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Geostatistical Modeling of Permeability With Annealing Cosimulation (ACS)

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Abstract

To provide accurate predictions of flow performance, the numerical model of permeability used by flow simulators must be consistent with all available geological and engineering data. The available data includes core permeability measurements, relevant permeability statistics (particularly, histograms, measures of spatial variability, and correlation with secondary variables such as porosity), and well test-derived permeability measurements. This paper documents an algorithm to generate 3-D permeability models that honor this variety of data. The algorithm, referred to as annealing cosimulation (ACS), generates stochastic permeability models using the numerical optimization algorithm known as simulated annealing. The application of simulated annealing to reservoir modeling is not new. The variety of information considered, however, and the practical example presented in this paper will be of interest to reservoir modelers.

Introduction

Geostatistical techniques are increasingly being used to generate the 3-D numerical models of porosity and permeability required for reservoir simulation.[1,2,3] The quality of a geostatistical model is directly related to how well it honors the available geological and engineering data. This paper documents the application of simulated annealing to the generation of 3-D permeability models. The originality of this paper is in the details of application that allow the *practical* and *simultaneous* integration of many sources of data.

One promise of geostatistics is a range of equiprobable models that may be used to quantify the uncertainty in the reservoir model. At times, there appears to be a wealth of data (core, well logs, seismic, production tests, and so on). Even in these ideal situations, however, the data are inadequate to provide a unique reservoir model; there is always uncertainty in the assignment of reservoir properties at unsampled locations.

To account for important geological variations, porosity and permeability must be modeled within homogeneous "rock types" that are based on a rational lithology/facies model constructed within a detailed sequence (or chrono-) stratigraphic framework.[4] There is a place for stochastic techniques in the construction of rock type models and stratigraphic frameworks, however, deterministic interpretive procedures dominate this aspect of reservoir description.

Uncertainty in the stratigraphic framework and geological concept are difficult to quantify. In many cases, however, these aspects of uncertainty are the most consequential. This is one reason why the integration of all available information is considered more important than the generation of multiple realizations and the subsequent quantification of uncertainty.

As stated above, porosity and permeability must be modeled within homogeneous rock types. The modeling, however, can not be performed independently within each rock type. Firstly, there may be spatial correlation in the porosity and permeability across rock type boundaries. Secondly, engineering-type data (production history, surveillance, and well test measurements) and geophysical-type data (seismic attributes) inform volumes of the reservoir that most likely cross stratigraphic and rock-type boundaries.[5]

Another consideration is that porosity and permeability data values are typically representative of core sized samples. In practice, *core* values are assigned to grid node locations and then considered representative of the entire cell containing that node. That is, we assign values representative of core or well log size samples to geological modeling cells usually no smaller than 1 by 50 by 50 feet. Proper accounting for this "missing scale" is an area for research.[6]

The problem addressed in this paper is the assignment of permeability (K) values to a 3-D grid network consisting of N nodes where the porosity (ϕ) and the rock-type (RT) have already been assigned. The final "model" consists of these three values at each grid node: $(RT_i, \phi_i, K_i), i=1,...,N$. The N grid nodes typically define a regular 3-D network of 10^5 - 10^7 nodes within the stratigraphic framework.

The rock type (RT) is modeled first. The RT variable is related to the lithology or facies and may also account for the depositional environment and diagenetic history. The precise definition of appropriate rock types depends on the specific problem being addressed. All geological populations with "significantly" different characteristics should be kept separate. Aspects of the RT model could be deterministic (e.g., the position of different stratigraphic sequences) and other aspects could be stochastic (e.g., positions of sandstone and shale within the sequences). Techniques for RT modeling will not be discussed here.

The porosity (ϕ) model is built such that it honors the existing RT model. Techniques for ϕ modeling will not be discussed here, however, the Gaussian, indicator, and annealing methods discussed below could also be applied to ϕ modeling.[7]

Permeability, in general, is more difficult than porosity to model and yet, has a greater influence on fluid flow behavior. Moreover, permeability is directionally dependent and should, theoretically, be modeled as a tensor. Current practice, however, is to model the primary directions (horizontal, K_H, and vertical, K_V) and neglect other aspects of the permeability tensor. When necessary, the directional permeabilities are modeled sequentially; the direction (K_H or K_V) with the most data is modeled first. Proper accounting for the tensorial aspect of permeability is an area for research.[6]

The Problem

The problem is to assign permeability values (say $K=K_H$) after the rock type (RT) and porosity (ϕ) have been modeled. That is, we need to assign K_i given RT_i and ϕ_i, $i=1,...,N$ where N is some large number of grid node locations. The quality of the numerical model, $(RT_i, \phi_i, K_i), i=1,...,N$, depends on how well it honors the available information or "data". For permeabilit modeling, the data include:

- local core measurements
- well test-derived measurements
- a histogram or probability distribution of K within each RT
- a cross plot or bivariate distribution of ϕ and K within each R
- measures of the K-spatial variability within each RT

These data should be honored to the extent that they are know there is always some intrinsic uncertainty due to measureme error, limits of the interpretive model, data paucity, and so on.

Some Conventional Solutions

The simplest approach to assign the N permeability values is develop a regression-type relationship between porosity an permeability for each rock type, for example,

$$\log K_i = a(RT_i)\,\phi_i + b(RT_i),\quad i=1,...,N. \qquad ...($$

where $a(RT)$ and $b(RT)$ are parameters determined by the cro plot of log K and ϕ for each rock type (RT). The regressio type relation could take more complicated forms (quadrati cubic, conditional average of K within a moving window of ϕ and could, at well locations, account for additional informatio available from well logs such as shalyness or indications of ro quality (FZI/RQI).[8]

These regression-type approaches are inadequate for a numb of reasons: 1) low and high permeability values tend to smoothed out, 2) the final permeability values do not show t correct measure of K-spatial variability, and 3) the uncertain in the permeability estimates is not accounted for.

There are a number of geostatistical techniques that could used for modeling permeability. Gaussian-related algorithm and soft indicator kriging (the Markov Bayes model) are t most common.[9] Some limitations of these methods will mentioned to motivate consideration of the annealir cosimulation procedure.

Gaussian techniques are the most straightforward for generati geostatistical realizations. In the case of permeability, t normal scores transform of porosity and permeability a assumed to follow a bivariate Gaussian distribution; t required parameters to implement the Gaussian approa include the linear correlation between the porosity a permeability transforms, the permeability variogram, t porosity variogram, and the cross porosity-permeabili variogram. Further assumptions can be made to remove t requirement for the porosity variogram and the cross porosit permeability variogram. The advantage of Gaussian-relat algorithms is unparalleled simplicity. There are a number disadvantages: 1) it is not straightforward to honor data different volume supports (e.g., core versus well test), 2) there is no flexibility to account for complex ϕ-K cross plots, and 3) there is no flexibility to account for complex measures of K-spatial variability (e.g., indicator variograms).

Problems with the soft indicator approach include greater mathematical complexity, a difficulty to handle data of different volume supports, and rather severe screening or Markov hypotheses.

The motivation for the annealing cosimulation (ACS) technique is to provide an algorithm with greater flexibility to integrate all of the available information.

The Annealing Cosimulation (ACS) Technique

Applying the numerical technique known as "simulated annealing" to geostatistical simulation is relatively new.[10,11,12,13] The name *Annealing Cosimulation (ACS)* is derived from: 1) through common usage simulated annealing has been shortened to "annealing", and 2) stochastic simulation of one attribute conditioned to others may be referred to as "cosimulation" (compare with *cokriging*). Interest in the annealing methodology is based on its ability to honor a wide variety of input data.

The technique of simulated annealing is based on an analogy with the physical process of annealing and is typically applied to global optimization problems. Annealing is the process by which a material undergoes extended heating and is slowly cooled. Thermal vibrations permit a reordering of the atoms/molecules to a highly ordered lattice, i.e., a low energy state. In the context of 3-D numerical modeling, the *annealing* process may be simulated by the following steps:

1. Create an initial 3-D numerical model (analogous to the initial alloy in true annealing) by assigning a permeability value to each grid node at random from the population distribution.

2. Define an energy or objective function (analogous to the Gibbs free energy in true annealing) as a measure of difference between desired features and those of the realization, e.g., the objective function could include the squared difference between the variogram of the realization and a model variogram derived from core data.

3. Perturb the model (analogous to the thermal vibrations in true annealing) by visiting a random location in the 3-D numerical model and assigning a new permeability value. The new value is a random drawing from the conditional distribution of permeability given the collocated porosity value.

4. Accept the perturbation (thermal vibration) if the objective function is decreased; reject it if the energy has increased.

5. Continue the perturbation procedure until a low energy state is achieved. Low energy states correspond to plausible 3-D numerical models of the reservoir.

In true simulated annealing, perturbations that increase the energy are accepted with a certain probability (the Boltzmann probability distribution). The parameter of the Boltzmann distribution (related to the temperature of true annealing) is then lowered according to some schedule. For the purposes of cosimulation, we have found that simply accepting all *good* changes and rejecting all *bad* changes works well.

In general, the objective function is made up of the weighted sum of N_c components:

$$O=\sum_{c=1}^{N_c} w_c \cdot O_c \,. \qquad \text{...(2)}$$

where O is the total objective function, w_c and O_c are weights and component objective functions respectively. Each component is designed to account for a source of data. The weights are calculated such that all components of the objective function are lowered to zero at the end of the annealing process. Details of how to calculate these weights are given in Deutsch and Cockerham.[13]

Components in the objective function are measures of mismatch between a reference property and the corresponding property of the candidate model. Any quantified geological or engineering measurement could be considered, e.g., variograms, seismic data, or well test data. From a practical point of view, though, it is necessary that each property component be easily updated after a perturbation; annealing techniques depend on perhaps millions of perturbations to arrive at an acceptable model.[12,13,14,15] All of the geological and engineering data considered below meet this "easy updating" criterion.

Referring back to "The Problem" statement, we will now discuss how the ACS approach to permeability modeling honors the different types of available data.

Local Core and Well Test Measurements

The known core permeability measurements are honored by assigning them to the nearest grid nodes and never perturbing them in the subsequent annealing procedure.

Well test data may be honored by adding a component to the objective function; this is beyond the scope of the present paper, see Deutsch, 1992 for details.[11]

Probability Distribution of Permeability

In the case of modeling permeability, the histogram or probability distribution of permeability is typically honored within each rock type by the constraint to honor the bivariate distribution of permeability with porosity. Alternatively, we could consider a cumulative distribution function (cdf) within each rock type discretized at a number of thresholds:

$$F(k_i, RT_r) = \text{Prob}\{K \leq k_i,\ K \text{Y} RT_r\},\ i=0,\ldots n_k,\ r=1,\ldots,n_{RT} \quad \ldots(3)$$

The reference cdf values could be obtained from proportions of core data less than each threshold; the values from the model could easily be calculated by proportions of nodal values less than the corresponding threshold. The component objective function is written:

$$O_F = \sum_r^{n_{RT}} \sum_i^{n_k} [F_{reference}(k_i, RT_r) - F_{model}(k_i, RT_r)]^2 \quad \ldots(4)$$

In general, this is unnecessary if the full bivariate relationship with porosity enters the objective function.

Cross Plot of ϕ and K

The relationship between ϕ and K must be quantified and then a component of the total objective function can be constructed.

One summary of the ϕ - K relationship is the linear correlation coefficient. A reference correlation coefficient $\rho_{reference}\ (RT)$ could be established for each rock type. The deviation from the model could then be measured by the sum of squared differences between the reference correlation coefficients and those of the model:

$$O_\rho = \sum_r^{n_{RT}} [\rho_{reference}(RT_r) - \rho_{model}(RT_r)]^2 \quad \ldots(5)$$

In many cases, it is desirable to capture more details from the cross plot. A discretized bivariate probability distribution may be used for this purpose. There are a number of ways to discretize a bivariate distribution. Figure 1 illustrates our preferred approach; the ϕ/K classes are established by defining porosity thresholds based on an equal number of data per class. The permeability thresholds (within each porosity class) are then established on the basis of an equal number of data per class.

The series of conditional cumulative distribution functions are denoted:

$$F(k_{i,j,r}, \phi_j, RT_r) = \text{Prob}\{K \leq k_{i,j,r},\ \phi_{j,r} \leq \phi \leq \phi_{j+1,r}\}, \quad \ldots(6)$$
$$i = 1,\ldots n_k,\ j = 0,\ldots,n_\phi,\ r=1,\ldots,n_{RT}$$

Where n_k and n_ϕ are the number of permeability and porosity thresholds respectively, n_{RT} is the number of rock types, ϕ_j, $j=0,\ldots,n_\phi$ are the porosity thresholds ($\phi_0=0$), and $k_{i,j,r}$, $i = 1,\ldots n_k$ are the permeability thresholds within porosity class j for rock type r. The component objective function is written:

$$O_{Biv.} = \sum_r^{n_{RT}} \sum_j^{n_\phi} \sum_i^{n_k} [F_{reference}(k_{i,j,r}, \phi_j, RT_r) - F_{model}(k_{i,j,r}, \phi_j, RT \quad \ldots(7$$

Considering conditional cumulative distributions within classe rather than the full bivariate histogram or bivariate cumulativ distributions, removes any bias that may be present in the co or reference data; there are often fewer lo porosity/permeability pairs than high porosity/permeabilit pairs. A second practical concern arises when there are too fe calibration data. The solution is to simply consider th correlation coefficient or fit a smooth bivariate model to th distribution. This is an area of research.

Perturbing a data value changes a number of cdf values within conditional distribution that may be easily recalculated.

Measures of K-Spatial Variability

The final type of data considered in this paper relate to th spatial arrangement of the permeability or K values. The flo performance of reservoirs is strongly dependent on the spati distribution of permeability. A random or homogeneous mod of permeability will have significantly different flo characteristics than a model showing spatial correlation.

In geostatistics, the variogram is commonly used as a measu of spatial correlation. A variogram model $\gamma(\mathbf{h})$, inferred fro the available core data and appropriate geological analog dat is a reservoir, attribute, and rock-type specific measure spatial correlation for all distance lags, $\mathbf{h}$. The variogram directionally dependent, that is, the vertical variogram typical has a much shorter range of correlation (say 5-50 feet) than th horizontal (range of 100-10000 feet). The horizontal variogra may also show directional anisotropy.

The permeability model should honor the variogram with each rock type. An objective function of the following form c be considered to achieve this:

$$O_\gamma = \sum_r^{n_{RT}} \sum_l^{n_h} [\gamma_{reference}(\mathbf{h}_l, RT_r) - \gamma_{model}(\mathbf{h}_l, RT_r)]^2 \quad \ldots($$

Where n_l is the number of variogram lags being considered (5 500), $\gamma_{reference}(\mathbf{h}_l, RT_r)$ is the reference variogram model (not to confused with the variogram from the permeability model) f lag $\mathbf{h}_l$ and rock type RT_r, and $\gamma_{model}(\mathbf{h}_l, RT_r)$ is the variogra from the permeability model.

In general, by constraining the permeability model to the ϕ - cross plot and the reference variogram (both by-rock-type) the is adequate control on the spatial distribution of permeabili

There are a number of instances, however, when a greater degree of control on the permeability distribution is required:

1. specific levels of permeability (high or low) have significantly *better* or *poorer* spatial correlation than the permeability values taken all together,
2. there is a complex spatial relationship between porosity and permeability that is not adequately captured by the cross plot alone, or
3. there is spatial correlation between the permeability values in one rock type with those in another rock type.

The first case may be handled by indicator variograms and the second and third situations may be handled by cross variograms.

An indicator transform $i(\mathbf{u};k_c)$ of the permeability value $k(\mathbf{u})$ at location $\mathbf{u}$ is set to 1 if $k(\mathbf{u})\leq k_c$, and to 0 if not. If the high permeabilities within a specific rock type r have a greater range of correlation than the median or low permeabilities, a high indicator threshold k_c would be considered. A variogram calculated using the indicator-transformed data, $\gamma_{I,reference}(\mathbf{h}_l,RT_r)$, would show greater correlation than the overall permeability variogram $\gamma_{reference}(\mathbf{h}_l,RT_r)$. In many cases, the available well data are inadequate to allow the direct calculation and modeling of indicator variograms; a reasonable estimate of the indicator variogram, consistent with the geological interpretation, is better than having the spatial correlation depend only on an *average* variogram.

An objective function of the following form could be considered to account for an indicator variogram:

$$O_{I,\gamma}=\sum_{r}^{n_{RT}}\sum_{l}^{n_l}\left[\gamma_{I,reference}(\mathbf{h}_l,RT_r)-\gamma_{I,model}(\mathbf{h}_l,RT_r)\right]^2 \qquad ...(9)$$

Where n_l is the number of variogram lags being considered (50-500), $\gamma_{I,reference}(\mathbf{h}_l,RT_r)$ is the reference indicator variogram model for lag $\mathbf{h}_l$ and rock type RT_r, and $\gamma_{I,model}(\mathbf{h}_l,RT_r)$ is the indicator variogram of the permeability model.

A number of indicator variograms could be considered simultaneously; for example, multiple indicator thresholds and different indicator variograms for different rock types could be used.

Cross-variograms are measures of spatial correlation between two different variables, e.g., porosity with permeability, permeability in rock type 1 with permeability in rock type 2, and so on. Occasionally, there arise situations when consideration of these measures is important to achieve a realistic permeability model. Once again, it is straightforward to account for cross variograms in annealing cosimulation.

$$O_{\gamma(\phi-K)}=\sum_{l}^{n_l}\left[\gamma_{(\phi-K),reference}(\mathbf{h}_l,RT_r)-\gamma_{(\phi-K),model}(\mathbf{h}_l,RT_r)\right]^2 ...(10)$$

Where n_l is the number of variogram lags, $\gamma_{(\phi-K),reference}(\mathbf{h}_l,RT_r)$ is the reference cross porosity (ϕ)-permeability (K) variogram model for lag $\mathbf{h}_l$ and rock type RT_r, and $\gamma_{(\phi-K),model}(\mathbf{h}_l,RT_r)$ is the cross variogram determined from the porosity and permeability models.

An Example

An illustrative example will now be presented from a shallow marine sandstone oil bearing formation. The univariate distribution of porosity and permeability for this example are displayed in Figure 2. Note the logarithmic scale for permeability and the bimodal aspect of the distributions; the bimodal aspect is also observable on the ϕ/K cross plot (see Figure 1). A rock-type differentiation at a porosity threshold of 10% was considered unnecessary due to a gradational change between the two populations and the fact that direct modeling of ϕ and K preserves the distinction of the rock >10% porosity and that <10% porosity.

For clarity, a single chrono-stratigraphic sequence will be shown; the actual model was 3-D and consisted of 20 layers. The intent here is to illustrate the flexibility and applicability of the ACS algorithm and not to present a full case study. A Gaussian simulation (`sgsim` from GSLIB)[9] was used to model porosity. Figure 3 shows a cross section through the porosity model and the corresponding vertical and horizontal variogram.

Figures 4, 5, and 6 show three alternative permeability models.

The linear regression model (see equation 1) shown in Figure 4 is a smooth log-linear rescaling of the porosity model. Note that the spatial structure of the resulting permeabilities is close to that of the porosities. The variogram does not reach the expected sill value since the variance has been reduced with the linear transform. The overall character of the permeability values appears too smooth and no assessment of uncertainty is possible.

Honoring the ϕ/K cross plot only, see Figure 5, with no explicit control on the spatial correlation of the permeability values results in the correct variance. There is some spatial continuity borrowed from the porosity model, however, the variogram does not match the target or reference variogram since no explicit variogram control was added.

The result of adding a variogram component to the objective function to control the spatial distribution of permeability is seen on Figure 6. This illustration is typical of the "conventional" application of ACS, i.e., with direct control on the ϕ/K cross plot and the K variogram.

A total of 40 lag direction vectors were considered in the ACS run for Figure 6. The lag vectors with the lowest reference variogram value are chosen. This implies that few lags (only the first) were chosen from the vertical direction and many lags were chosen from the horizontal direction. The variogram plot to the right in Figure 6 shows an excellent reproduction of the horizontal variogram and perhaps a poorer reproduction of the vertical variogram (except for the first lag). The correlation with porosity causes this vertical continuity to be imparted to the permeability values. Adding more lags to the ACS run could ensure that the reference variogram model is honored more precisely; this was considered unnecessary.

Figure 7 shows ϕ/K cross plots of the 2-D K values shown on Figures 4, 5, and 6 with the porosity values of Figure 3. The linear regression was correctly applied (the cross plot reflects the linear transform) and both ACS models reproduce the reference ϕ/K cross plot.

To compare these models, each should be entered in a flow simulator. Then, the predicted reservoir performance may be compared. This was not done. We did, however, calculate the effective permeability of the three 2-D sections shown on Figures 4, 5, and 6:

	Effective Horizontal Permeability	Effective Vertical Permeability
Linear Regression	34 md	1.7 md
Cross Plot Only	30 md	0.51 md
Cross Plot and Variogram	47 md	0.42 md

Note that there are significant differences in the effective horizontal and vertical permeability values. The predicted flow performance is expected to be significantly different.

Conclusions

An algorithm, based on simulated annealing, has been presented for constructing geostatistical models of permeability. The annealing cosimulation (ACS) algorithm provides great flexibility to integrate a variety of geological and engineering information.

The 3-D permeability model is constructed after the porosity has been modeled within homogeneous rock types. The modeling, however, can not be performed independently within each rock type; there may be spatial correlation in the porosity and permeability across rock type boundaries and many data inform volumes that cross stratigraphic and rock-type boundaries.

We have presented the ACS technique and details of how to account for local permeability measurements, a permeability histogram, the bivariate distribution between porosity and permeability, a permeability variogram, indicator variograms, and cross variograms. It would be possible to account for other types of data such as well test data and other production information.

Nomenclature

γ	=	variogram function
$\mathbf{h}$	=	lag separation vector
I	=	indicator transform
i	=	indicator transform data value
K	=	absolute permeability
N	=	number of grid nodes in a numerical model
n_{RT}	=	number of rock types
n_{ϕ}	=	number of porosity thresholds
n_K	=	number of permeability thresholds
ϕ	=	porosity
RT	=	rock type (usually based on lithology or facies)
ρ	=	correlation coefficient
$\mathbf{u}$	=	location coordinates vector

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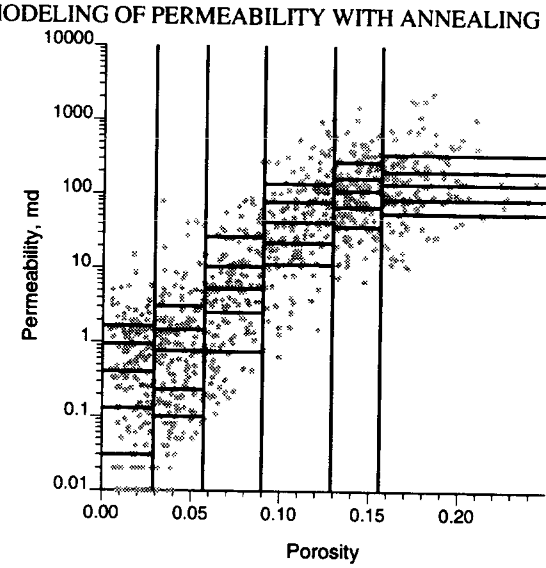

Figure 1: Cross plot of permeability versus porosity with core data shown as gray dots and class definition for the bivariate distribution shown by the black lines. Five porosity and five permeability cutoffs were used.

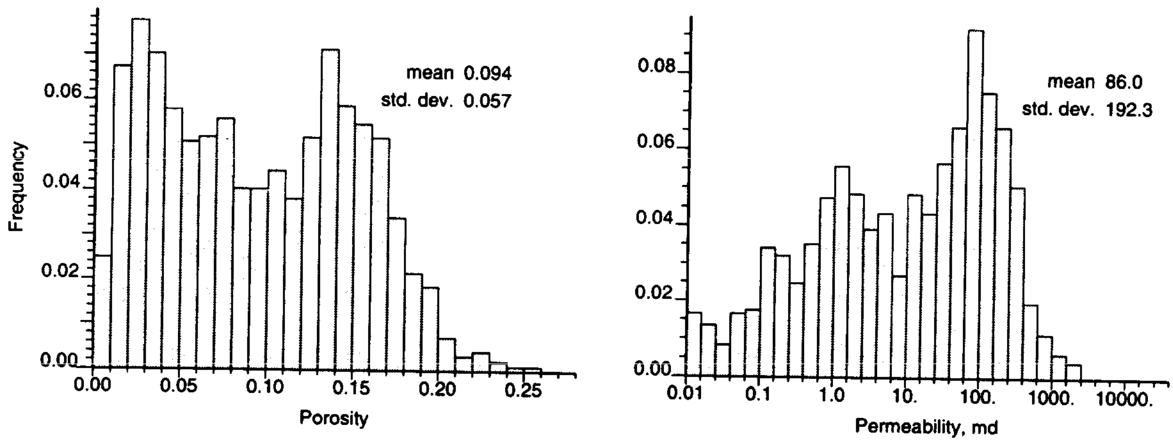

Figure 2: Histogram of the core porosity and core permeability data.

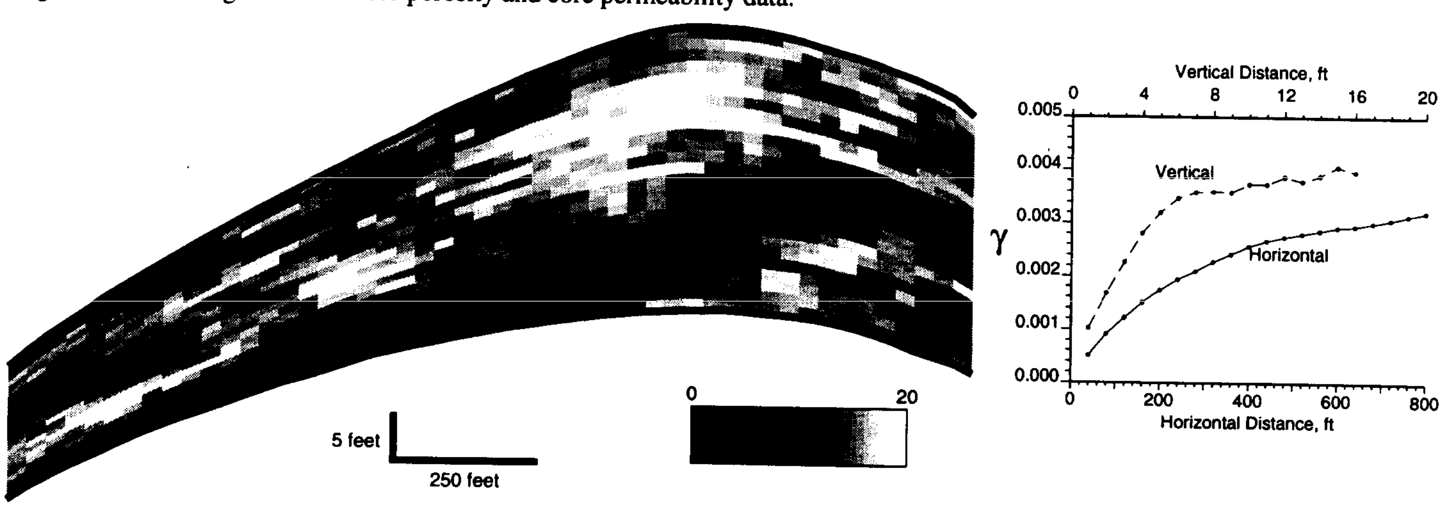

Figure 3: Cross section through the porosity model with the corresponding vertical and horizontal variograms.

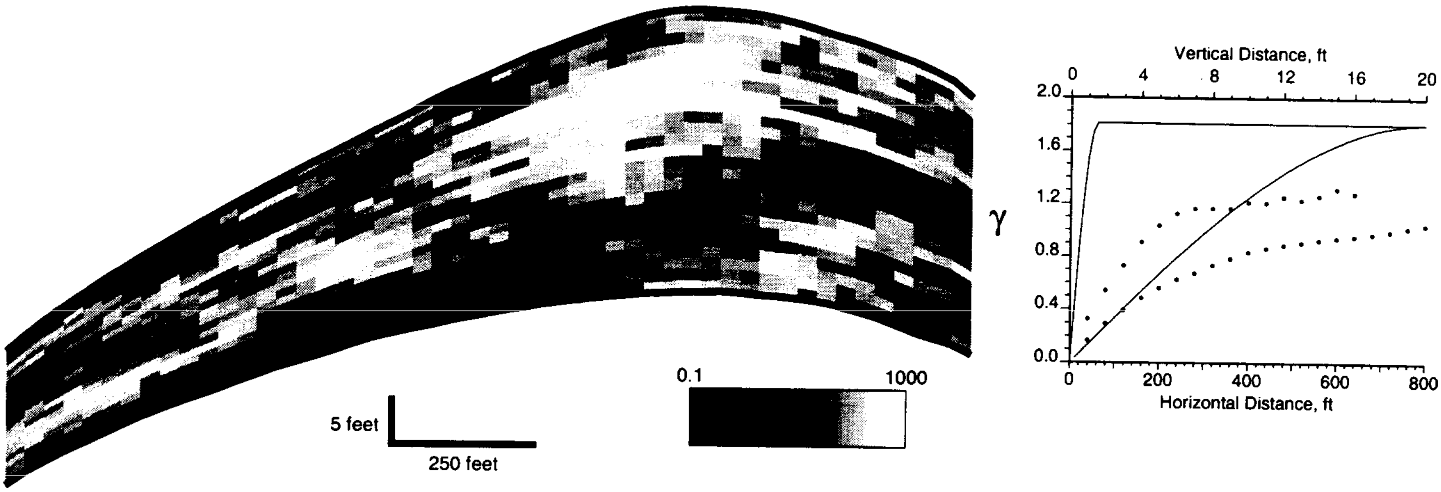

Figure 4: Cross section through the permeability model based on a linear transform of the porosity model.

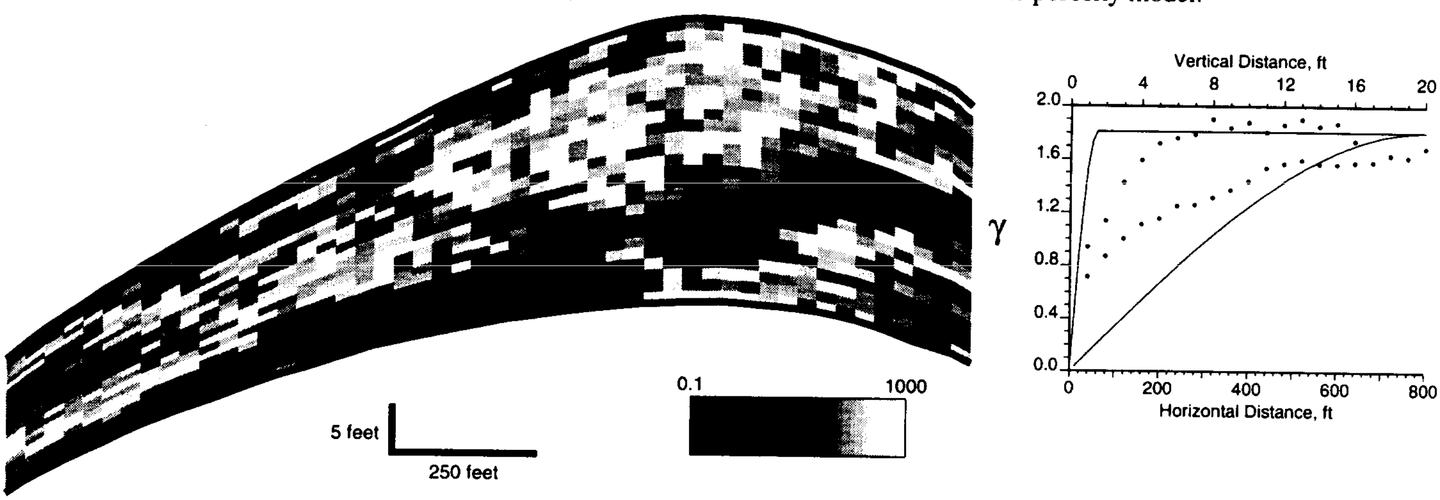

Figure 5: Cross section through the permeability model based on a reproduction of the porosity-permeability cross plot.

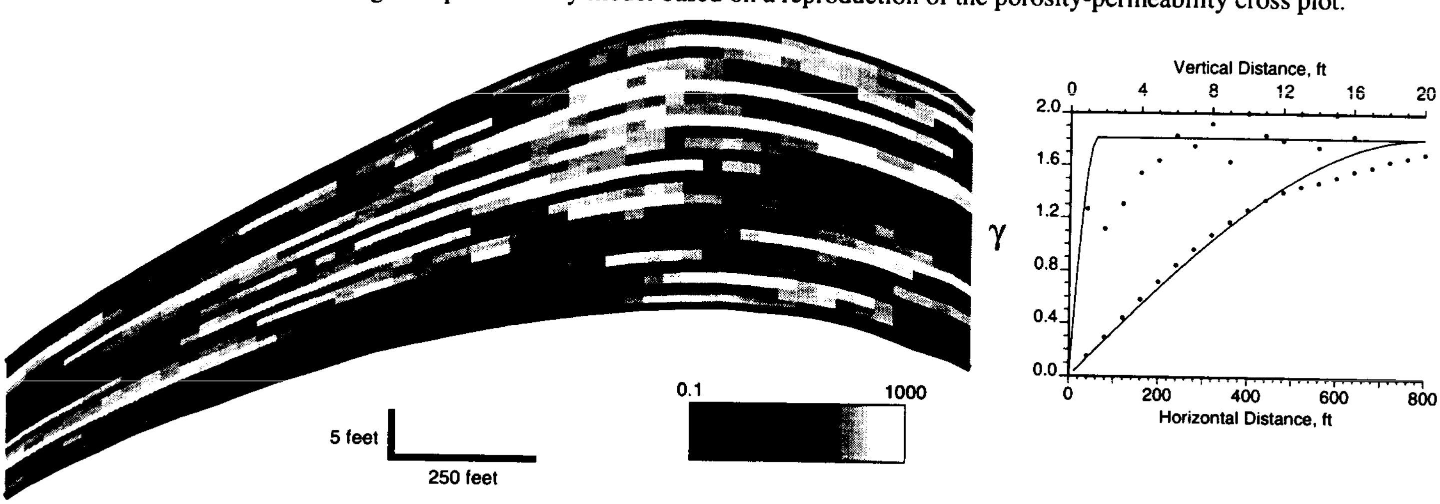

Figure 6: Cross section through the permeability model based on a reproduction of the porosity-permeability cross plot and the permeability variogram.

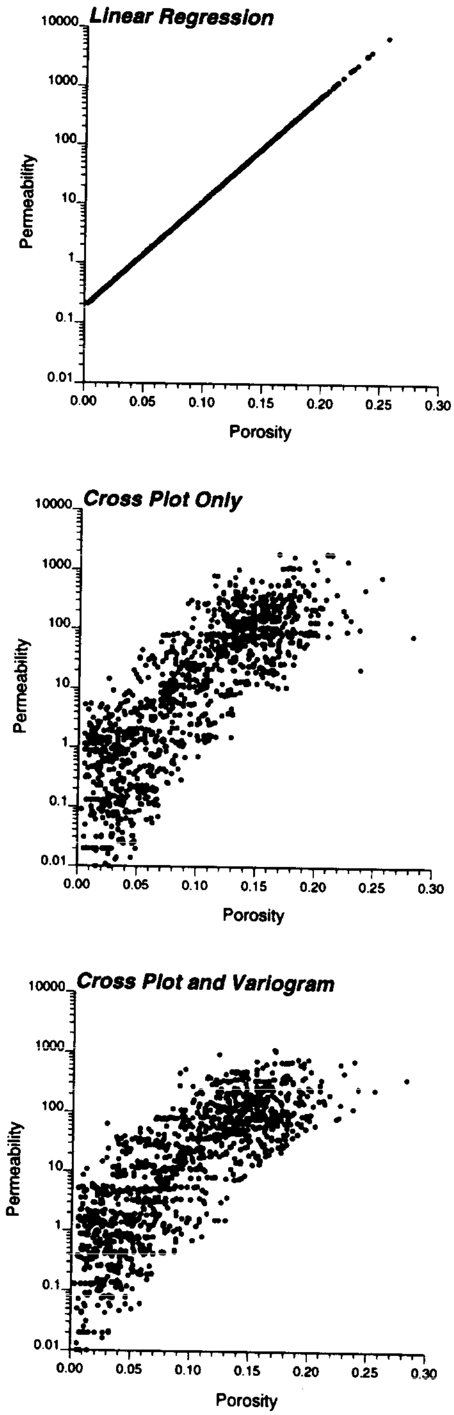

Figure 7: Reproduction of the porosity-permeability cross plot (compare with core data shown on Figure 1).