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Power Averaging for Block Effective Permeability

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ABSTRACT

A binary type permeability distribution with spatial autocorrelation is introduced to model the transition between shales and sand in a reservoir. The contrast between the two modal permeability values can be made realistically high, and the autocorrelation ranges can be made realistically large and anisotropic. A steady state, single phase flow simulation is run over a network whose block grids are informed from the previous permeability distribution. The resulting network effective permeabilities are plotted vs. the shale proportion and show that a power averaging process would yield a good estimate much more accurate than either the arithmetic average (power 1) or geometric average (power 0) traditionally used. Connections with percolation theory results are indicated.

INTRODUCTION

One of the most pervasive problems in the description of an heterogeneous medium is the problem of averaging from one scale to another. A medium property is observed at one scale on a particular support (volume) of measurement, but the value of that property "averaged" over a different volume size at a different location is needed.

By "average" it is meant the unique value of the latter volume that could replace the set of all smaller measurements that could be obtained and processed within it, were there no limitations of resolution, money, and/or CPU time.

For example, actual measurements can be performed on a small support such as permeabilities on core plugs and the effective permeability of a simulation block is required.

If the averaging process of the particular variable under study were known the problem would be much alleviated. For example, the average porosity of a volume is simply the arithmetic average of the porosities of all the samples that constitute it. The same arithmetic averaging process holds true for additive variables such as saturations and more generally grades or volume/weight percentages of various phases.

Unfortunately, many other medium characteristics are not additive, i.e. the corresponding averaging process is not a mere arithmetic average and, worse, most often it is unknown. For example, the average or effective permeability of a block remains unknown even if all the thousands of core plugs that constitute it were accessible for analysis: it is neither the arithmetic average nor the geometric average of these core permeabilities, although in many situations practice has adopted the second averaging process.

Compounding the problem is the fact that not all plugs (or supports) constituting the block are available for measurements. This second problem calls for interpolation of the unknown plug values from neighboring known values. Unfortunately, interpolating values before knowing how they average is like putting the cart before the horse and it does not matter how fancy the cart is, whether called kriging or splines. The key problem is the averaging process not the choice of the interpolation process.

Another word of caution should be addressed to the people in charge of data gathering, geologists and geophysicists. The goals of planning and monitoring production are in many ways fundamentally different from those of exploration. Qualitative and causal description in geological jargon of the various scales of heterogeneities of a medium is of little help to the simulation engineer: he is more interested in the consequences of the geological processes than in their origins, and he needs numbers (at present deterministic) at actual locations and at the scale relevant to his monitoring tools. One should not be surprised if petroleum engineers tend to generate their own data through well-testing and history matching, if the information they get from upstream are geological arguments about what happened millions of years ago, and high-tech well logs that go into the atomic structure of the well walls with no indication how that information may affect the megascopic properties of a simulation block.

If traditional geologists are not all aware of the key importance of quantification when dealing with scales of heterogeneities and their averaging, mathematicians tend to overlook the actual media conditions when setting working hypotheses. Typically, numerical approaches to effective medium properties rely on such hypotheses as:

- Small spatial variability with no spatial dependence
- Unbounded, statistically isotropic medium
- Specific, continuous and normal-related distributions such as the lognormal distribution for permeability
- Whenever spatial correlation is considered, the corresponding range is unique and finite; most often the range is considered very small with regard to the averaging dimensions which essentially amounts to a no-correlation hypothesis
- Random spherical inclusions in a uniform matrix

etc.

Notwithstanding the importance of these researches and their outstanding feature of being quantitative, they have yet failed to produce usable results, at least in the oil industry and more particularly for effective permeability. This should not come as a surprise: permeability fields are known to be

- Highly variant: core plugs permeability can vary by several order of magnitudes from impermeable shales to open fractures
- Highly anisotropic: shales intrusions are not spherical
- Highly spatially correlated with nested ranges of correlation covering multiple scales of variability; more likely the process has no finite range without necessarily being fractal, i.e. self-repetitive at all scales,

etc.

Actual use of geological information in reservoir description and monitoring requires an understanding and numerical modelling of the scale problem and related averaging processes. The model(s) should feature practicality rather than mathematical tractability and thus may require such engineering tools as simulations rather than analytical perturbation-type developments.

An Empirical Approach

Rather than trying to analytically derive the averaging process, we will observe it through repeated flow simulations performed on realizations of permeability fields. This forward modelling, common to petroleum engineering practice, allows consideration of any permeability field, not restricted by such hypothesis as continuity or small variance.

Before getting into more sophisticated recovery processes with multiphases flow, we will consider a single phase, steady state, flow equation solved by finite difference methods at each grid block, see Desbarats (1986). Because of hardware limitations, the system was limited to a maximum of 500 grid blocks. Constant pressure boundaries were imposed in the direction of flow, and no-flow boundaries were specified in the two other directions perpendicular to flow. The effective permeability, denoted K_e, of the network is obtained by dividing the total volumetric flux across a section perpendicular to flow by the imposed pressure gradient. This flow simulation was repeated for each realization of the grid block permeability field, yielding each time an effective K_e. The relations of K_e with the characteristics of the input (grid block) permeability field are observed.

The Input Permeability Field

Experience and previous simulation works in shaly reservoirs have shown that the recovery, hence the flow conditions do not depend on the fine details of the permeability spatial distribution, but rather on the spatial connectivity of the extreme permeability values, either low such as impervious shale barriers, or high such as open fractures, ref. Haldorsen and Chang (1985).

In other words, and in first approximation for recovery purposes in single phase steady state flow conditions, the permeability field can be approximated by its two extreme modes, K_{shale} for the essentially impervious phase and K_{sand} for the much coarser grained rock. Thus, the permeability distribution considered hereafter is binary and characterized by the 3 parameters:

$$K_{sh} << K_{ss}$$

$$p \in [0,1] : \text{ volumetric proportion of shales.} \quad (1)$$

However, these two modes are not distributed at random in the space. Rather their spatial distribution obeys the following bivariate distribution:

$$\text{Prob}\{x \in \text{shale}, x+h \in \text{shale}\} = S(h), \quad (2)$$

S(h) being a given function of the 3-dimensional interdistance vector h.

REMARKS

The function S(h) fully characterizes the bivariate spatial distribution of the binary permeability field, in particular:

$$\text{Prob}\{x \in \text{sand}, x+h \in \text{sand}\} = 1 - 2p + S(h)$$

$$\text{Prob}\{x \in \text{sand}, x+h \in \text{shale}\} = p - S(h) \quad (3)$$

The random function permeability K(x) is stationary with moments:

$$E\{K(x)\} = p\,K_{sh} + (1-p)\,K_{ss}$$

$$\mathrm{Var}\{K(x)\} = \left[K_{ss} - K_{sh}\right]^2 \cdot p(1-p)$$

Correlation between K(x), K(x + h) is:

$$\rho(h) = \left[S(h) - p^2\right]/p(1-p).$$

However, the function S(h) does not inform high order connectivity characteristics such as the trivariate probability:

$$\mathrm{Prob}\{x \in \text{shale},\ x+h \in \text{shale},\ x+2h \in \text{shale}\}.$$

Several techniques exist to generate realizations of a binary random field with a given univariate and bivariate spatial distribution, c.f. relations (1) and (2). The technique used here is that of indicator simulation, described in Journel and Isaaks (1984), whereby an appropriate threshold is applied on an appropriate gaussian field generating two spatially correlated phases called sand (above threshold) and shale (below threshold).

If 500 grid blocks were retained for the flow simulation, 500 binary-type permeability values were generated for each simulation.

The study consists in varying the input statistics: p, S(h) and the contrast K_{ss}/K_{sh}, and observe the resulting network effective permeabilities K_e.

- A dispersion of shale and sandstone permeability values can easily be generated around their mode values K_{sh} and K_{ss}, thus generating a continuous bimodal distribution. However, the binary distribution allows studying the impact of the sole transition shale-sandstone, low-high, independently of other and minor sources of variability.

This binary model, although still limited and provisional, goes a long way beyond the limitations of previously used stochastic models, such as small contrasts and no spatial correlation. In the present study contrasts K_{ss}/K_{sh} up to 10^6 and correlation ranges from nought to several times the network dimensions were considered.

SOME RESULTS

In the following, an exponential model with a vertical to horizontal anisotropy 1/15 has been considered for the correlation function ρ (h):

$$\rho(h) = \exp\left[-\frac{3h'}{a}\right]$$
$$\text{with: } h' = \sqrt{h_x^2 + h_y^2 + (15h_z)^2} \qquad (4)$$

h_x, h_y, h_z being the 3 rectangular coordinates of vector h.

Thus, the practical range, i.e. the distance beyond which spatial correlation vanishes is a in the horizontal plane, and a/15 in the vertical direction.

Figure 1 gives the results for horizontal flow in the x-direction in a network of dimensions (L_x, L_y, L_z) such that: $L_x/a = 1.7$, $L_y/a = .8$, $15L_z/a = 3$, i.e. of dimensions 1.7, .8 and 3 times the respective correlation ranges. The network is discretized into $n_x = 12$, $n_y = 6$, $n_y = 6$, i.e. a total of 432 grid blocks. Each grid block was assigned a permeability (diagonal isotropic tensor) equal to $K_{sh} = 10^{-2}$ md. or $K_{ss} = 10^4$ md.; the contrast is thus $c = K_{ss}/K_{sh} = 10^6$.

The figure graphs the network effective permeability (in the x-direction) vs. the proportion p of shales. Since the generation of the permeability field is stochastic in nature, an average of 30 to 50 realizations (with each 432 permeability values) were retained for each preassigned shale proportion p ; the flow simulation was run for each such realization to derive the corresponding effective permeability K_e ; finally, it is the arithmetic average of these 30 to 50 realizations of K_e which is plotted vs. p on Figure 1.

Also plotted as references on Figure 1 are the arithmetic, geometric and harmonic averages of K_{shale} and K_{sand}, respectively:

$$k_a = pK_{sh} + (1-p)K_{ss}$$
$$k_g \text{ s.t. } \mathrm{Ln}k_g = p\mathrm{Ln}K_{sh} + (1-p)\mathrm{Ln}K_{ss} \qquad (5)$$
$$k_h \text{ s.t. } \frac{1}{k_h} = \frac{p}{K_{sh}} + \frac{(1-p)}{K_{ss}}$$

with: $k_h \le k_g \le k_a$, for all $p \in [0,1]$.

From Figure 1 it appears that the effective permeability K_e is always greater than the geometric average k_g, a limit towards which it tends for very large proportion of shales. The curve K_e vs. p features a dramatic drop (percolation behavior) for p around .84. This percolation limit proportion was found to be around .69 when the correlation range is made nought, a value consistent with the results of percolation theory, c.f. Hammersley and Welsh (1980), and Desbarats (1986).

A power-average, with power w = .57 is shown to provide an excellent fit to the curve K_e vs. p for proportion of shales below the critical limit p_1 .84. Recall the power w-average is k_w such that:

$$\left[k_w\right]^w = pK_{sh}^{\ w} + (1-p)K_{ss}^{\ w} \qquad (6)$$

Next, vertical flow (in the z-direction) was simulated over a network of dimensions (L_x, L_y, L_y) such that: $L_x/a = L_y/a = 1$, $15\,L_z/z = 3.25$. Recall that the vertical range of correlation is a/15.

The network is discretized into $n_x = n_y = 6$, $n_z = 12$, i.e. a total of 432 grid blocks. Thus, the vertical distance between two grid blocks is 3.25/12 = .27 times the vertical range.

The corresponding results are given on Figure 2. The results for high proportion of shales $p > .6$ show extreme fluctuations from one realization of the permeability field to another and cannot be considered yet as significant (more runs are needed).

Again, the average effective permeability K_e appears as significantly greater than the geometric average k_g. The percolation behavior seems to start around $p \approx .6, .7$, a critical limit much lower than that ($p_1 \approx .84$) found for horizontal flow with a range of correlation 15 times larger. A power-average with a low power $w = .12$ provides a reasonable fit for the range of $p \in [0, .5]$.

In preliminary conclusions of Figures 1 and 2, it appears that:

- For practical values of the shale proportion $p < .5$, a power w-average provides a good fit to the curve K_e vs. p.

- The averaging power w increases with the correlation range in the direction of flow, with for upper bound $w = 1$ corresponding to arithmetic averaging for perfection correlation and laminar flow.

Thus, the practice consisting in using the arithmetic average k_a for horizontal flow and the geometric average for k_g for vertical flow is sound but could be considerably improved with appropriate power w-averaging.

The next problem is, of course, the estimation of that averaging power w.

ON-GOING RESEARCHES

Sensitivity of the "effective" averaging power w to parameters characterizing the geometry of shale intrusions (low K, small proportion p) in a sand matrix is being investigated. The descriptive parameters retained are purposely chosen simple so that they could be inferred in practice from available well-logs and geophysical information, such as:

- Volumetric proportion p

- Average aspect ratio

- Correlation range(s) of the indicator of shale presence which is linked to both the shale size distribution and the correlation range of shale centers.

Other parameters influencing the averaging power w are the dimensions of the block whose effective permeability is requested. It appears already that the relative dimension - with regard to the indicator correlation range - in the direction of flow will be most important.

Another field of research will consist in investigating the case of large proportions p beyond the percolation limits, i.e. the case of sand intrusions in a matrix of shale, or in more practical terms the case of high permeability streaks in a sand-shale matrix.

Last, the results obtained from a binary model for the permeability distribution will have to be extended to continuous, although clearly multimodal distributions. Generation of such distributions can easily be done by adding a random component to previously generated and spatially correlated mode values.

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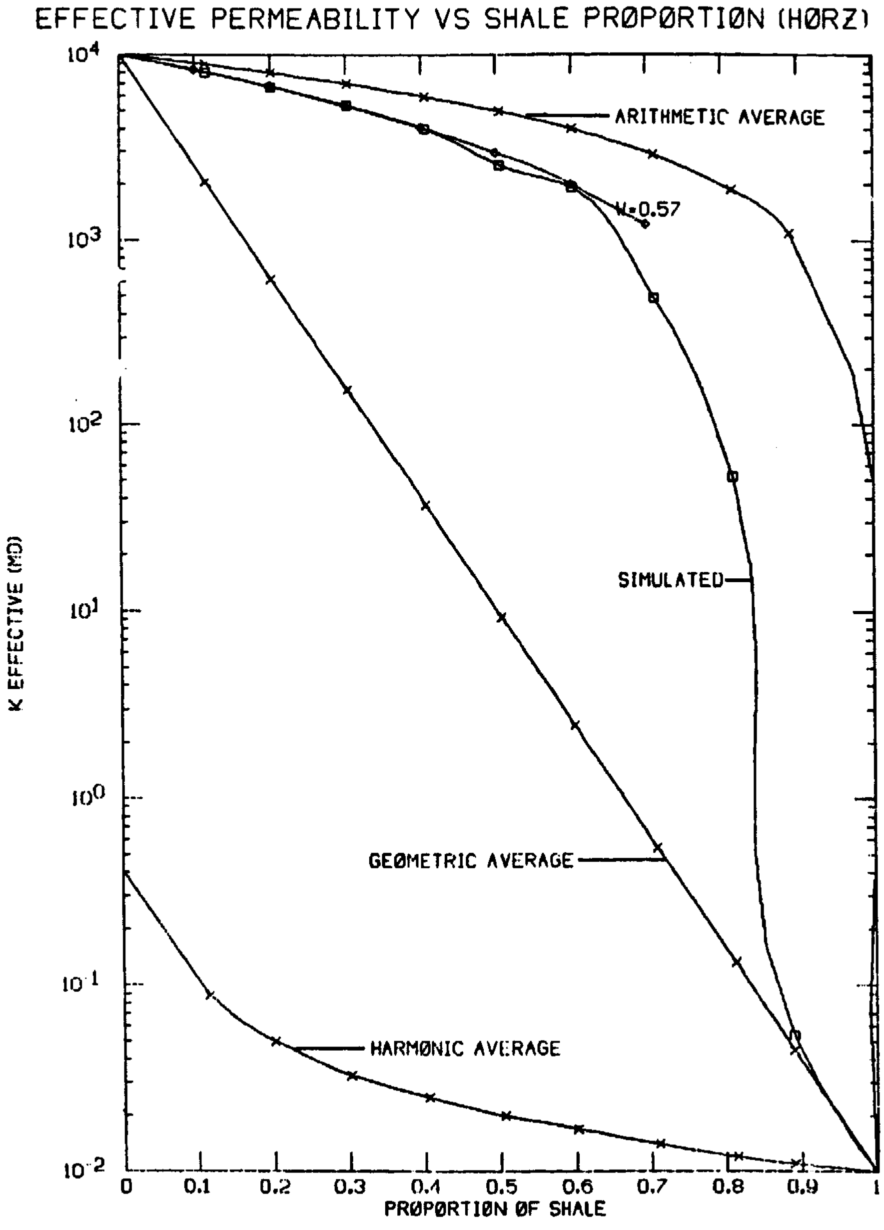

Fig. 1—Effective permeability vs. shale proportion (horizontal flow).

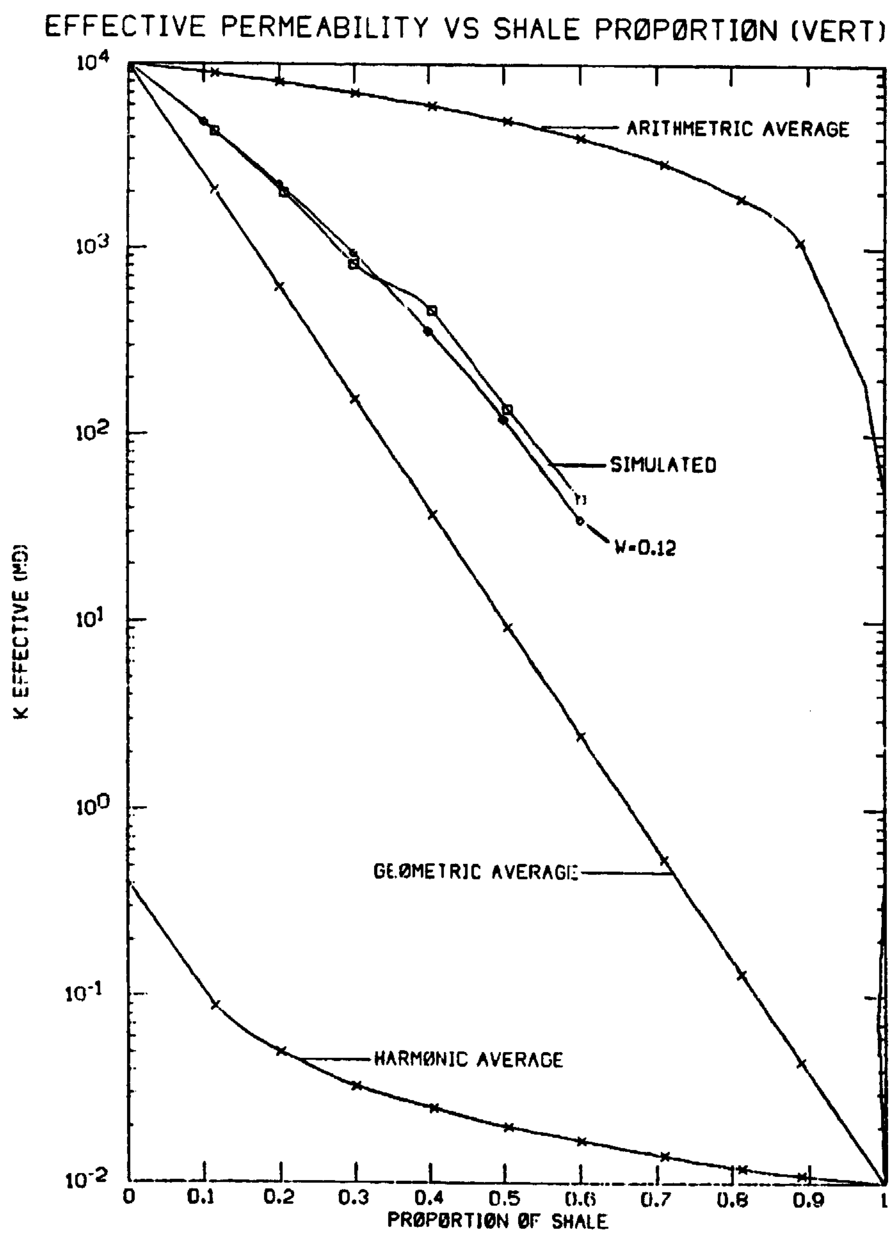

Fig. 2—Effective permeability vs. shale proportion (vertical flow).