Chapter 16

ADVANCES IN THE PRACTICAL IMPLEMENTATION OF INDICATOR GEOSTATISTICS

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ABSTRACT

Indicator Kriging (IK) is being commonly applied to mineral resource assessment problems. More recently, indicator simulation has also been used for sensitivity analysis and difficult change of support problems. The essential theory underlying indicator geostatistics has not changed since its introduction 10 years ago; however, the last decade has produced many important developments in its implementation. The objective of this paper is to consider some of the more important implementation details. Examples from the Porgera gold deposit in Papua New Guinea are used to illustrate the steps in an indicator geostatistical study and the importance of various implementation decisions.

Specifically, in many precious metals deposits the continuity of the grades is typically quite anisotropic. This anisotropy varies in both magnitude and direction throughout the deposit. Building a model of this anisotropy can considerably improve the local kriged or simulated points. One approach to build this model is presented. In addition, there is typically more than one mineralization style and geological rock type; the spatial character of the mineralization can be quite different within each geological rock type. The rational treatment of anisotropy and geological subdivisions is common to all mineral resource assessment procedures; the importance of this qualitative geological information is not diminished with more sophisticated techniques such as indicator kriging.

This paper presents an overdue comprehensive look at the practical aspects of indicator methodology. Many of the techniques and procedures documented in this paper have been developed during the course of detailed mine feasibility studies.

INTRODUCTION

This paper focuses on indicator kriging (IK) because the method has enough flexibility to model the grade distribution in most deposits and is simple to explain and implement without a great deal of training. More straightforward manual methods or geostatistical methods like ordinary kriging do not have the same flexibility, e.g., IK allows the variograms to change with the magnitude of the grade. Furthermore, IK provides a probabilistic framework to evaluate change of support problems, and recoverable reserves for various selective mining unit sizes.

The first step in any geostatistical study is to consider the geological controls of the mineralization. It is vital to directly incorporate as many of these controls as possible into the numerical model of the deposit. The actual implementation of the interpolation or simulation algorithm is important once the detailed geology model is in place. In practice, the implementation details of a particular geostatistical methodology are as important as its theoretical foundation. Most methods will give acceptable results if enough geological controls are accounted for and there are enough conditioning data; therefore, a comparative study of different methods is not worthwhile.

One important component of the geological controls is information on preferential directions of continuity. These preferential directions may be due to structural features such as fracture or shear zones, or lithological features such as the rock type, the contact between the different rock types, or gradational composition changes. In any case, the magnitude of the anisotropy is usually quite strong and important. Where this anisotropy is significant the orientation of the anisotropy is also very important. Rarely can the orientation be considered as constant throughout the entire deposit; often, the direction must be aligned locally. An approach to achieve this local alignment will

be presented.

A second important component of the geological controls is the presence of numerous rock types. In most cases the data must be separated into different rock types or geological zones. Moreover, there may be overlapping controls to the mineralization, i.e., a rock type model with a co-dependent structural model. Although there is no general approach to treat these geological controls some thoughts and methods that have worked in practice, will be presented.

Indicator kriging is described in numerous references with some pointers on how to implement the kriging and the follow-up steps necessary to obtain a mineral resource estimate. However, many practical problems have never been fully discussed, for example, choosing the cutoffs, incorporating data of different types, modelling the indicator variograms, correcting order relations, and accounting for selective mining unit volume support. These practical problems will be discussed with examples from the Porgera gold deposit in Papua New Guinea.

The topics discussed in this paper will roughly correspond to the order in which they are performed in practice. Of course, some steps overlap, are independent, or are performed iteratively with other steps. The idea behind the definition of geological domains will be discussed first. Secondly, the motivation for and procedure to build anisotopy models will be discussed. Thirdly, IK will be described in some detail. Finally, some additional topics such as the transition to mine planning, indicator simulation, an alternative IK procedure, and testing the validity of parametric geostatistical methods will be discussed.

GEOLOGICAL MODELS

The idea behind geological models is to define the 3-D geometry of zones of mineralization which can be considered to have the same properties. Subsequent geostatistical studies or manual evaluation methods will use these 3-D models to constrain how data are combined. The block size is either chosen small enough or some type of block splitting is used to adequately resolve geological boundaries. Solids modelling techniques will undoubtedly be used more extensively in the future.

The stationary or homogeneous geological domains may be based on such features as rock type, the level of oxidation, level of fracturing, or the mineralization style. Moreover, the geological model may change for different metals, e.g., silver may be affected more by oxidation than gold; hence, there may be additional zonation near the surface. Additional models may be built for metallurgical classification and specific gravity.

The definition of so-called stationary or homogeneous geological domains depends on the amount of data and the goals of the study. In some cases the scale of the study permits all of the data to be considered as one stationary population. In other situations, where there are enough data, it is essential to use several different domains.

The Porgera gold deposit is located in the highlands of Papua New Guinea, 600 km north-west of Port Moresby. Epithermal gold-silver mineralization occurs in and around small late Miocene basic stocks and dykes intruded into Cretaceous carbonaceous shales and calcareous siltstones. The deposit has proven and probable mining reserves of 50.8 mt at 7.6 g/t gold, including 6.6 mt at 24.5 g/t gold in a high grade underground mining section. Production commenced in 1990, and the deposit is expected to produce 900,000 ounces gold per year for the first [illegible] years of production.

Mineralization is strongly structurally controlled by intrusive contacts and a variety of fault and breccia zones, the most important of which is the Roamane Fault zone (Figure 1). The earliest phase of mineralization was low silica, high suplhide and consisted of carbonate-quartz-pyrite-sphalerite-galena veins and disseminated pyrite. Gold is closely associated with the sulphides, often within the lattice of the pyrite. Superimposed on this is a lower temperature high-silica assemblage of vuggy quartz, carbonate and roscoelite with minor sulphides, coarse gold and various tellurides. Though this assemblage occurs throughout the Porgera deposit it has produced bonanza gold grades in breccias in and adjacent to the Roamane Fault. Several other mineralization types characterize other parts of the mineralized system.

The variety of mineralization styles and structural directions in the mineralized system dictate that mineralization be modeled by dividing the deposit into several relatively homogeneous domains. Even within these domains, continuity directions vary. The deposit is best modeled by varing continuity directions within these domains. The large data base (more than 60000 2m samples) has gold grades up to 5000 g/t.

ANISOTROPY MODELS

The continuity of the mineralization typically varies with direction. During the formation of the orebody, or some later remobilization of the ore minerals, there are preferential directions of continuity formed by zones of weakness or host rock mineralogy. This anisotropic permeability field causes the mineralizing fluids to follow preferential paths. If nothing else, gravity causes temperature and pressure gradients which influence the deposition of most ore minerals.

Wherever the continuity of the mineralization is direction dependent it is very important to have the direction correctly defined. A misalignment of 10 degrees can cause extrapolation of ore into waste and waste into the ore zones. If an average direction of continuity is used artificial lenses can be created wherever the mineralization has curvilinear patterns of anisotropy. Figure 1 shows a simplistic 2-D example of what can happen. In this case neither the grade nor the tonnes of ore will be estimated correctly.

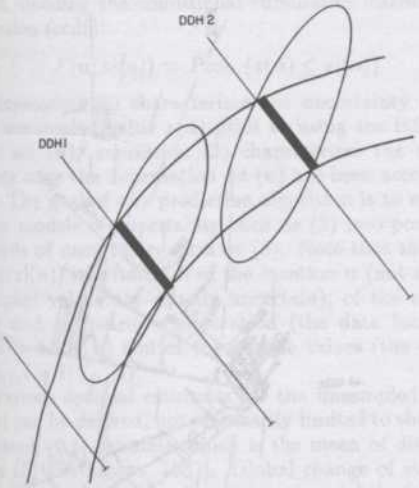

Figure 1: An example of the discontinuous lenses created by using an average direction of continuity in presence of curvilinear structures.

Rather than build 3-D models of these anisotropy parameters it may be possible to transform the coordinates to unfold the mineralization. This transformation is straightforward in certain cases [Dagbert, et.al., 1984; Deutsch, 1989]; however, it is not usually feasible in 3-D because of the difficulty in working out the analytical expressions for the coordinate transformation.

The best approach is to construct a block model of the parameters that define the anisotropy. In this way every location could have a different anisotropy definition for kriging and/or simulation. The block size can be chosen to adequately resolve the fluctuation in each parameter defining the anisotropy. Typically, the anisotropy angles can vary in space but not the anisotropy ratios which are left fixed for each geological domain.

Although the anisotropy model can be used to orient the search ellipse and for computing variogram/structural distances in kriging, it can also be used with inverse distance or closest sample estimation schemes.

A similar scheme to handle variable anisotropy has been proposed independently by Soares, 1990. However, the goal was different; he only considered one indicator cutoff to establish the geometry of 2-D rock strata.

Figure 2 illustrates the three angles and two anisotropy factors required in 3-D. Many software packages take a shortcut and only use two angles and two anisotropy factors. The third angle, which is necessary to accommodate the geological concept of a plunge or rake, is required when the ellipsoid defining the geometric anisotropy does not have any horizontal axis. Modeling the geometric anisotropy within the limbs of a plunging syncline typically requires three angles. In general, modeling anisotropy with three angles is not

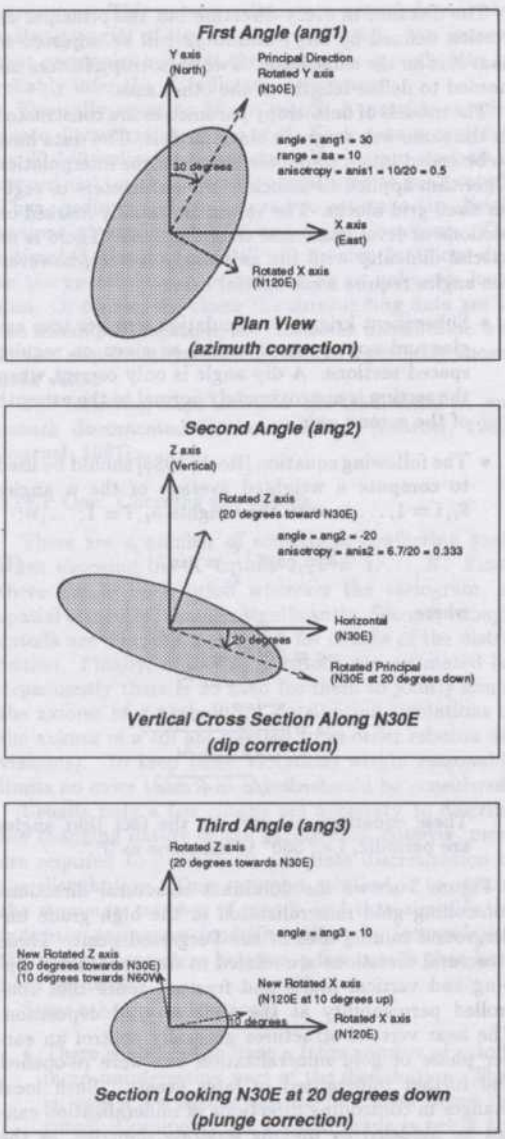

Figure 2: An illustration of the three angles and two anisotropy factors required to define anisotropy in three dimensions.

easy.

The easiest way to describe the three angles and two anisotopy factors is to imagine the rotations and squeezing that would be required to transform a sphere to an ellipsoid. The outside shell of this ellipsoid represents the physical distance at which the mineralization has the same level of continuity; if the mineralization is one half as continuous in a direction the ellipse will be one half as large. This ellipse can also be thought of as the range of the variogram. A longer range implies a better continuity as measured by the variogram.

The distance in every direction but the principal direction defined by $ang1$ and $ang2$ will be adjusted so that it is on the same basis. Two anisotropy factors are needed to define lengths of the other axes.

The models of anisotropy parameters are constructed in the same way as grade block models. The data have to be coded into a computer file and some interpolation algorithm applied to associate the parameters to regular sized grid blocks. The values are usually marked on sections or level plans and then digitized. There is no special difficulty with the anisotropy factors; however, the angles require some special care:

- Subsequent kriging or simulation requires true angles and not apparent angles as given on regular spaced sections. A dip angle is only correct when the section is approximately normal to the azimuth of the second axis.
- The following equation [Rock, 1988] should be used to compute a weighted average of the n angles $\theta_i, i = 1, \ldots, n$ with the weights $\lambda_i, i = 1, \ldots, n$:

$$\overline{\theta} = cos^{-1}\frac{c}{r} = sin^{-1}\frac{s}{r} \qquad (1)$$

where,

$$c = \frac{\sum_{i=1}^{i=n} \lambda_i cos(\theta_i)}{\sum_{i=1}^{i=n} \lambda_i}$$

$$s = \frac{\sum_{i=1}^{i=n} \lambda_i sin(\theta_i)}{\sum_{i=1}^{i=n} \lambda_i}$$

$$r = \sqrt{c \cdot c + s \cdot s}$$

These equations account for the fact that angles are periodic, i.e., 360° is the same as 0°.

Figure 3 shows the dominant structural directions controlling gold mineralization in the high grade underground mining area of the Porgera deposit. These structural directions are related to steep southerly dipping and vertical faults and fracture zones that controlled permeability at the time of gold deposition. The near vertical structures generally control an earlier phase of gold mineralization but were re-opened and further mineralized in later events. Such local changes in controlling directions of mineralization cannot be modeled by making separate domains, as the number of such domains would be unmanageably large. The use of anisotropy models for dip azimuth, dip, and if required, plunge, allows grades to interpolated where they are expected to occur based on geological controls. This is particularly important for underground resource estimates. In addition the correct data will be used for estimation; high grade blocks will be kriged using high grade data, rather than the data that might have been used if average filter directions were used. The method is most applicable where geological controls are well understood, and there is sufficiently detailed data to estimate blocks without needing the search to extend too far from the blocks being kriged.

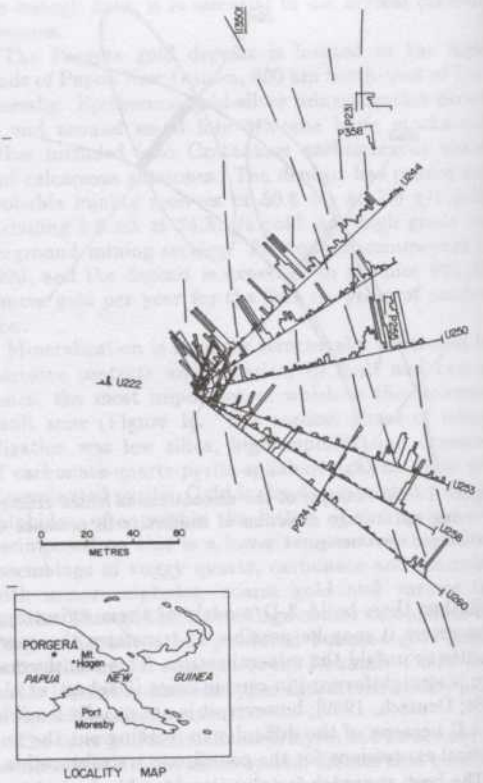

Figure 3: Directions of continuity of mineralization in relation to drill hole intersections Porgera Gold Deposit, section 22350E.

INDICATOR KRIGING

Essentially, geostatistics is a collection of statistical tools that are used to understand and model spatial variability. The models of spatial variability are then used for descriptive and predictive purposes. In the context of mineral resource assessment the goal is almost always predictive, i.e., to estimate the quantity of metal in a deposit with an associated measure of uncertainty.

The basic approach of all predictive statistics is to turn any unsampled (unknown) value z into a random variable Z. The probability distribution of Z then characterizes the uncertainty about the unknown true value z. The probability distribution of Z depends on the location $\mathbf{u}$ (a three dimensional vector with east, north, and elevation coordinates) and the level of information near $\mathbf{u}$. The cumulative distribution function (cdf) of $Z(\mathbf{u})$ is denoted:

$$F(\mathbf{u}; z) = \text{Prob}\{z(\mathbf{u}) \leq z\} \qquad (2)$$

when the cdf $F(\mathbf{u}; z)$ is conditioned to a particular set of information, e.g., n nearby data values $Z(\mathbf{u}_i) = z(\mathbf{u}_i), i = 1, \ldots, n$. The notation "conditional to (n)" is

used, defining the conditional cumulative distribution function (ccdf):

$$F(\mathbf{u}; z|(n)) = \text{Prob}\{z(\mathbf{u}) \leq z|(n)\} \quad (3)$$

Expression (2) characterizes the uncertainty about the unsampled value $z(\mathbf{u})$ prior to using the information set (n); expression (3) characterizes the uncertainty once the information set (n) has been accounted for. The goal of any prediction algorithm is to update prior models of uncertainty such as (2) into posterior models of uncertainty such as (3). Note that the ccdf $F(\mathbf{u}; z|(n))$ is a function of the location $\mathbf{u}$ (not all unsampled values are equally uncertain), of the sample size and geometric configuration (the data locations $\mathbf{u}_i, i = 1, \ldots, n$) and of the sample values (the values $z(\mathbf{u}_i), i = 1, \ldots, n$).

Various optimal estimates for the unsampled value $z(\mathbf{u})$ can be derived, not necessarily limited to the least squared error estimate which is the mean of distribution (3) [Srivastava, 1987]. Global change of support techniques such as the affine correction and the indirect correction through permanence of a lognormal distribution can also be applied at the local level to estimate the local recoverable reserves. Finally, realizations can be drawn from the local distribution to provide a conditional simulation.

The important concept behind indicator geostatistics is the direct coding of experimental data in the form of cumulative distribution functions (cdf). The cdf of a single data $z(\mathbf{u}_j)$ would appear as a step function; 0 for all grade thresholds less than $z(\mathbf{u}_j)$ and 1 for all grade thresholds greater than $z(\mathbf{u}_j)$. This 0/1 coding at a specified threshold cutoff z_k defines an indicator function:

$$i(z(\mathbf{u}_j); z_k) = \begin{cases} 0, & \text{if } z_k < z(\mathbf{u}_j) \\ 1, & \text{if } z_k \geq z(\mathbf{u}_j) \end{cases} \quad (4)$$

Kriging of the indicator variable provides an estimate of the ccdf $F(\mathbf{u}; z|(n))$:

$$\begin{aligned} E\{I(\mathbf{u}; z)|(n)\} &= 1 \text{ x Prob}\{I(\mathbf{u}); z) = 1|(n)\} \\ &+ 0 \text{ x Prob}\{I(\mathbf{u}); z) = 0|(n)\} \\ &= \text{Prob}\{I(\mathbf{u}); z) = 1|(n)\} \\ &= F(\mathbf{u}; z|(n)), \text{ as defined in(2)} \end{aligned}$$

Thus, kriging applied to indicator data provides a least squares estimate of the ccdf (2). Note that indicator kriging is not designed to directly estimate either the unsampled value $z(\mathbf{u})$ or its indicator transform $i(\mathbf{u}; z)$; it is designed to provide a ccdf model of uncertainty about $z(\mathbf{u})$. The IK approach is said to be non-parametric in the sense that it does not estimate the ccdf through its parameters but, rather, the ccdf for various threshold values of z are estimated directly.

As opposed to parametric techniques, IK calls for K kriging systems if there are K cutoffs z_k discretizing the ccdf $F(\mathbf{u}; z_k|(n))$ and for an equal number of indicator variograms. Its advantage over parametric approaches is that it captures much more from the original z-data than a mere z-variogram: K indicator variograms are extracted from the data rather than a single variogram. This greater reliance on actual data is the main strength of the indicator approach. An important prerequisite is that there must be enough data to reliably infer the K indicator variograms.

Typically, seven to fifteen cutoffs K provide an adequate discretization of the ccdf. Each data is replaced by K 1/0 indicator transforms. Variogram analysis is carried out on the indicator transforms at each cutoff. Then, ordinary kriging is used to estimate the distributions at any number of unsampled locations. The estimated distributions will not be step functions due to the uncertainty in estimating at an unknown location. Of course, the closer the surrounding data are to the unsampled location, the closer the distribution will be to a step function characteristic of a perfectly known data value.

The following steps describe the traditional IK approach documented in the literature [Journel, 1983; Journel, 1987]:

STEP One: Choosing the cutoffs

There are a number of sometimes conflicting goals when choosing the K cutoffs $z_k, k = 1, \ldots, K$. First, there should be a cutoff wherever the variogram, or spatial structure, changes significantly. Second, enough cutoffs are required to account for details of the distribution. Finally, since the K cutoffs are estimated independently there is no need for them to jointly honor the axioms of a probability distribution (violations of the axioms of a cdf are referred to as order relation deviations). To keep these violations within reasonable limits no more than 9-15 cutoffs should be considered.

Usually only a few cutoffs are necessary to describe the changing nature of the variograms; however, more are required to provide an adequate discretization of the distribution. One common solution is to choose the *maximum* number of cutoffs and then simplify the indicator variogram modelling effort by interpolating the model parameters between a few cutoffs that have been modeled.

Some other considerations:

- There is no need to have a large number of cutoffs in an uninteresting part of the distribution. That is, only a few cutoffs need to be chosen below the cutoff. Commonly the median grade is below the economic cutoff. In this case, it is unnecessary to choose the first four deciles as cutoffs.

- The grade cutoffs change after volume support correction (accounts for mining selection). Therefore, it is unnecessary to choose the economic cutoff. Furthermore enough cutoffs should be chosen both below and above the economic cutoff to provide details in the important part of the distribution after volume support correction.

- Choosing deciles of the distribution is not a good choice because resolution is lost in the important upper tail of the distribution. Moreover, cutoffs which give classes with an equal quantity of

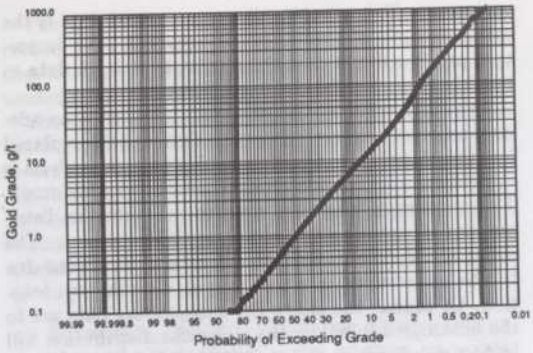

Figure 4: Lognormal probability plot of gold data from one domain of the Porgera gold deposit.

metal are unacceptable because there will be too few classes in the lower tail. The best *automatic* choice would be to choose the first half of the cutoffs such that they separate the distribution into classes with roughly the same number of data. The second half of the cutoffs can then be chosen such that the classes contain roughly the same quantity of metal. For typical "lognormal" type precious metal distributions each cutoff will be roughly double the preceding cutoff (e.g., 0.1, 0.2, 0.4, 0.8, 1.2, 2.4, 4.8, 10.0, 20.0, g/t).

- Looking at probability plots can provide some insight into the selection of appropriate cutoffs. For example, significant changes in slope or discontinuities are logical thresholds for cutoffs. Figure 4 shows a probability plot for data from one domain of the Porgera deposit.

- The alternative IK approach presented in Appendix A avoids the problems associated with defining artificial class boundaries.

- Indicator variograms for cutoffs less than the 10th percentile and above the 90th percentile are very difficult to infer due to the small proportion of indicator data lesser and greater than the cutoff. Moreover, the 10th percentile of data for precious metal deposits is usually at or near the limit of analytical accuracy where data are noisy, hence very low cutoffs should be avoided.

STEP Two: Indicator Transformation

If all the assays represent samples collected and assayed by the same technique then there is no difficulty in constructing the indicator transformations. For a particular cutoff z_k and data value $z(\mathbf{u}_j)$ the indicator transform $i(z(\mathbf{u}_j); z_k)$ is 1 if the grade $z(\mathbf{u}_j)$ is less than the cutoff z_k and 0 otherwise.

The K indicator transforms for a particular data $z(\mathbf{u}_j)$ will be zeros until the data is less than a cutoff and then all the indicator transforms will be ones. The indicator transform is equivalent to coding the data in the form of a cdf, i.e., when the indicator transform is zero there is a 0% probability that the grade is less than the cutoff and when the transform is one there is a 100% probability that the grade is less than the cutoff.

When evaluating a mineral deposit it is common to have various types of assay data. For example, old drillholes, chip or bulk samples, different sampling techniques, and sometimes just different assaying methods for the same sampling technique. In many cases it is possible to code *secondary* data as soft data. That is, the indicator transforms will not show the characteristic sharp 0 to 1 transition. The soft or fuzzy indicator transforms are obtained through calibration scatterplots.

STEP Three: Variogram Modelling

A variogram model is needed for each of the K cutoffs chosen. These variograms are typically well behaved and easily interpreted because, by definition, there are no outliers. Moreover, there is some continuity imparted to the K variograms due to their common physical origin. Therefore, they should not be modelled independently. Note that the variance of an indicator variable is $p(1-p)$ where p is the proportion of 1's, that is, the average indicator value at a cutoff: $p_k = \frac{1}{n}\sum_{j=1}^{n} i(\mathbf{u}_j; z_k)$. Prior to modeling the indicator variograms it is suggested that the variogram values for all indicator thresholds k be divided by their respective $p_k(1-p_k)$ variance. The sill of each variogram will the be approximately 1.0 and it will be simpler to jointly model all indicator variograms.

It is good practice to build the K models from a common pool of L structures [Journel, 1987], say $Sph_{a_l}(\mathbf{h}), l = 1, \ldots, L$,

$$\gamma_I(\mathbf{h}; z_k) = \sum_{l=0}^{L} C_l(z_k) Sph_{a_l(z_k)}(\mathbf{h}) \qquad (5)$$

Where $Sph_{a_l}(\mathbf{h})$ is the spherical variogram model for a range parameter a_l and distance vector $\mathbf{h}$. The variability of the amplitude parameters $C_l(z_k)$ and any varying anisotropy parameters should be plotted and appear reasonably smooth. There should not be sudden large changes in the variogram parameters unless there is a good physical explanation (for example, above a certain level z_k the mineralization seems to be controlled by a different process).

This approach to modelling indicator variograms accounts for the common origin of the indicator variables, makes it easy to infer the variogram model for additional cuttofs (interpolate the model parameters), and reduces order relation deviations. The scaling of the variogram by the variance $p(1-p)$ does not affect the kriging weights.

Figure 5 shows the indicator variograms from one domain of the Porgera deposit.

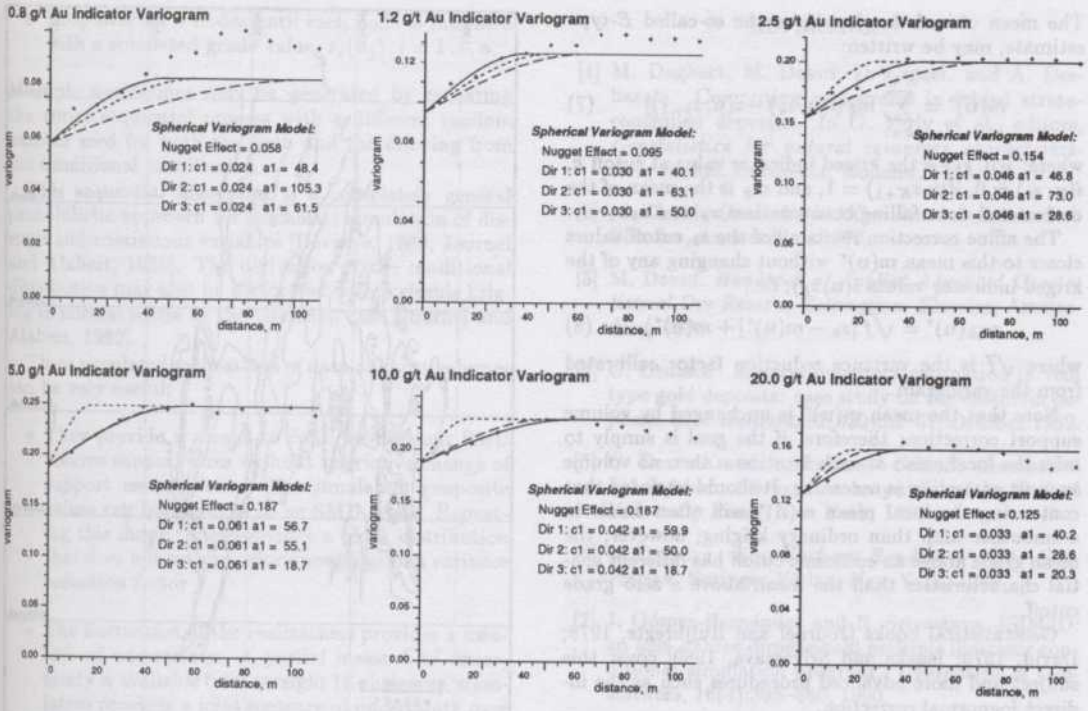

Figure 5: Indicator variograms from one domain of the Porgera gold deposit.

STEP Four: Kriging

At each location the ordinary kriging system is constructed and solved for all K cutoffs. The search radius and the number of data must remain the same from one cutoff to the next; a histogram or ccdf distribution is constructed with a set of data independent of the magnitude of the data values. Since only the variogram changes from cutoff to cutoff an iterative matrix solver may be used to reduce the computer requirements.

After kriging it is necessary to ensure that the estimated ccdf follows the axioms of a cdf. The order relations need to be corrected:

$$F(\mathbf{u}; z|(n)) \geq F(\mathbf{u}; z'|(n)), \text{ for any } z > z' \quad (6)$$
$$\text{and: } F(\mathbf{u}; z|(n)) \in [0, 1]$$

The ccdf values estimated by IK may not satisfy these order relations. The amplitude of such deviations is not usually large [Journel, 1987], deviations greater than 0.01 possibly signal an implementation error such as non-acceptable covariance models.

The following algorithm corrects the estimated ccdf $F^0(\mathbf{u}_0; z_k|(n)), k = 1, \ldots, K$ without introducing any bias:

1. Correct all the order relations by going upward from the lowest cutoff z_1 to the highest z_K to produce a correctly ordered sequence $F^1(\mathbf{u}_0; z_k|(n)), k = 1, \ldots, K$.
2. Correct all the order relations by going downward from the highest cutoff z_K to the lowest z_1 to produce a correctly ordered sequence $F^2(\mathbf{u}_0; z_k|(n)), k = 1, \ldots, K$.
3. Retain the series created by the average of the two:

$$F(\mathbf{u}_0; z_k|(n)) = \frac{F^1(\mathbf{u}_0; z_k|(n)) + F^2(\mathbf{u}_0; z_k|(n))}{2},$$
$$k = 1, \ldots, K$$

There are other algorithms available but a simple sequential correction should be avoided because a bias will be introduced to the estimated average grade.

STEP Five: Volume Support Correction

The IK distributions represent the same volume support as the data (the composited core volume). It is possible to correct these local distributions so that they represent the volume of a selective mining unit (SMU). Volume support correction is important when the drilling data are too widely spaced to clearly identify the high grade portions of the orebody. In these situations the smoothing effect of kriging will provide a biased reserve assessment above any elevated cutoff.

Classical volume support correction procedures correct the quantiles of the IK-derived distribution by reducing the variance and leaving the mean unchanged.

$$m(\mathbf{u})^* = \sum_{k=1}^{K+1} m_k \left[i(\mathbf{u}; z_k) - i(\mathbf{u}; z_{k-1}) \right] \qquad (7)$$

where: $i(\mathbf{u}; z_k)$ is the kriged indicator value at cutoff k, $i(\mathbf{u}; z_0) = 0$, $i(\mathbf{u}; z_{K+1}) = 1$, and m_k is the mean of the declustered data falling between class z_k and z_{k-1}.

The affine correction resets all of the z_k cutoff values closer to this mean $m(\mathbf{u})^*$ without changing any of the kriged indicator values $i(\mathbf{u}; z_k)$, i.e.,

$$z_{V,k}(\mathbf{u})^* = \sqrt{f} \left[z_k - m(\mathbf{u})^* \right] + m(\mathbf{u})^* \qquad (8)$$

where $\sqrt{f}$ is the variance reduction factor calibrated from the variogram.

Note that the mean $m(\mathbf{u})^*$ is unchanged by volume support correction; therefore, if the goal is simply to infer the local mean at each location $\mathbf{u}$ then no volume support correction is necessary. It should be noted that contouring the local mean $m(\mathbf{u})^*$ will often result in a smoother map than ordinary kriging; however, the mean grade above an economic cutoff has different spatial characteristics than the mean above a zero grade cutoff.

Geostatistical books [Journel and Huijbregts, 1978; David, 1979; Isaaks and Srivastava, 1989] cover this subject and more advanced procedures such as the indirect lognormal correction.

STEP Six: Calculation of Recoverable Reserves

In many cases the local mean values (6), or E-type estimates, are the end goal; however, an estimate of the local probability or proportion of recoverable ore $p(\mathbf{u}; z_c)^*$ and the local mean $m(\mathbf{u}; z_c)^*$ above an economic cutoff z_c may also be computed.

The probability or proportion of recoverable ore in an area may be directly obtained from the volume support corrected distribution:

$$p(\mathbf{u}; z_c)^* = 1 - i(\mathbf{u}; z_{V,c})^* \qquad (9)$$

It is unlikely that a cutoff $z_{V,k}(\mathbf{u})^*$ falls exactly on the economic cutoff z_c; a linear interpolation between the known cutoffs is used.

The recovered ore grade is simply the grade above the economic cutoff z_c,

$$m_V(\mathbf{u}; z_c)^* = \frac{\sum_{all z_{V,k} > z_c}^{K+1} m_k \left[i(\mathbf{u}; z_{V,k}) - i(\mathbf{u}; z_{V,k-1}) \right]}{p(\mathbf{u}; z_c)^*} \qquad (10)$$

There may be a partial class between z_c and the first cutoff $z_{V,k}$ greater than z_c. This is handled by linear interpolation.

The chosen level of selectivity (SMU size V) can then be used for reserve reporting and project economics. This is attractive because it offers the possibility of adapting the reserve to the mining selectivity.

Figure 6 shows the E-type estimate for one section through the Porgera deposit.

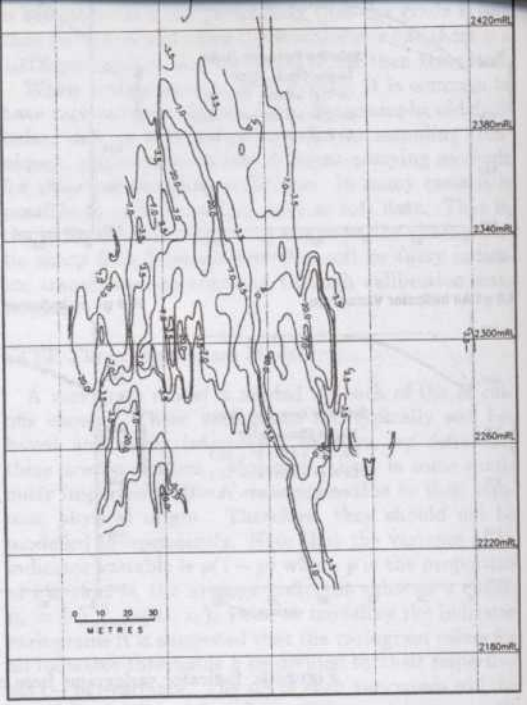

Figure 6: A contour map of the IK E-type estimate for one section through the Porgera gold deposit.

SIMULATION

Until now the focus has been on IK to construct a single orebody model; however, the simulation approach allows alternate equally probable orebody models to be created that match the indicator variograms.

The sequential indicator simulation algorithm [Gomez and Srivastava, 1990; Journel, 1989; Journel and Alabert, 1990] can be employed to create realizations that match the indicator variogram models. The sequential indicator simulation algorithm to simulate the grade $Z(\mathbf{u}_j)$ for n grid nodes consists of the following steps:

1. Start at any node $\mathbf{u}_1$ and derive the conditional distribution of the grade $Z(\mathbf{u}_1)$ given all the nearby data.

2. Draw a realization from that conditional distribution, say, $z_s(\mathbf{u}_1)$, and consider that value as conditioning data for the simulation of all subsequent nodes.

3. Move at random to a second node, say $\mathbf{u}_2$. Derive the conditional distribution of the grade $Z(\mathbf{u}_2)$ given all available original data and the first result, $i_s(\mathbf{u}_1)$. Then, draw a realization from it, say, $z_s(\mathbf{u}_2)$.

4. Loop over all n nodes until each node is informed with a simulated grade value, $z_s(\mathbf{u}_j), j = 1 \ldots n$.

Multiple simulations may be generated by repeating the entire sequential process with a different random number seed for both the path and the drawing from the conditional distribution.

This sequential algorithm is a completely general probabilistic approach for stochastic simulation of discrete and continuous variables [Devroye, 1986; Journel and Alabert, 1989]. The derivation of the conditional distribution may also be performed with a simple kriging of normal scores in the Gaussian case [Journel and Alabert, 1989].

These simulated realizations of data-support volumes can be very useful:

- They provide a means to consider different SMU volume support sizes without restrictive change of support assumptions. The simulated composite values can be averaged to an SMU grade. Repeating this many times provides a block distribution that does not require prior knowledge of a variance reduction factor f.

- The fluctuation of the realizations provides a measure of uncertainty. A partial measure of uncertainty is available from straight IK; however, simulation provides a joint measure of uncertainty over volumes different that the composite data-support volumes.

- They provide a way to predict the mill head grade variation with time. This can be useful to design blending facilities.

CONCLUSIONS

We have attempted to describe one particular approach to orebody modelling based on indicator kriging with a maximum amount of geological input in the form of anisotropy and geological rocktype models. Detailed examples from the Porgera deposit in Papua New Guinea illustrate the concepts discussed.

Acceptable results in terms of realistically representing the orebody and matching production after mine start-up have been obtained when special care was first taken to account for geological constraints and then the geostatistical implementation details.

ACKNOWLEDGEMENTS

We would like to thank the Porgera Joint Venture for allowing publication of this paper, Placer Dome Inc for the availability of computer facilities, and the people who commented on the paper before publication for their suggestions.

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APPENDIX A: A TEST FOR THE VALIDITY OF PARAMETRIC METHODS

Parametric methods like disjunctive kriging, multi-Gaussian kriging, lognormal kriging, and Gaussian based simulation procedures are often proposed as alternatives to indicator methods. A simple check exists for checking the validity of the multivariate normal assumption which underlies all of these methods. The check consists of computing the normal scores variogram, theoretically deriving the corresponding indicator variograms, computing the experimental indicator variogram models, and then evaluating the closeness of the two. A large deviation implies that parametric methods based on the Gaussian assumption are inappropriate.

Computing the normal scores variogram is straightforward; the variogram is computed on the standard normal score transforms of the data [Isaaks and Srivastava, 1989, page 469].

The theoretical derivation of the corresponding indicator variograms is given in Xiao, 1985 and also described with source code in Deutsch and Journel, 1992. For any given quantile, or indicator cutoff, the Gaussian-based indicator variogram may be computed by integrating the bivariate Gaussian distribution.

One property of the multivariate Gaussian distribution is that the indicator variograms show symmetric destructuration around the median, e.g., the indicator variogram for the 10th percentile is the same as the 90th percentile. Moreover, the median indicator variogram presents the most spatial structure, i.e., all other indicator variogram rise more steeply to their sill values.

It would be straightforward to evaluate parametric methods based on multivariate distributions other than the Gaussian distribution. The same procedure could be followed; the difference would be the alternate derivation of the theoretical indicator covariance.

APPENDIX B: ALTERNATIVE IK IMPLEMENTATION

The advantage of the traditional IK approach documented above is that it allows the straightforward incorporation of soft data and constraint intervals. The disadvantages are that resolution is lost within the classes of the distribution, selecting the appropriate cutoffs is not straightforward, and a variogram model must be inferred for each cutoff. The alternative approach described below attempts to overcome these disadvantages but the implementation is not straightforward when soft data and constraint intervals are used.

The idea is to reset the cutoffs defining the estimated distribution locally to the data values falling into the local neighborhood. Points on the cumulative distribution function are estimated where data are available rather than at artificial class boundaries. Figure 7 illustrates this approach.

As shown on Figure 7 there is no probability to be less than the minimum data value or to exceed the maximum data value; however, there is certainly a chance to encounter them. The same procedure as the conventional IK approach can be adopted between classes and beyond the minimum and maximum data values.

Some practitioners who use the conventional approach attempt to use many classes so that resolution of high values is not lost. This solution causes the kriging to become tedious and also creates many order relations problems. This alternative approach eliminates both or these problems, i.e., we have detailed resolution in the upper tail of the distribution and order relations are seldom found because the indicator coding changes with every kriging.

The output is not as neat as the conventional approach, i.e., the number of cutoffs and the actual cutoffs can change at every point being estimated. This is viewed as an advantage because it gives a better indication of what data are being used. Note: even with the conventional approach the cutoffs change after volume support correction; therefore, the probability of exceeding an interesting cutoff is not directly available even if the interesting cutoff was chosen as one of the original cutoffs.

This approach requires that the variogram model be defined with the same number and type of nested struc-

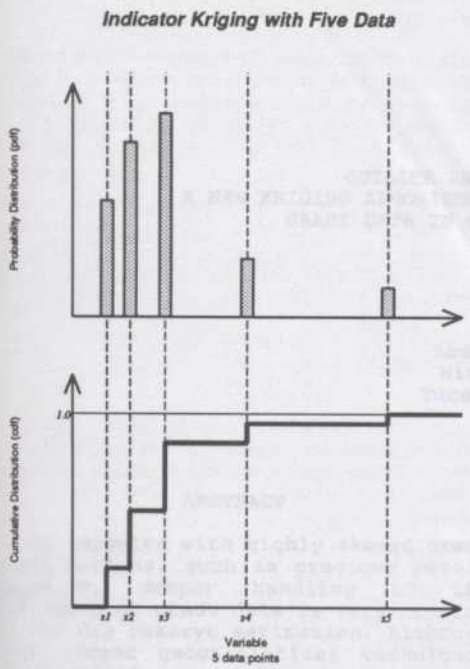

Figure 7: An illustration of IK with the cutoffs identified to the (five) data values retained in the neighborhood.

tures for all cutoffs. The idea is to model the indicator variograms, that have been normalized to a sill of 1.0, where a significant change in the spatial structure can be identified. The model parameters are then linearly interpolated between cutoffs. The practitioner must ensure consistency in the models, a "discontinuity" can be explicitly handled by adding a cutoff in the variogram definition. Note that a completely different variogram structure for a certain range of the data values can be handled by setting certain coefficients to zero.

Both methods can benefit from an iterative matrix solution method after the first matrix inversion. An iterative method will be especially quick if the variogram model changes continuously from cutoff to cutoff. Of course, if constraint intervals are being used then the size of the kriging system could change at different cutoffs. This will reduce the efficiency of an interative matrix solver.