the study area. These features lead to positive correlation (variogram $\gamma(\mathbf{h})$ below the sill variance σ^2) for large horizontal distances. Although large-scale geologic layers are handled explicitly in the modeling, there can exist layering and features at a smaller scale that cannot be handled conveniently by deterministic interpretation. This type of variogram behavior is also called *zonal anisotropy* because it is manifested in a directional variogram that does not reach the expected sill variance.
6. *Geologic cyclicity*: Geological phenomenon often occur repetitively over geologic time leading to repetitive or cyclic variations in the facies and petrophysical properties. This imparts a cyclic behavior to the variogram—that is, the variogram will show positive correlation going to negative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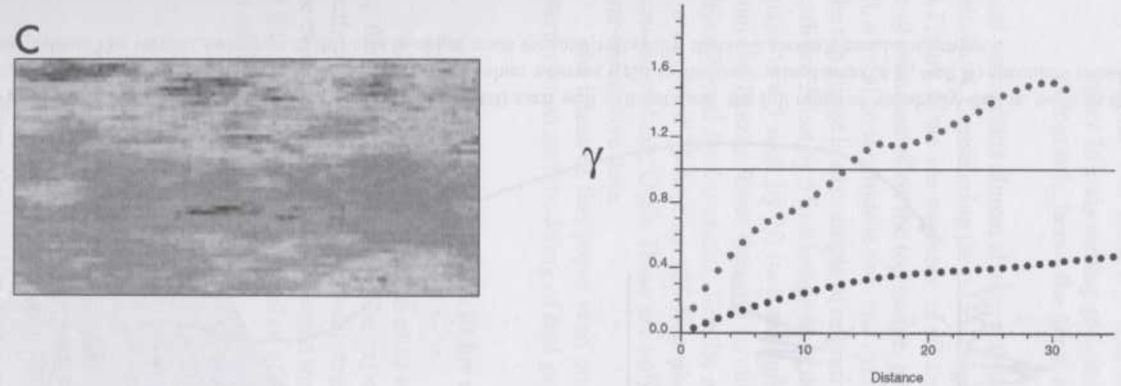


Figure 2. A, image showing geometric anisotropy with corresponding vertical (red) and horizontal (black) variograms. The image is 100 pixels by 50 pixels and the variograms were calculated to a total distance of 35 pixels in the both directions. B, image showing cyclicity in the vertical direction (red variogram) and zonal anisotropy in the horizontal direction (black variogram). The image is 100 pixels by 50 pixels and the variograms were calculated to a total distance of 35 pixels in both directions. C, image showing a trend in the vertical direction (red variogram) and zonal anisotropy in the horizontal direction (black variogram). The image is 100 pixels by 50 pixels and the variograms were calculated to a total distance of 35 pixels in both directions.

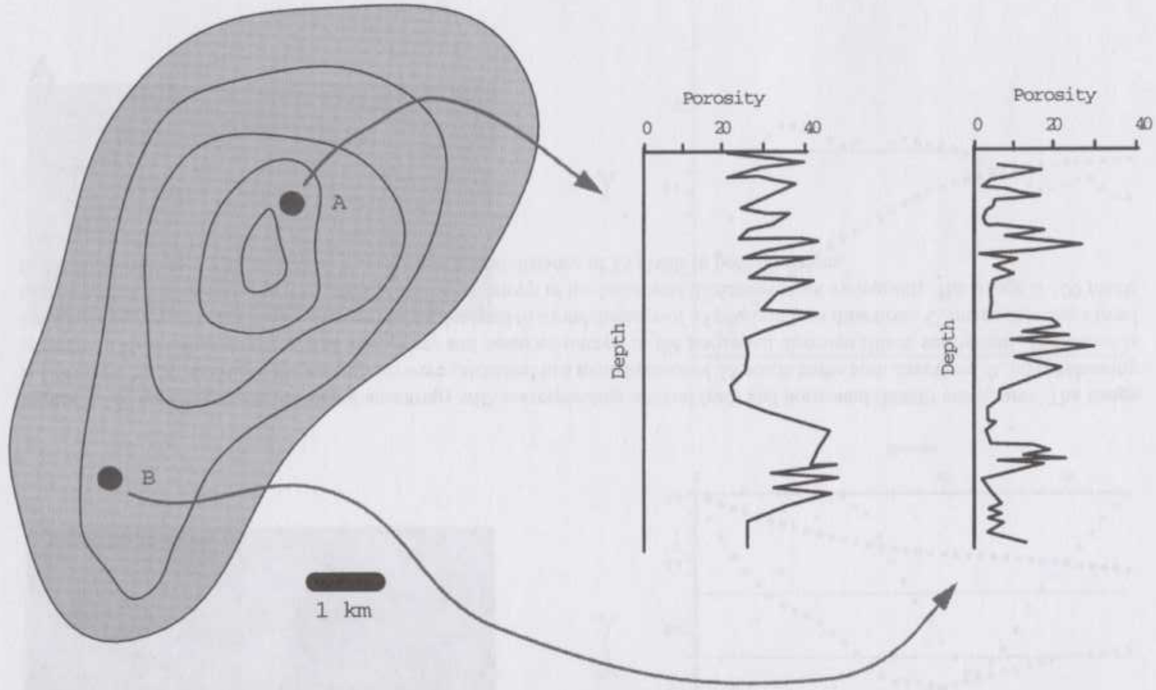


Figure 3. In presence of areal trends (illustrated at the left) each well will not "see" the full range of variability, that is, wells in the higher valued areas (e.g., well A) encounter mostly high values whereas wells in the lower valued areas (e.g., well B) encounter mostly low values. The vertical variogram in this case does not reach the total variability, that is, it shows a *zonal anisotropy*.

correlation at the length scale of the geologic cycles going to positive correlation and so on. These cyclic variations often dampen out over large distances as the size or length scale of the geologic cycles is not perfectly regular. For historical reasons, this is sometimes referred to as a *hole effect*. In early mining geostatistics, cyclicity was observed in "down-hole" variograms; hence the name *hole effect*.

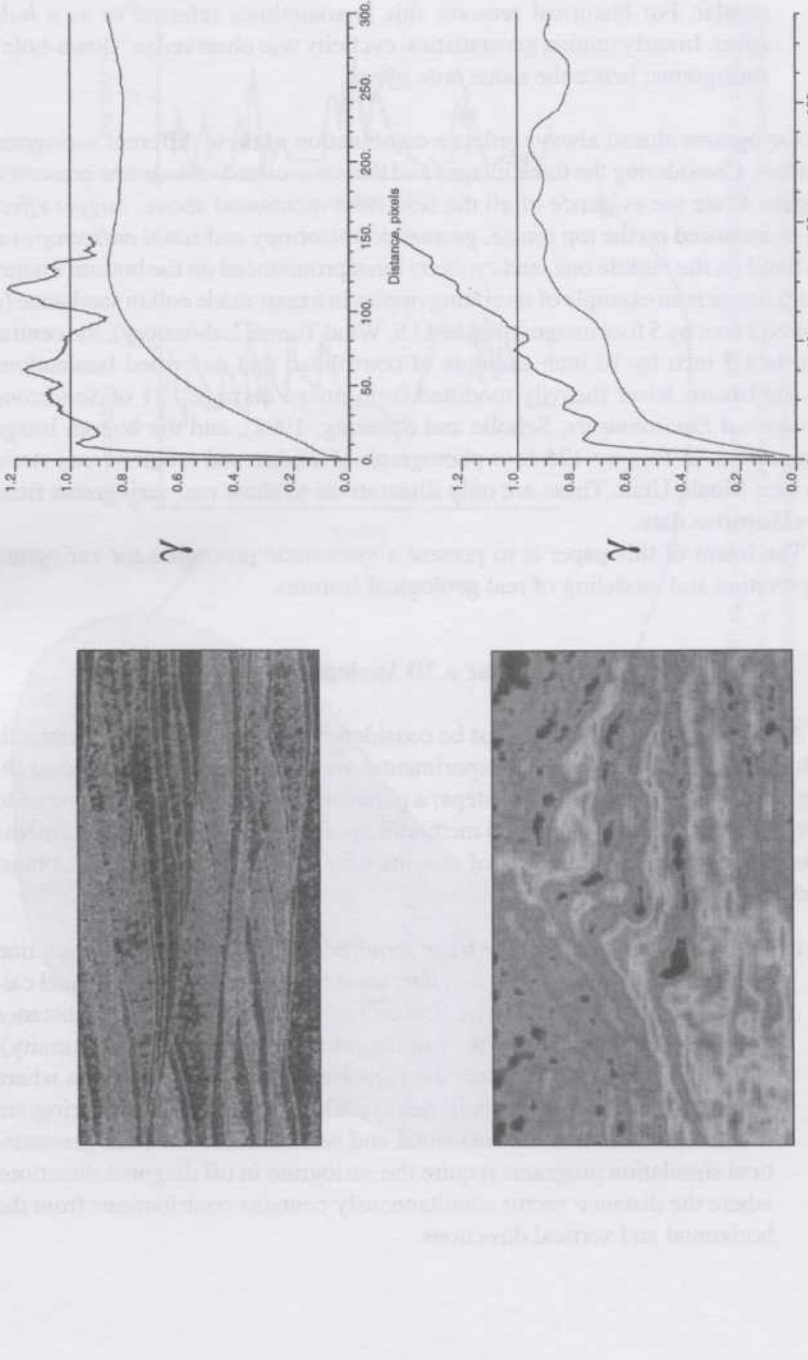
Real variograms almost always reflect a combination of these different variogram behaviors. Considering the three images and their associated variograms presented in Figure 4, we see evidence of all the behaviors mentioned above: *nugget effect* most pronounced on the top image, *geometric anisotropy* and *zonal anisotropy* on all, a trend on the middle one, and *cyclicity* most pronounced on the bottom image. The top image is an example of migrating ripples in a man-made eolian sandstone (a modified 2 foot by 5 foot image from the U.S. Wind Tunnel Laboratory), the central image is a 5 inch by 10 inch example of convoluted and deformed laminations from the Brazos River (heavily modified from image on page 131 of *Sandstone Depositional Environments*, Scholle and Spearing, 1982), and the bottom image is a modified 75 foot by 125 foot photograph of wedge and tabular cross strata from near Moab, Utah. These are only illustrations to show real variograms from near-exhaustive data.

The intent of this paper is to present a systematic procedure for variogram interpretation and modeling of real geological features.

Requirement for a 3D Variogram Model

All directional variograms must be considered simultaneously to understanding the variogram behavior. The experimental variogram points are not used directly in subsequent geostatistical steps; a parametric variogram model is fitted to the experimental points. A detailed methodology for this fitting is a central theme of this paper. There are a number of reasons why experimental variograms must be modeled:

1. The variogram function $\gamma(\mathbf{h})$ is required for all distance and direction vectors $\mathbf{h}$ within the search neighborhood of subsequent geostatistical calculations; however, we only calculate the variogram for specific distance lags and directions (often, only in the principal directions of continuity). There is a need to *interpolate* the variogram function for $\mathbf{h}$ values where too few experimental data pairs are available. In particular, the variogram is often calculated in the horizontal and vertical directions, but geostatistical simulation programs require the variogram in off diagonal directions where the distance vector simultaneously contains contributions from the horizontal and vertical directions.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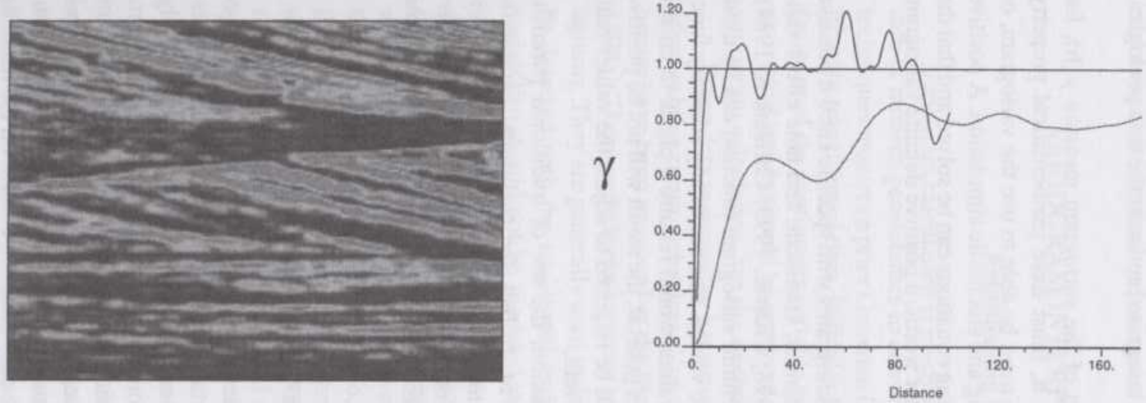

Figure 4. Three different geologic images with the corresponding directional variograms. Note the cyclicity (the undulations in both directional variograms on the bottom example), trends (particularly noticeable in the vertical direction of the middle variogram), geometric anisotropy (the shorter range of correlation in the vertical direction on all variograms), and zonal anisotropy (the horizontal variogram not raching the sill, which is most noticeable on the top variogram).

2. There is also a need to introduce geological information regarding anisotropy, trends, sampling errors, and so on in the model of spatial correlation. As much as possible, we need to filter artifacts of data spacing and data collection practices and make the variogram represent the true geological variability.
3. Finally, the covariance counterpart of the variogram measure $\gamma(\mathbf{h})$, for all distance and direction vectors $\mathbf{h}$, must have mathematical property of *positive definiteness*, that is, we must be able to use the variogram, or its covariance counterpart, in kriging and stochastic simulation. A positive definite model ensures that the kriging equations can be solved and that the kriging variance is positive—in other words, a positive definite variogram is a legitimate measure of distance.

For these reasons, geostatisticians have fit variograms with specific known positive definite functions like the spherical, exponential, Gaussian, and hole effect variogram models (Journel and Huijbregts, 1978; Cressie, 1993; Christakos, 1984). It should be mentioned that any positive definite variogram function can be used, including tabulated variogram or covariance values. The use of any arbitrary function or nonparametric table of variogram values would require a check to ensure positive definiteness (D. E. Myers, 1991). In general, the result will not be positive definite and some iterative procedure would be required to adjust the values until the requirement for positive definiteness is met.

With a "correct" variogram interpretation, the use of traditional parametric models is adequate to achieve a good fit. In fact, the traditional parametric models permit all geological information to be accounted for and realistic variogram behavior to be fit. Moreover, the use of traditional variogram models allows straightforward transfer to existing geostatistical simulation codes (Deutsch and Journel, 1997).

Importance of the Variogram in Geostatistics

The variogram is used by most geostatistical mapping and modeling algorithms. Object-based facies models and certain iterative algorithms, such as simulated annealing, do not use variograms. Not only is the variogram used extensively, it has a great effect on predictions. Occasionally there are enough data to control the appearance and behavior of the numerical models; however, these cases are infrequent and of lesser importance than the common case of sparse data control. The available data are too widely spaced to provide effective control on the numerical model. The variogram provides the only effective control on the resulting numerical models.

The lack of data, which makes the variogram important, also makes it difficult to calculate, interpret, and model a reliable variogram (Cressie and Hawkins, 1980;

Genton, 1998a). Practitioners have been aware of this problem for some time with no satisfactory solution. Variogram modeling is important and the "details" often have a crucial impact on prediction. In particular, the treatment of zonal anisotropy and systematic vertical or horizontal trends is critical.

VARIOGRAM INTERPRETATION AND MODELING

Establishing the Correct Variable

Variogram calculation is preceded by selection of the variable of interest. It is rare in modern geostatistics to consider untransformed data. The use of Gaussian techniques requires a prior Gaussian transform of the data and the variogram model of these transformed data. Indicator techniques require an indicator coding of the facies or of the continuous variable at a series of thresholds. Also, systematic areal or vertical trends should be removed from the variable prior to transformation and variogram calculation; see the next section.

When data is skewed or has extreme high or low values; estimated variograms often exhibit erratic behaviors. Various robust alternatives to the traditional variogram have been proposed in the literature (Cressie and Hawkins, 1980; Genton, 1998a). These include madograms, rodograms, general and pairwise relative variograms. They are generally used to determine ranges and anisotropy, which cannot be detected with the traditional variogram. However, these measures of spatial variability should not be modeled, as they cannot serve as input for subsequent estimation or simulation algorithms. Instead, it is recommended to transform the data to Normal space before performing variogram calculations, e.g., with the normal score transform (Deutsch and Journel, 1997). This has some important advantages: (1) the difference between extreme values is dampened and (2) the theoretical sill is known to be 1. Also, some algorithms (e.g., p-fields) may require that variogram calculations be performed on a Uniform score transformation of the data. However, the Uniform score and Normal score variograms are generally so similar that the latter can be used most of the time.

Removing the Trend

As mentioned above, the first important step in all geostatistical modeling exercises is to establish the correct property to model and to make sure (inasmuch as it is possible) that this property is stationary over the domain of the study. Indeed, if the data shows a systematic trend, this trend must be modeled and removed before variogram modeling and geostatistical simulation. Variogram analysis and all subsequent estimations or simulations are performed on the residuals. The trend is added back to estimated or simulated values at the end of the study.

There are problems associated with defining a reasonable trend model and removing the deterministic portion of the trend; however, it is essential to consider deterministic features such as trends deterministically. The presence of a significant trend makes the variable nonstationary, that is, it is unreasonable to expect the mean value to be independent of location. Residuals from some simple trend model are easier to consider stationary. Removing a trend by estimaiton from the data themselves can introduce a bias; however, this bias is considered less significant than the errors introduced by leaving the trend alone.

Trends in the data can be identified from the experimental variogram, which keeps increasing above the theoretical sill, see earlier discussion. In simple terms, this means that as distances between data pairs increase the differences between data values also systematically increase.

To illustrate the above, consider the porosity data shown in Figure 5, which clearly exhibits a trend in the porosity profile along the well. Porosity increases with depth due to a fining-upward of the sand sequence. The (normal-score) variogram corresponding to this porosity data is shown in Figure 6. It shows a systematic increase well above the theoretical sill of 1. One could fit a "power" or "fractal" variogram model to the experimental variogram, however, since these models do not have a sill value (it is infinite), they cannot be used in simulation algorithms

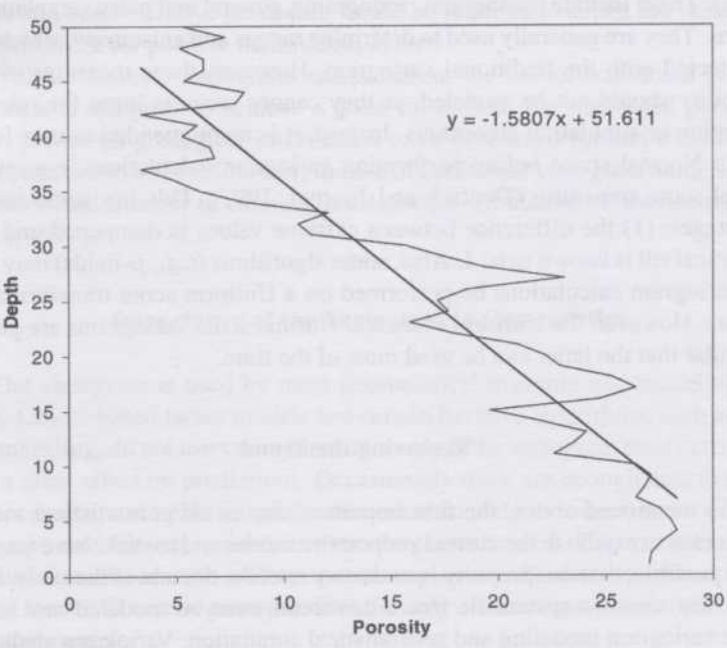

Figure 5. Porosity profile along a vertical well with a clear vertical trend. A linear trend model is fitted to the data (solid line).

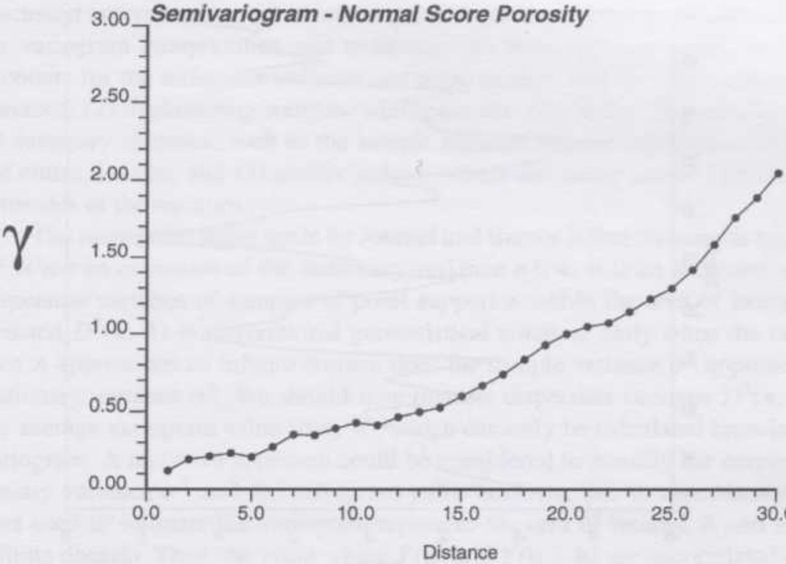

Figure 6. Variogram of the normal score transform of the porosity values shown in Figure 5. The vertical trend in porosity reveals itself as a continuous increase in the variogram above the sill value of 1.0.

such as sequential Gaussian simulation. But above all, they are not representative of the property of interest.

A linear trend was fitted to the porosity profile (see Figure 5) and then removed from the data. The resulting residuals constitute the new property of interest and their profile is shown in Figure 7. The (normal-score) variogram of the residuals is shown in Figure 8, which now exhibits a clearer structure reaching the theoretical sill of 1 at about 7 distance units.

Variance for Variogram Interpretation

There is often confusion about the correct variance to use for variogram interpretation. It is important to have the variance σ^2, or $C(\mathbf{0})$ value, to correctly interpret positive and negative correlation. Recall that a semivariogram value $\gamma(\mathbf{h})$ above the sill variance implies negative correlation between $Y(\mathbf{u})$ and $Y(\mathbf{u}+\mathbf{h})$, whereas a semivariogram value $\gamma(\mathbf{h})$ below the sill implies positive correlation. There has been some discussion in the literature about the correct variance to use for the sill variance and for variogram interpretation. This discussion was summarized by Goovaerts (1997, page 103), who references Journel and Huijbregts (1978, page 67) and the article by Barnes (1991). There are three issues that must be

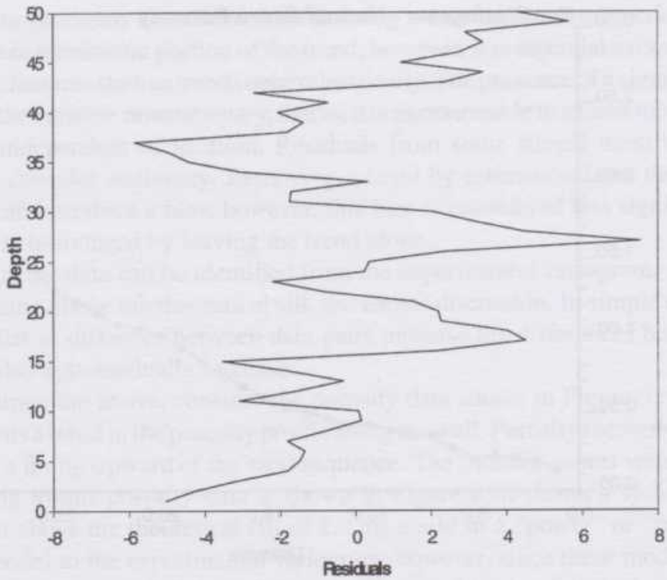

Figure 7. Vertical profile of the residual porosity values (after removal of the linear trend).

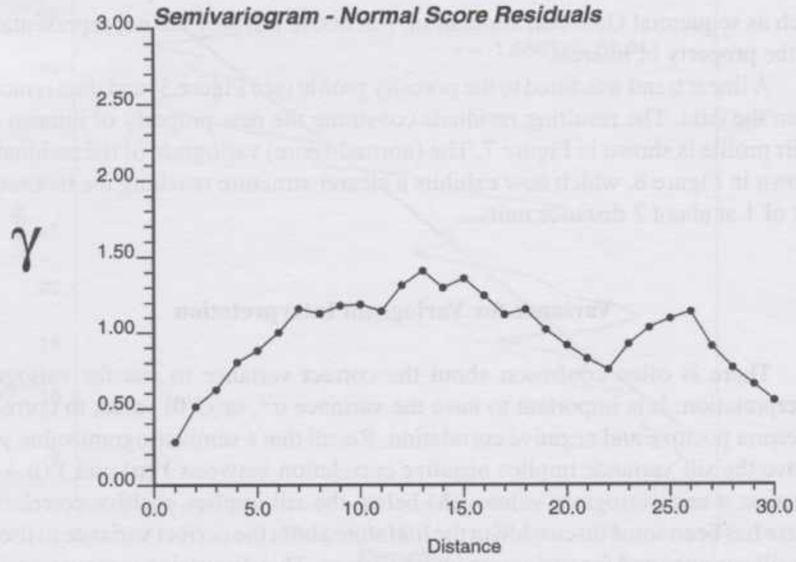

Figure 8. Variogram of the normal score transform of the residual of porosity values shown above. The variogram reaches the expected sill value of 1.0.

discussed before making a recommendation regarding the correct variance to use for variogram interpretation and modeling: (1) the *dispersion variance*, which accounts for the difference between our finite domain and the infinite stationary variance; (2) *declustering* weights, which account for the fact that our data, and all summary statistics, such as the sample variance $\hat{\sigma}^2$, are not representative of the entire domain, and (3) *outlier values*, which can cause erratic and unstable estimates of the variance.

The main point being made by Journel and Barnes is that the sample variance $\hat{\sigma}^2$ is *not* an estimator of the stationary variance σ^2, $\bullet$, it is an estimator of the dispersion variance of samples of point support $\bullet$ within the area of interest A, denoted $D^2(\bullet, A)$ in conventional geostatistical notation. Only when the sample area A approaches an infinite domain does the sample variance $\hat{\sigma}^2$ approach the stationary variance σ^2. We should note that the dispersion variance $D^2(\bullet, A)$ is the average variogram value $\bar{\gamma}(A, A)$, which can only be calculated knowing the variogram. A recursive approach could be considered to identify the *correct* stationary variance σ^2 and the variogram $\gamma(\mathbf{h})$; however, this is unnecessary. The data used to estimate the variogram represent the area of interest A and not an infinite domain. Thus, the point where $Y(\mathbf{u})$ and $Y(\mathbf{u}+\mathbf{h})$ are uncorrelated is the dispersion variance $D^2(\bullet, A)$. In other words, we should use the sample variance as the sill of the sample semivariogram and acknowledge that, in all rigor, it is a particular dispersion variance.

The second issue relates to the use of the naïve sample variance or the sample variance accounting for declustering weights. The use of declustering weights is very important to ascertain a reliable histogram, mean, variance, and other summary statistics. Although the use of declustering weights is important, they are not used in the calculation of the variogram, that is, there are more experimental pairs in areas of greater sampling density. Therefore, the sill (or semivariogram value corresponding to zero correlation) is reached at the naïve sample variance. Efforts have been made (Omre, 1984) to incorporate declustering weights into the variogram calculation; however, they provide no better variogram and are difficult to implement in practice. As a result, declustering weights should not be used in data transformation or variance calculation for variogram calculation, interpretation, and modeling.

The last issue that must be addressed is the influence of outlier sample values. It is well known in statistics that the variance, being a squared statistic, is sensitive to outlier values. For this reason, the sample variance may be unreliable. We should note, however, that this is not a problem with transformed data; the Gaussian and indicator transform remove the sensitivity to outlier data values. The resulting Gaussian distribution has no outliers by construction and there are only 0's and 1's after indicator transformation.

The correct variance for variogram interpretation is the naï ve equal-weighted variance.

METHODOLOGY FOR VARIOGRAM INTERPRETATION AND MODELING

The methodology advocated in this paper is classical in that it assumes the regionalized variable is made up of a sum of independent random variables. Each constituent random variable has its own variogram structure. The component variogram structures may be added arithmetically to create a complete 3D variogram model to be used for geostatistical algorithms. Since each variogram structure corresponds to a specific underlying geological phenomenon, the actual modeling phase (traditional curve fitting exercise) is preceded by a necessary *interpretation* stage.

In an approach similar to the well-established model identification part of well test interpretation (Gringarten, 1986), where the pressure response is partitioned into different time regions, the total variance of the phenomenon under study is divided into *variance regions*. The behavior of the variogram in each region is then identified as being of a specific *structure type*. This is analogous to the *model recognition* step of well test analysis where the pressure-derivative signature of each time region is associated to a specific flow regime. Well test analysis defines three time regions: (1) early time corresponding to near-wellbore effects (e.g., wellbore storage, hydraulic fractures); (2) middle time characterized by the basic reservoir behavior (e.g., single or double porosity, composite); and (3) late time accounting for reservoir boundaries (e.g., faults, no-flow, or constant pressure boundaries). Similarly, three major variance regions can be defined for variogram analysis: (1) short-scale variance (*nugget effect*); (2) intermediate-scale variance (geometric anisotropy—there may exist more than one such structure); and (3) large-scale variance (zonal anisotropy, hole effect). The methodology proceeds sequentially, identifying first the short-scale variance, then intermediate-scale structures, and finally large-scale features. For the variogram interpretation to be consistent in 3D, all structures contributing to the total variance must exist in all directions and their variance contributions must be equal.

The steps of the methodology are as follows:

1. Compute and plot experimental variograms in what are believed to be the principal directions of continuity based on *a priori* geological knowledge (variogram calculation is not covered in this paper; see Deutsch and Journel, 1997). If geological information is ambiguous, one can use 2D variogram maps to determine major horizontal directions of continuity [A variogram map is a plot of experimental variogram values in a coordinate system (h_x, h_y) with the center of the map corresponding to the variogram at lag 0.0 (Goovaerts, 1997)]. Consider the horizontal and vertical experimental variograms shown in Figure 9A.

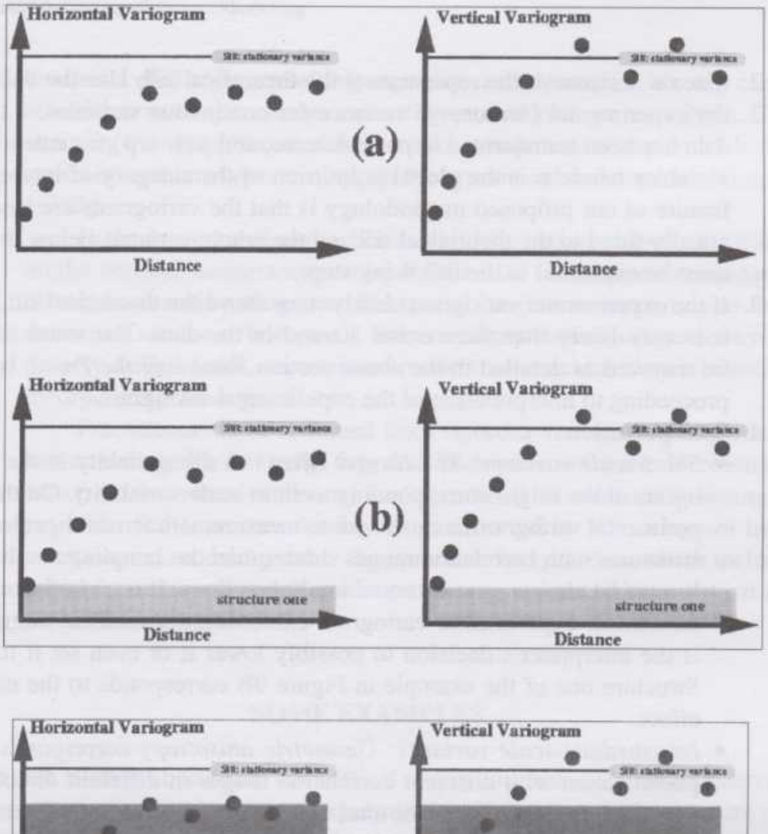

Figure 9. A, experimental horizontal and vertical variograms; B, structure one represents a nugget effect; C, structure two represents geometric anisotropy; D, structure three represents zonal anisotropy.

2. Place a horizontal line representing the theoretical sill. Use the value of the experimental (stationary) variance for continuous variables, 1 if the data has been transformed to normal score, and $p(1-p)$ for categorical variables where p is the global proportion of the category of interest. A feature of our proposed methodology is that the variograms are systematically fitted to the theoretical sill and the whole variance below the sill must be explained in the following steps.
3. If the experimental variogram clearly rises above the theoretical sill, then it is very likely that there exists a trend in the data. The trend should be removed as detailed in the above section *Removing the Trend*, before proceeding to interpretation of the experimental variogram.
4. **Interpretation**
   - *Short-scale variance*: The *Nugget effect* is a discontinuity in the variogram at the origin corresponding to short scale variability. On the experimental variogram, it can be due to measurement errors or geological structures with correlation ranges shorter than the sampling resolution. It must be chosen as to be equal in all directions. It is picked from the directional experimental variogram exhibiting the smallest nugget. It is the interpreter's decision to possibly lower it or even set it to 0.0. Structure one of the example in Figure 9B corresponds to the nugget effect.
   - *Intermediate-scale variance*: *Geometric anisotropy* corresponds to a phenomenon with different correlation ranges in different directions. Each direction encounters the total variability of the structure. There may exist more than one such variance structure. Structure two in Figure 9C represents geometric anisotropy with longest correlation range in the horizontal direction.
   - *Large-scale variance*: (1) *Zonal anisotropy* is characterized by directional variograms reaching a plateau at a variance lower than the theoretical sill, i.e., the whole variability of the phenomenon is not visible in those directions. Structure three in Figure 9D corresponds to zonal anisotropy, only the vertical direction contributes to the total variability of the phenomenon at that scale; (2) *hole effect* is representative of a "periodic" phenomenon (*cyclicity*) and characterized by undulations on the variogram. The hole effect does not actually contribute to the total variance of the phenomena; however, its amplitude and frequency must be identified during the interpretation procedure, also, it can only exist in one direction.
5. **Modeling**
   Once all the variance regions have been explained and each structure has been related to a known geological process, one may proceed to variogram modeling by selecting a licit model type (spherical, exponential,

Gaussian, etc.) and correlation ranges for each structure. This step can be referred to as the *parameter estimation* part of variogram analysis. Constraining the variogram model by a prior interpretation step with identification of structure types can help fit the experimental variograms (Genton, 1998b).

In the presence of sparse horizontal data, variance structures visible on the vertical variogram must be forced onto often nonexistent experimental horizontal variograms. Horizontal ranges corresponding to these structures are then "borrowed" from ancillary data (analogue outcrops, densely drilled fields, depositional models, seismic), as shown in section Incorporating Analogue Data.

Practitioners have refrained from rigorous variogram interpretation and modeling due to sparse data and inadequate software. Increasingly, data from horizontal wells and analogue fields or outcrops is becoming available. Software can also be designed to aid in 3D variogram interpretation and modeling rather than promote bad practice, which includes misinterpretation of trends, zonal anisotropy, and not linking vertical and horizontal variograms.

SOME EXAMPLES

Figure 10 shows horizontal and vertical variograms from a Canadian reservoir. The variogram was fitted with two exponential models (short-scale structure and long range structure) and a dampened hole-effect model in the vertical direction. Note that the good horizontal variogram is due to the availability of horizontal wells.

Five variance regions were used for the horizontal and vertical variogram of Figure 10 (Table 1). The first small component is an isotropic nugget effect. The next three are exponential variogram structures with different range parameters. Three exponential structures are required to capture the inflection point at a variance value of about 0.3 on the vertical variogram, and the long range structure in the vertical variogram in the variance region 0.8 to 1.0. The fifth dampened hole effect variogram structure only applies in the vertical direction and adds no net contribution to the variance. Note that the dampening factor is five times the range.

Figure 11 shows horizontal and vertical variograms for the Amoco data, which was made available to the Stanford Center for Reservoir Forecasting for testing geostatistical algorithms (Journel, 1991). Note the zonal anisotropy in the vertical directions due to systematic areal variations in the average porosity.

Three variance regions were for the horizontal and vertical variogram of Figure 11 (Table 2). The first two components are anisotropic spherical variogram structures. The last variogram captures the zonal anisotropy in the vertical direction.

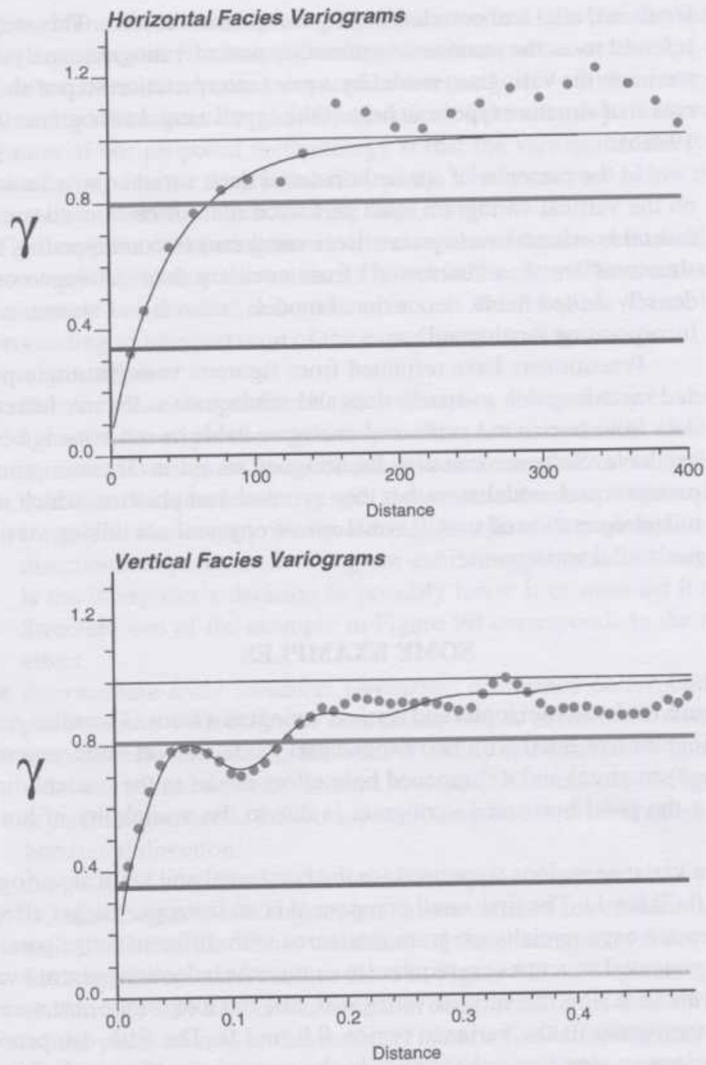

Figure 10. Horizontal and vertical variogram fitted with a combination of exponential and dampened hole effect variograms. Calculated from a Canadian reservoir.

Figure 12 shows facies (presence of limestone coded as 1, dolomite and anhydrite coded as zero) variograms calculated from a major Saudi Arabian reservoir; see Benkendorfer and others (1995) for the original variograms. Note the interpretable horizontal variograms and the consistent vertical and horizontal variograms.

Table 1. Parameters for Five Variance Regions of Variogram on Figure 10[a]

Variance contribution	Type of variogram	Horizontal range, m	Vertical range, m
0.05	Nugget		
0.29	Exponential	100.0	0.015
0.46	Exponential	175.0	0.450
0.20	Exponential	100.0	0.500
0.20	Dampened Hole Effect		0.060

[a]The variance contribution, a variogram type, a range in the horizontal direction, and a vertical range specify each variance region. No horizontal anisotropy is considered.

Two variance regions were identified for the facies variogram on Figure 12 (Table 3). Note that the sill in this case is 0.24 (related to the relative proportion of limestone to dolomite). Both are anisotropic exponential variograms.

SOFTWARE IMPLICATIONS

The methodology and examples presented above have a number of implications on software design for variogram calculation, interpretation, and modeling. Firstly, it is evident that multiple directions must be considered simultaneously. It is poor practice to consider each direction independently and attempt to merge one-dimensional variograms after modeling. Variance contributions identified in one direction must automatically persist in all directions.

One can imagine a semiautomatic fitting procedure whereby once the total variance has been divided into different nested structures, the parameters of the nested structures (i.e., model type and ranges) are estimated with some type of constrained optimization procedure. Provided the contribution of all structures is consistent in all three major directions, this is all that is required to construct a licit 3D-variogram model. A completely automatic variogram-fitting algorithm, even if it generates consistent 3D models, can potentially lead to nongeological models. In addition, it would not allow the incorporation of external information in the presence of sparse data.

Table 2. Parameters for Three Variance Regions of Variogram on Figure 11[a]

Variance contribution	Type of variogram	Horizontal range, ft	Vertical range, ft
0.50	Spherical	750.0	6.0
0.40	Spherical	2000.0	50.0
0.10	Spherical	7000.0	∞

[a]The variance contribution, a variogram type, a range in the horizontal direction, and a vertical range specify each variance region. No horizontal anisotropy is considered.

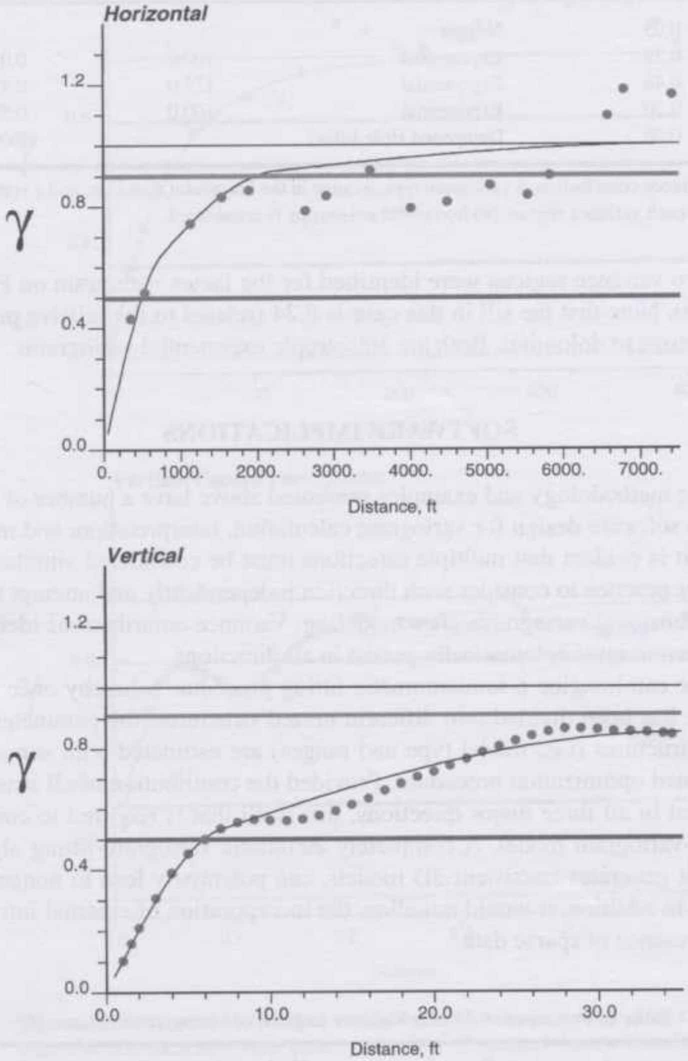

Figure 11. Horizontal and vertical variogram fitted with a combination three spherical variograms. These variograms were calculated from the "Amoco" data, made available for testing geostatistical algorithms. Note the zonal anisotropy evident in the vertical direction and the trend in the horizontal variogram.

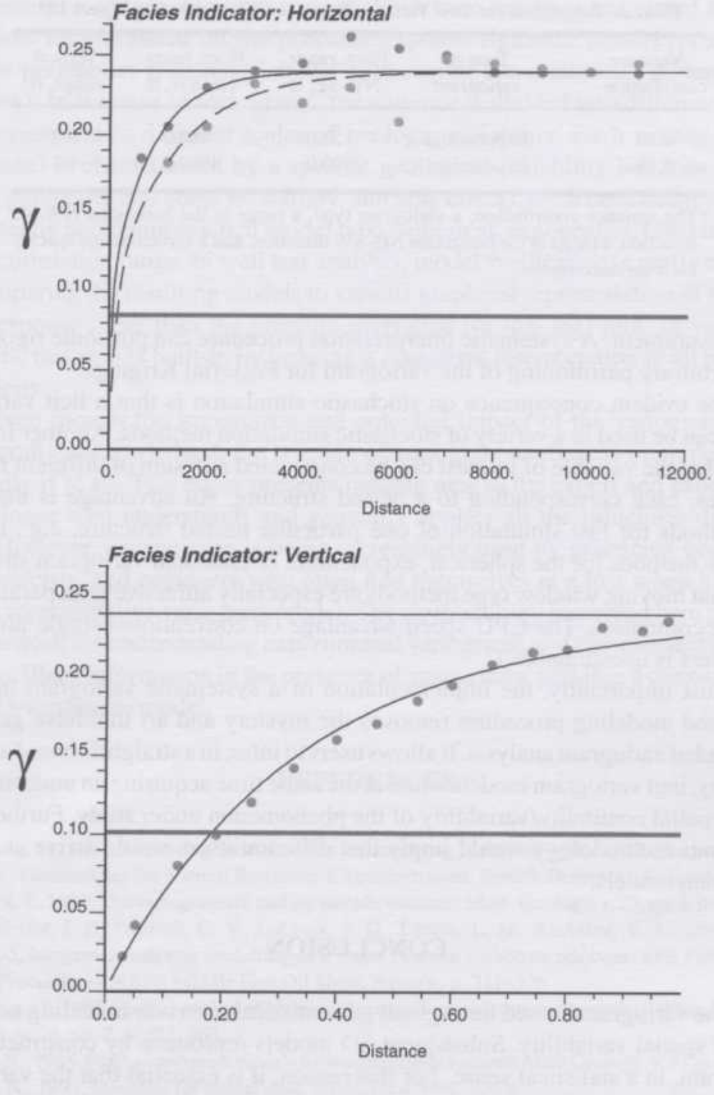

Figure 12. Horizontal and vertical facies variogram for a major Arabian carbonate reservoir (SPE 29869). There are two horizontal variograms on the upper figure; the dashed line is in the NE–SW direction and the solid line is in the NW–SE direction.

Kriging-based softwares that make use of the variogram models proposed here should be able to isolate or filter any particular nested structure. This could be of some advantage when considering, for example, seismic data, which is a low-pass filter; high frequency or short-scale components can be filtered out in

Table 3. Parameters for Two Variance Regions of Variogram on Figure 12[a]

Variance contribution	Type of variogram	Horz. range, NW-SE, ft	Horz. range, NE-SW, ft	Vertical range, ft
0.10	Exponential	150.0	400.0	0.8
0.14	Exponential	2500.0	4000.0	1.2

[a]The variance contribution, a variogram type, a range in the horizontal NW-SE direction, a range in the horizontal NE–SW direction, and a vertical range specify each variance region.

the measurement. A systematic interpretation procedure can put some rigor in the often arbitrary partitioning of the variogram for Factorial Kriging.

The evident consequence on stochastic simulation is that a licit variogram model can be used in a variety of stochastic simulation methods. Another implication is that the variable of interest can be constructed as a sum of different random variables, each corresponding to a nested structure. An advantage is that there are methods for fast simulation of one particular nested structure, e.g., moving window methods for the spherical, exponential, or Gaussian variogram structures (note that moving window type methods are especially attractive with parallel processing computers. The CPU speed advantage on conventional single processor computers is questionable.).

Most importantly, the implementation of a systematic variogram interpretation and modeling procedure removes the mystery and art that have generally surrounded variogram analysis. It allows users to infer, in a straightforward and reliable way, licit variogram models while at the same time acquiring an understanding of the spatial continuity/variability of the phenomenon under study. Furthermore, a rigorous methodology would imply that different users would arrive at similar variogram models.

CONCLUSION

The variogram is used throughout geostatistical reservoir modeling as a measure of spatial variability. Subsequent 3D models reproduce by construction the variogram, in a statistical sense. For this reason, it is essential that the variogram be representative of the true heterogeneity present in study area. It represents the modeler's quantitative understanding of the spatial variability of the property of interest given the data, and any related additional geological and geophysical information that may be available.

The interpretation methods presented in this paper are reminiscent of the revolution in well testing that came about with the pressure derivative and the development of a rigorous analysis methodology based on the principles of *model identification* and *model verification*. Model identification consists of partitioning the pressure response into time regions. Each time region is associated to a specific flow regime based on the pressure response signature (*model recognition*) and the parameters required to model each regime are evaluated (*parameter estimation*). In the case of variograms, the variance is divided into different regions that correspond to different scales of geologic variability. Each variance region (structure) is characterized by a specific geological variability behavior (nugget effect, geometric and zonal anisotropy, and hole effect). Each behavior is modeled analytically and requires a licit model type (spherical, exponential, Gaussian, etc.) and a correlation range. In well test analysis, model verification is partly achieved by comparing the resulting models to various graphical representation of the pressure response other than the pressure derivative (or log–log) plot. In variogram analysis, this step is built-in by insuring a consistent interpretation in all principle directions.

Even though the importance and potential impact of the variogram model is generally acknowledged, the practice of variogram analysis is often done half-heartedly if at all. This paper presents nothing new to the expert and experienced practitioner who understands and generally applies all the principles discussed here. However, geostatistics is being increasingly used by practicing geologists, geophysicists, and engineers who often find themselves at a loss when having to infer a representative variogram model. The methodology presented here provides a framework for understanding experimental variograms and complementing them with ancillary information in the presence of sparse data, yielding a consistent and licit 3D-variogram model.

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