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Improved Reservoir Management Through Ranking Stochastic Reservoir Models

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Abstract

Significant uncertainty exists in the detailed 3-D distribution of lithofacies, porosity, and permeability in every reservoir. Understanding and modeling the heterogeneous 3-D distribution of these rock properties is critical for improved oil recovery and reservoir management. Geostatistical techniques are being increasingly used to generate alternative heterogeneous 3-D reservoir models that are consistent with the available data.

Although a large number of stochastic reservoir models or *realizations* may be available, a small number of realizations are considered in practice. Due to computer limitations, it is only possible to visualize and perform fine-scale full-field flow simulation on a limited number of realizations. Techniques are reviewed in this paper for ranking a suite of geostatistical realizations so that *low-side*, *expected*, and *high-side* realizations may be reliably chosen. Detailed analysis/flow simulation may then be performed on these realizations that somehow bound the uncertainty in the reservoir. Reservoir management is improved when *expected* and *bounding* cases are considered rather than using a limited number of "random" realizations.

This paper reviews a number of methods for ranking geostatistical reservoir models. These methods may be classified into three categories. The first category includes statistical methods such as simple statistics, 3-D measures of connectivity, and connectivity to specific well locations. Methods in the second class are based on approximations to flow simulation, e.g., random walk-type results. The third category is for flow-simulation based methods for a simpler process than that being considered for improved oil recovery, e.g., tracer simulation and flow simulation with coarsened models. The applicability of a number of ranking methods is illustrated with a small example.

There is no unique ranking index when there are multiple flow response variables and no ranking measure is perfect. Nevertheless, the value of ranking realizations will be quantified by examining the expected loss knowing an economic loss function and the true distribution of uncertainty.

Introduction

The primary objective of the application of geostatistical tools for reservoir modeling is to create realistic numerical geological models or *realizations* of the 3-D spatial distribution of lithofacies, porosity, and permeability. An often disturbing fact of geological reservoir modeling is that there are alternative realizations that honor all of the available data equally well and yet yield different reservoir performance predictions. In fact, it is often advertised that "a different realization can be obtained by simply changing a random number seed".

A fundamental principle of geostatistics is that of data integration, i.e., all known data should be honored "by construction" and not left to chance. Of course, this is not always possible; it is difficult to constrain detailed 3-D geological realizations to seismic and historical production data. We are restricting ourselves to *plausible* realizations. There are times when certain realizations would be rejected on the basis of familiarity with the reservoir or data not used in the geostatistical modeling such as production-related historical observations. We are not considering the rejects; only those realizations that meet all basic requirements of reasonableness are considered.

Exact prediction / decision making would require exhaustive knowledge of the spatial distribution of porosity and permeability. At any specific instant in time, there is such a single true distribution of petrophysical properties. This true distribution was created by the complex interaction of many different chemical, physical, and biological processes over geological time and would be accessible only through exhaustive sampling. In all practical situations, the unique true distribution will remain unknown.

This paper addresses the uncertainty due to incomplete information. We do not consider uncertainty due to (1) the volume support or scale difference between core measurements and geological modeling cells, (2) the limited flexibility of our geostatistical modeling techniques to reproduce complex nonlinear features, or (3) numerical approximations in the subsequent flow simulator.

Ranking Realizations

An excellent reference on stochastic modeling is available in the paper by Haldorsen and Damsleth[1]. The idea of ranking stochastic realizations has been around for a while and was first published in the context of stochastic reservoir modeling in 1992[2]. The early paper of Berteig[3] presents an example of uncertainty quantification.

Consider the case where we are interested in the economic ultimate recovery (EUR) of a given improved oil recovery scheme. The true EUR remains unknown until, perhaps, some time in the future when the reservoir is depleted. The uncertainty in the EUR may be estimated, however, by the following sequence of steps:

1. generate L geostatistical realizations of the rock and fluid properties $\{z^{(l)}(\mathbf{u}), \mathbf{u} \in A\}, l = 1, \ldots, L$, where z is a vector random variable representing lithology, porosity, and permeability,
2. perform a detailed flow simulation representing the actual recovery process on each realization $l = 1, \ldots, L$, and
3. construct a histogram of the response variable of interest, $\xi(l), l = 1, \ldots, L$ where $\xi(l)$ is, for example, the EUR of the l'th geostatistical realization.

Risk-qualified decisions can be made knowing this uncertainty and loss functions that quantify the impact of making a mistake for a given error. This risk-qualified decision making requires L to be large enough to adequately define the histograms of each critical response variable. Given the complexity of simulating an improved recovery scheme, this may be a prohibitively large number of realizations.

The ranking of the realizations from highest to lowest EUR $r_\xi(l), l = 1, \ldots, L$ may be determined from the EUR of each realization. For example, if $r_\xi(26) = 1$ then realization number 26 is the realization with the lowest EUR and if $r_\xi(7) = L$, then realization number 7 leads to the lowest EUR. The central idea behind ranking realizations is to use some simpler measure to rank realizations and then run the full flow simulation with fewer realizations, say, the *5%-low*, *50%-expected*, and *95%-high* ones. This would allow bounding the uncertainty without performing a large number of fine-scale flow simulations.

The simpler measure is considered a good ranking statistic when it correctly identifies low and high realizations. That is, for ranking statistic k (ranking index denoted $r_k(l)$), the goodness of a ranking statistic is measured through the correlation between $r_\xi(l)$ and $r_k(l), l = 1, \ldots, L$. Many flow responses depend on the connectivity of the reservoir-quality rock; there are many measures of connectivity that may be quickly calculated without running a full simulator. The goodness of a ranking measure can only be validated by running the detailed flow simulations (there is no longer a need for the ranking measures at that point). Nevertheless, this rigorous validation would be appropriate until confidence in certain ranking measues is established.

Before describing a number of ranking measures, consider some cases where ranking geostatistical realizations is problematic:

1. when each realization leads to nearly the same answer – that is, when the presence of stochastic heterogeneities is more important than the specific differences between the realizations,
2. when the aspect of uncertainty being assessed is easy to calculate – for example, the uncertainty in pore volume may be directly assessed by calculating the pore volume of all realizations,
3. when there are many independent reservoir responses of interest – that is, no single ranking index could lead to a unique reliable ranking.

There are times, however, when significant professional or CPU time is required to evaluate each realization and the number of realizations considered must be limited. In these situations, it is worthwhile to consider ranking the realizations to limit the number of fine scale simulations and yet to obtain an idea of uncertainty in the flow response.

Almost all flow response variables of interest depend on some measure of continuity, connectivity, or tortuosity. The ranking tools increase in complexity and required knowledge of the specific problem; more information must be known about the reservoir and production plan to rank realizations based on the more complex ranking measures. Certain statistical ranking measures do not even require knowledge of the well locations.

The statistical ranking measures require each cell in the geostatistical model to be assigned a *net* indicator:

$$i(\mathbf{u}) = \begin{cases} 1, & \text{if location } \mathbf{u} \text{ is } net \text{ or reservoir quality} \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

in practice, this indicator is defined on the basis of some combination of (1) the lithofacies, (2) a porosity threshold, and (3) a permeability threshold. Sensitivity studies should be considered when there are no evident thresholds for porosity and permeability.

Ranking with Statistical Measures The simplest approach to rank realizations is with simple summary statistics. Given $j = 1, \ldots, N$ geostatistical cells in the reservoir model, one could consider the net-to-gross ratio for each realization, $l = 1, \ldots, L$:

$$ntg(l) = \frac{1}{N} \sum_{j=1}^{N} i^{(l)}(\mathbf{u}_j) \tag{2}$$

the net pore volume:

$$npv(l) = \frac{1}{N} \sum_{j=1}^{N} i^{(l)}(\mathbf{u}_j) \cdot \phi^{(l)}(\mathbf{u}_j) \cdot V(\mathbf{u}_j) \tag{3}$$

where $i^{(l)}(\mathbf{u}_j)$ is the net indicator (1), $\phi^{(l)}(\mathbf{u}_j)$ is the porosity for cell j and $V(\mathbf{u}_j)$ is the gross volume of cell j. Another simple summary statistic would be the the average permeability:

$$\bar{k}(l) = \frac{\sum_{j=1}^{N} i^{(l)}(\mathbf{u}_j) \cdot k^{(l)}(\mathbf{u}_j)}{\sum_{j=1}^{N} i^{(l)}(\mathbf{u}_j)} \tag{4}$$

where $k^{(l)}(\mathbf{u}_j)$ is the horizontal permeability for cell j.

Realizations may be ranked according to static measures of continuity. One measure of this type is available by determining the sets of *net* geological modeling cells that are connected in 3-D space (e.g., as available in Stratamodel's SGM program with the GEOBODY option). The two-step procedure for determining GEOBODIES are (1) assign each cell in a 3-D model a 1 or a 0 code depending on whether the cells are *reservoir quality* or not, and (2) scan through the 3-D binary model aggregating those reservoir quality cells that are connected. The result will be $N_{geobody}$ GEOBODIES or connected bodies each with an associated volume $V_{geobody,j}, j = 1, \ldots, N_{geobody}$.

The distribution of GEOBODY volumes can be used for ranking in a number of ways including (1) the fraction of reservoir quality cells within the first n GEOBODIES (any arbitrary number can be chosen): the higher this fraction the more "connected" the model, (2) the number of GEOBODIES that include 75% (or some other arbitrary fraction) of the total reservoir quality cells, or (3) the totuosity (defined as the surface area to volume ratio) may also be calculated and used for ranking.

If the well locations are known, the connected pore volume within some radius could rank the available realizations. The volume connected to a well location is easily calculated once the GEOBODIES have been determined. The number of cells connected to a well location in 3-D space ranks the realizations: the larger the number of connected cells the "higher" the model ranks.

The cumulative volume connected between multiple wells, for example, the volume of *net* cells connected between injector-producer well patterns could also be used for ranking realizations. This could be calculated by determining which GEOBODIES are jointly intersected by both wells and then limiting the contribution within some practical drainage radius.

Ranking with Simple Flow Models There are random walk algorithms that measure "dynamic" continuity between injecting and producing locations. These methods often call for a solution to the pressure field (single phase flow) given assumed well rates. Particles are then tracked through the media and the distribution of "times" or "lengths" between injecting and producing wells provides a measure of connectivity that could be used for ranking.

There are other relatively simple and fast flow models including (1) tracer simulation, (2) simulation based on a network of 1-D stream tubes, and (3) a water flood simulation in lieu of a more complex miscible or compositional-type simulation. The time of first water arrival is likely a good measure for the breakthrough of other miscible components in a more complex process.

Another ranking approach is to use the correct physics or flow equations but with the geological models scaled to such a coarse resolution that the computer time is acceptable. The coarseness of the underlying grid will compromise the direct usefulness of the results. Nevertheless, the relative ranking of the results may be used to rank the underlying geological realizations.

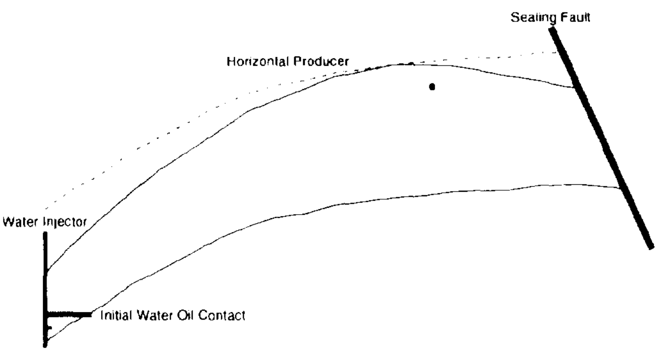

Figure 1: Example 2-D flow scenario with vertical water injector and horizontal producing well.

An Example

For illustration, a 2-D cross sectional example will be shown to illustrate the ranking measures described above. Although the example is synthetic, it shares many features with realistic reservoir examples. This example is fashioned after a fairly high net-to-gross (about 80%) fluvial environment with discontinuous shale remnants. The shales have no effective porosity or permeability. The sand/shale (lithology), porosity, and permeability distributions are constructed to honor the profile of properties at the water injection well and the horizontal production well, see Figure 1. This representative cross section is 1000 meters long (200 - 5 meter blocks) and about 20 meters thick (40 - 0.5 meter blocks). In practice, this is one section of many connected to the horizontal production well.

The flow characteristics of this cross section are not known exactly due to uncertainty in the geologic model. 250 realizations of lithology, porosity, and permeability were generated and flow simulation performed on all 250 realizations. The flow response variables of interest are (1) the time of first water arrival at the horizontal producer, (2) the water cut after the injection of one pore volume of water, and (3) the fraction of oil recovered when the water cut reaches 90%, see below.

The 250 realizations of lithology were constructed with sequential indicator simulation[4]. Although the indicator data at the well locations is honored; the average shale proportion in each realization varies due to ergodic fluctuations, see Figure 2. To be realistic, the vertical variogram range was considered known at 1.0 m and the horizontal variogram range was considered to be follow a triangular distribution (minimum of 50 m mode of 100 m and maximum of 250 m).

Porosity models were generated with sequential Gaussian simulation for the sand lithology only; shales were assigned zero effective porosity. The 250 porosity realizations were constrained to the well data, a histogram from the well data, a known vertical variogram, and an uncertain horizontal variogram. Permeability was assigned to each realization with simulated annealing to honor the permeability at the well locations, a porosity-permeability cross plot (0.6 correlation coefficient), and a variogram model.

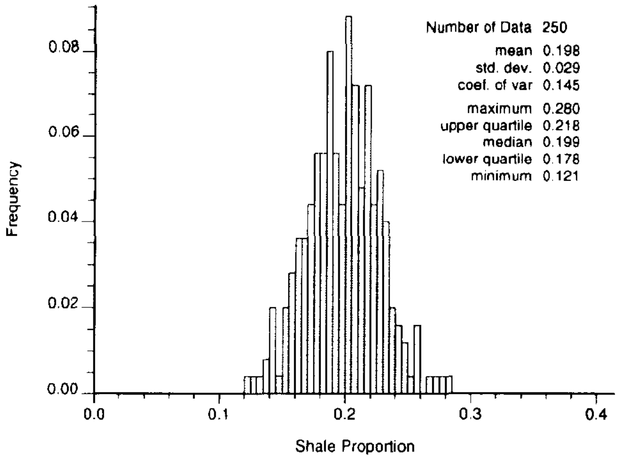

Figure 2: Histogram of shale proportion in the 250 geostatistical realizations.

Flow Modeling ECLIPSE[5] was used to perform a water flooding simulation at the detailed resolution (8000 blocks). A single vertical injector and a horizontal producer perpendicular to the flow direction were assumed. Both wells were assumed to be pressure controled. The initial water-oil contact is shown on Figure 1 and realistic water and oil relative permeabilities were chosen. In this case it is possible to perform the detailed flow simulation on all 250 realizations within a reasonable time. In general, with larger 3-D models and a greater number of wells, it would only be possible to consider a limited number of flow simulations.

The following five flow response variables were used to summarize the full flow simulation responses:

1. ξ_1 = *breakthrough* = the fractional pore volume injected at water breakthrough to the horizontal producing well,

2. ξ_2 = *ultimate recovery* = the recovery when the water cut at the producer exceeds 90%, and

3. ξ_3 = *final rate* = the total fluid rate when the water cut at the producer exceeds 90%.

4. ξ_4 = *oil rate* = the oil production rate at breakthrough,

5. ξ_5 = *intermediate recovery* = the recovery after 2 pore volumes of water injection,

Histograms of the 5 response variables for all 250 realizations are shown on Figure 3. The relatively large variance of each realization is due to the 2-D (versus 3-D) example and the heterogeneous distributions of rock properties. A priori we expect this large variance to make ranking easier and more important.

The 250 realizations may be ranked according to each of these five response variables. Table 1 shows the correlation between the ranking indices provided by $\xi_i, i = 1, \ldots, 5$. Some remarks: (1) the negative correlation between r_{ξ_1} (breakthrough) and r_{ξ_3} (final rate) is explained by noting that the faster the breakthrough the faster the well is shut in and the lower the total fluid rate

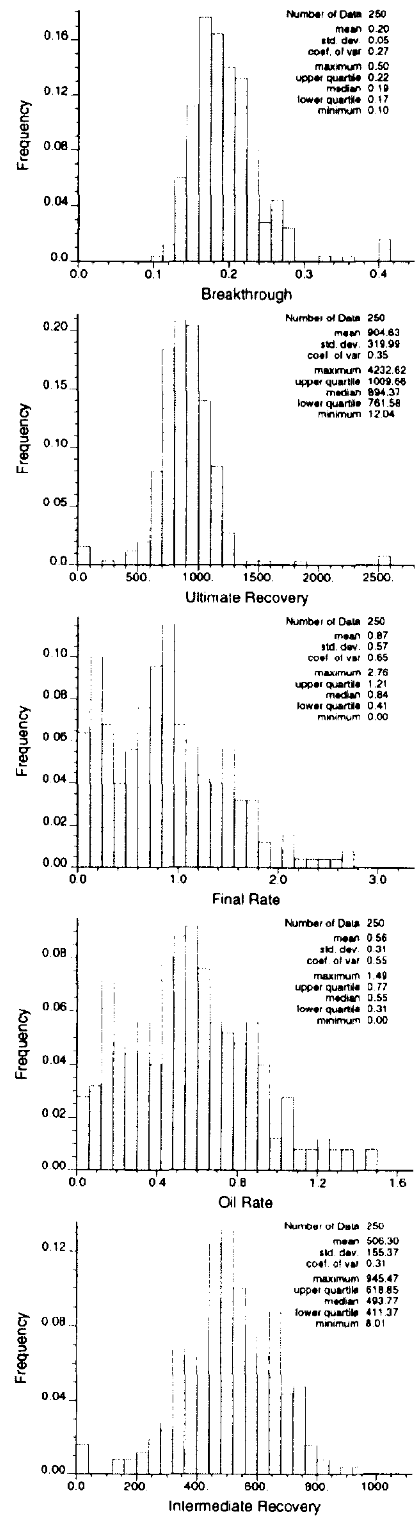

Figure 3: Histogram of the response variables for all 250 geostatistical realizations.

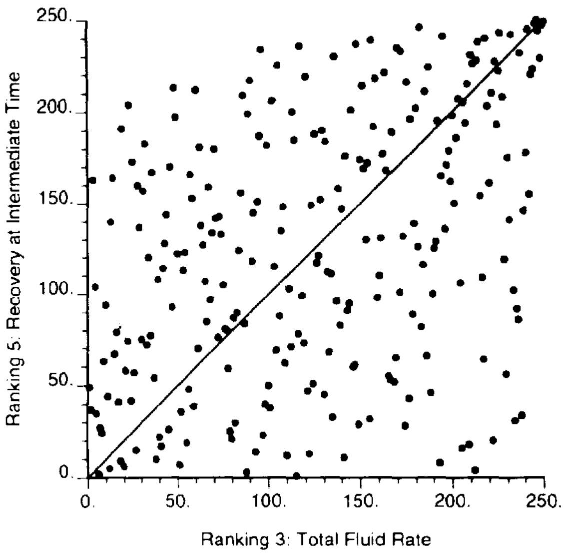

Figure 4: Scatterplot between the ranking index due to response 5 (intermediate recovery) versus the ranking index due to response 3 (final rate). The linear correlation coefficient is 0.39.

	Response Variable Ranking				
	r_{ξ_1}	r_{ξ_2}	r_{ξ_3}	r_{ξ_4}	r_{ξ_5}
r_{ξ_1}	1.00	0.24	-0.16	-0.08	0.20
r_{ξ_2}		1.00	0.15	0.21	0.78
r_{ξ_3}			1.00	0.98	0.39
r_{ξ_4}				1.00	0.46
r_{ξ_5}					1.00

Table 1: Correlation coefficients between the ranking indices provided by the five reference flow simulation responses.

will be at that time, (2) the strong positive correlation between r_{ξ_3} (final rate) and r_{ξ_4} (oil rate) is explained by noting that both ξ_3 and ξ_4 are measures of effective permeability of the system, and (3) the good correlation between r_{ξ_2} and r_{ξ_5} is explained by noting that ξ_2 and ξ_5 are both measures of oil recovery at two different times. The scatterplots between the ranking measures should be checked to ensure that a few outlier points are not destroying an otherwise good correlation. As an example, this is not the case on Figure 4.

An important point to note is that, in general, there is poor correlation between these reference ranking measures. The implication is that there is no unique ranking. A different ranking will have to be used for each response variable of interest.

Figure 8 (at the end of the paper) shows gray scale maps of the permeability for the low and high ranking realization for all five response variables. The high ranking realizations have more connected dark pixels (high permeability). The results appear reasonable, i.e., the realization with the fastest breakthrough (upper right) has a connected high permeability streak from the injection well.

	Statistical Ranking Measures						
	r_{s_1}	r_{s_2}	r_{s_3}	r_{s_4}	r_{s_5}	r_{s_6}	r_{avg}
r_{ξ_1}	0.16	0.15	0.07	0.19	0.00	0.18	0.12
r_{ξ_2}	0.38	0.37	0.00	0.32	0.27	0.20	0.27
r_{ξ_3}	0.51	0.52	0.32	0.43	0.24	0.36	0.41
r_{ξ_4}	0.54	0.55	0.32	0.46	0.27	0.39	0.44
r_{ξ_5}	0.48	0.46	0.12	0.42	0.40	0.32	0.38
	0.41	0.41	0.17	0.36	0.24	0.29	0.32

Table 2: Correlation between statistical ranking measures and reference flow response ranking measures.

Statistical Ranking Measures For brevity, the results from a limited number of statistical ranking measures are shown here:

1. r_{s_1} = net to gross ratio (2)
2. r_{s_2} = net pore volume (3)
3. r_{s_3} = arithmetic average of permeability (4)
4. r_{s_4} = total connected volume within 5 GEOBODIES
5. r_{s_5} = total volume connected to the injecting and producing well locations
6. r_{s_6} = total volume connected between the injecting and producing wells

Table 2 shows the correlation between the true ranking and the statistical ranking. Some remarks: (1) the net-to-gross ratio is the best overall ranking measure, (2) the total connected volume provides the best ranking for breakthrough, (3) the ranking provided by the average permeability and connectivity to well locations do not perform well in this case, and (4) r_{avg} (the ranking based on the average of $r_{s_i}, i = 1, \ldots, 6$) does not perform better than the net-to-gross (r_{s_1}) or connectivity ranking (r_{s_4}).

Flow-Based Ranking Measures The realizations were scaled up so that the flow simulation could be run much faster. A 16:1 scale-up resulted in a 24:1 savings in CPU time. Upscaling of absolute permeability was performed by applying no-flow boundary conditions and solving the single-phase steady-state flow equations. No changes were made to the relative permeability curves. The coarse-scale flow response variables are not considered trustworthy due to numerical dispersion; however, the realization ranking provided by these responses performs as well as the statistical ranking measures, see Table 3. The poorest correlation is for the breakthrough time; the connectivity of the high and low permeability streaks are not captured in the scale-up while the average permeability characteristics are maintained fairly well (probably due to the single-phase flow-based upscaling).

Figure 5 shows the correlation between coarse and fine scale flow responses for a number of scale-up ratios. There is surprisingly little difference between the results at 64:1 and 16:1. The correlation drops significantly at a 256:1 scale-up ratio.

breakthrough	0.27
ultimate recovery	0.38
final rate	0.56
oil rate	0.56
intermediate recovery	0.40

Table 3: Correlation between reference flow response ranking and coarse scale flow response ranking.

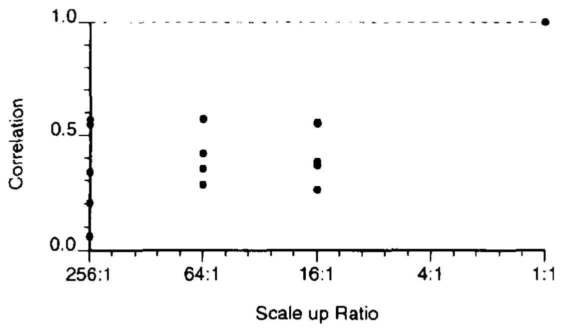

Figure 5: Correlation between the reference flow response ranking and coarse scale flow response ranking versus the scale up ratio.

Decision Making with Loss Functions One motivation for ranking realizations is to pick *low-side, expected,* and *high-side* realizations. Almost always we want to know the cumulative probability to assign to the low and high realizations, e.g., is the low realization the 1% quantile or the 10% quantile? Less often, we want to use a limited number of realizations to define the full CDF of the response variable so that optimal decisions can be *made*[6,7].

Figure 6 illustrates this idea with the distribution of ultimate recovery. The histogram and CDF of the 250 ultimate recovery values are shown at the top. An assumed loss function is shown in the center of Figure 6. This loss function quantifies the economic impact of estimating too much or too little recovery. This loss function was chosen arbitrarily for this example. Finally, knowing the CDF and the loss function we can calculate the expected loss associated to any estimate of ultimate recovery (bottom of Figure 6). Note that the optimal estimate of ultimate recovery is 770 (arbitrary units).

It is possible to use a limited number of realizations, chosen by some ranking measure, to define some points on the CDF curve. Assuming some interpolation (say, linear) between these CDF points we can make the same L-optimal estimate. Of course, this estimate is poorer than we would get with the procedure shown in Figure 6 since the ranking is not perfect and the interpolation between the known CDF points is an approximation. Figure 7 illustrates the "Ranked" approximation to the true cdf. In this case, the net-to-gross ratio was the best statistical ranking for the ultimate recovery. The L-optimal estimate from the "Ranked" CDF is 800 (versus 770 from the true distribution) and the expected loss (read from the true curve) increases by 1.1% from 53.29 to 53.87. Depending on the units of the loss function (dollars in all practical problems), this amount quan-

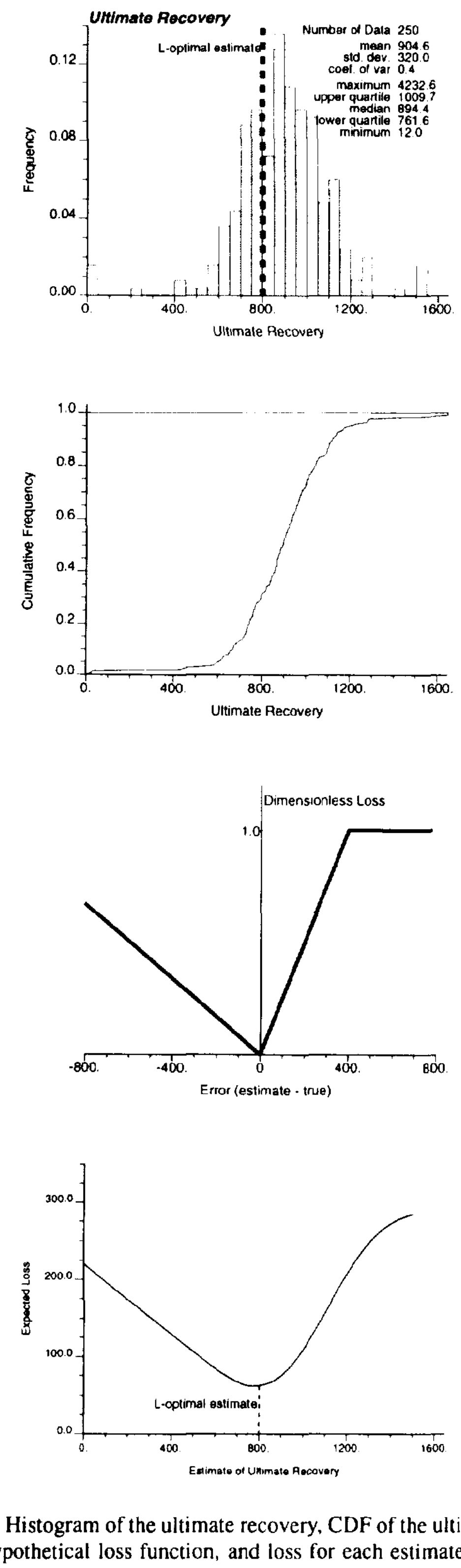

Figure 6: Histogram of the ultimate recovery, CDF of the ultimate recovery, hypothetical loss function, and loss for each estimate, the L-optimal estimate is 730.

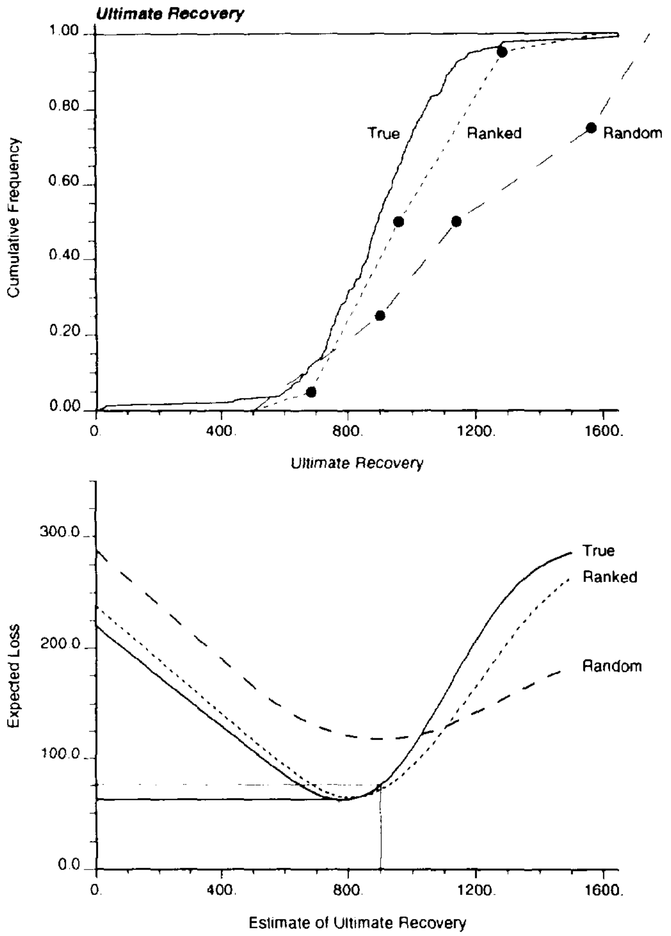

Figure 7: CDF and expected loss for the true distribution of ultimate recovery, the approximation obtained when using the three ranked realizations, and the approximation obtained using the first three realizations.

tifies the value of running more flow simulations to more precisely quantify uncertainty.

Figure 7 also shows an estimated CDF and expected loss curve labeled "Random". In this case, the first three realizations were taken, sorted from smallest to largest, and then assigned to the 25%, 50% and 75% quantile. Note that the CDF deviates considerably from the true cdf and the L-optimal estimate is 900 (versus 770 from the true distribution) with an expected loss (read from the true curve) of 64.80 (a 20% from the true value). Note that the ranked CDF is closer to the true CDF and the L-optimal estimate is better. Considering loss functions allows a value to be placed on ranking the realizations.

Discussion

This paper has made a number of points regarding the ranking of geostatistical realizations:

- Ranking and selecting geostatistical realizations is necessary since it is often impossible to perform reservoir simulation on a large number of geostatistical realizations.
- Ranking must be performed according to a scalar quantity; assumptions must be made to rank realizations when there are multiple flow response variables.
- Ranking realizations is important when there is a significant variation in the response variable, e.g., the range in the breakthrough time was from 0.17 to 0.5 pore volumes of water injected.
- For the example presented here the net-to-gross ratio is a good ranking index. Flow simulation at a coarse scale also worked well even with scale-up ratios as high as 64:1.
- The use of loss functions and the expected loss is an approach that could be used to measure the value of ranking realizations or to justify the CPU-expense of running multiple fine-scale detailed flow simulations.

Future Work

Any fast flow simulation, regardless of implicit assumptions, is a good candidate for ranking. Promising techniques based on streamlines and random walk simulation have not been demonstrated in this paper. Although no ranking measure can overcome the non-uniqueness inherent with multiple response variables, these techniques have potential for improved ranking.

A greater number of case studies are required to determine those ranking measures which are simple and robust. When possible, we may want to avoid the task of ranking realizations and pay the CPU-cost of running more detailed flow simulations. In this case, we might still consider ranking the realizations and then running a limited number of *even more* detailed flow simulations. Given the significant geological heterogeneity and significant size of most reservoirs (relative to the scale of core data), the related problems of scale-up and ranking realizations will be the subject of future work.

Nomenclature

γ	=	semivariogram
$\mathbf{h}$	=	lag separation vector
$\mathbf{u}$	=	location coordinates vector
ϕ	=	pososity
k_h	=	horizontal permeability
i	=	node index
j	=	node index
l	=	realization number index
$r_\xi(l)$	=	rank order of (l) according to ξ
$\xi(l)$	=	flow response variable

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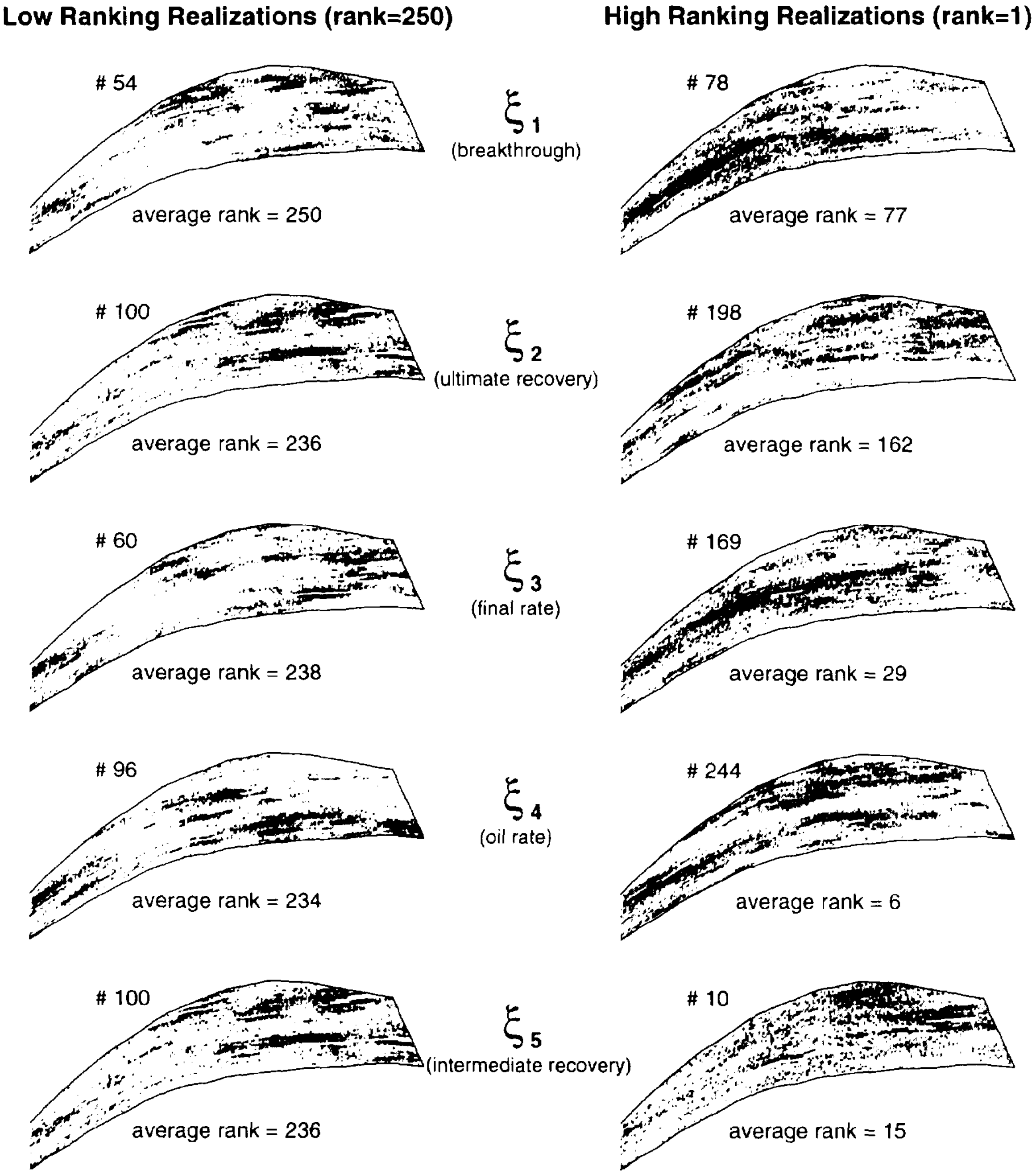

Figure 8: Lowest and highest ranking realizations for each of the five response variables. The gray scale maps are of permeability (white is low and black is high permeability). In all cases there are more high permeability cells in the high ranking realizations.