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The Quality Map: A Tool for Reservoir Uncertainty Quantification and Decision Making

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Abstract

The parameters that govern fluid flow through heterogeneous reservoirs are numerous and uncertain. Even when it is possible to visualize all the parameters together, the complex and nonlinear interaction between them makes it difficult to predict the dynamic reservoir responses to production. A flow simulator may be used to evaluate the responses and make reservoir management decisions, but normally only one deterministic set of parameters is considered and no uncertainty is associated with the responses or taken into account for the decisions.

This paper introduces the concept of a “quality map”, which is a two-dimensional representation of the reservoir responses and their uncertainties. The quality concept may be applied to compare reservoirs, to rank stochastic realizations and to incorporate reservoir characterization uncertainty into decision making, such as choosing well locations, with fewer full field simulation runs.

The data points necessary to generate the quality map are obtained by running a flow simulator with a single well and varying the location of the well in each run to have a good coverage of the entire horizontal grid. The “quality” for each position of the well is the cumulative oil production after a long time of production.

The geological model uncertainty is captured by multiple stochastic realizations. One quality map is generated for each realization and the difference between the realization maps is a measure of the uncertainty in the flow responses. For each cell, the lower quartile of the local distribution of quality is extracted to build a map, which can be used for decision making accounting for uncertainty.

The methodology for building the quality map is presented in detail and the applications of the map are demonstrated with fifty realistic reservoir models.

Introduction

The uncertainty in any geological model is unavoidable given the sparse well data and the difficulty in accurately relating geophysical measurements to reservoir-scale heterogeneities. The two largest static uncertainties, the reservoir geometry as defined by surfaces and the petrophysical property distributions, may be quantified by geostatistical methods.

When no uncertainty is considered, reservoir decisions may be made using a deterministic geological model and some optimization algorithm (Ref. 3 and 4). Once a decision is made, such as the number, type and location of the wells, there are techniques to transfer the uncertainty in the static parameters to the flow responses through flow simulations (Ref. 2 and 8).

However, uncertainty ought to be taken into account for decision making. Stochastic models of the static parameters may be used directly for decisions that do not require flow simulations (Ref. 9 and 10). However for most of the reservoir decisions, the value of each option needs to be estimated using a flow simulator.

For a limited number of production scenarios and geological models, the “full approach” of obtaining one flow response for each reservoir model and each production scenario and then choosing the scenario that is the best in average over all the models, may be applied (Ref. 7, 11 and 13). Decisions with more complex problems and with a larger number of models may be made based on response surfaces generated with experimental design techniques (Ref. 1 and 5).

Multiple geostatistical realizations can be ranked to decrease the number of models to process through a flow simulator for decision making. Deutsch and Srinivasan[6] present several ranking techniques, showing that there are limitations in all of them but the best results are obtained with some flow simulation based technique.

Vasantharajan and Cullick.[12] present the concept of a quality measure of the reservoir for locating wells. The measure is a combination of static characteristics of the reservoir and does not account properly for the dynamic and nonlinear interaction between the parameters.

The quality map, introduced in this paper, uses a flow simulator to integrate all the parameters that affect the flow of fluids through heterogeneous reservoirs and to ensure that the proper dynamic interactions between them is taken into account.

Modeling the geological uncertainty through multiple geostatistical realizations and building one quality map for each one of the realizations, uncertainty can be integrated into decision making, such as well location.

The methodology for generating the quality value of each cell of the map for a specific realization is presented. Then it is shown that a few data points are sufficient to interpolate by kriging a map for each realization. How to generate the mean quality map, the map of the quality uncertainty and the lower quartile quality map using the local quality distribution over the quality map of all the realizations is also presented.

One very large case study using fifty different realistic reservoirs is presented to demonstrate the following uses of the quality maps: (1) well location for a specific realization using the quality map of that realization; (2) well location in a manner that is robust with respect to the uncertainty and reducing the number of scenarios in the "full approach", using the lower quartile quality map; (3) reduction of the number of realizations in the "full approach" to just one, by identifying a representative realization for each scenario, (4) ranking of realizations for several purposes using the quality map of all realizations; (5) reservoir comparisons using the average values of the mean quality and uncertainty quality maps.

Quality Maps

The quality map is, by construction, a map of "how good the area is for production". The quality at a location is a measure of the expected oil production if a well were to be placed at that location (with no other wells in the reservoir). The map is generated by running a flow simulator multiple times and varying the position of a single well in each run to cover the entire horizontal grid. Each run evaluates the quality for the horizontal cell where the well is located.

The quality unit is the cumulative oil production (N_p) after a certain time of production. The total time of production depends on the size of the reservoir but must be long enough to allow the well to produce all possible oil, given the operational controls of the well.

In the simulations, the well is completed in all oil layers with automatic shut down of the layer when some water (or gas) cut limit is reached. No rate limits are imposed, allowing the well to produce the maximum it can. Only a minimum bottom hole pressure (BHP) and a minimum oil rate must be specified in accordance with the real operational limitations of the well.

Considering the production of a vertical well from the three-dimensional geological model translates three-dimensional results into a two-dimensional grid. The flow simulator accounts for all the interactions between variables and returns one single value of quality (N_p) for each well position. The greater the horizontal transmissibility around the well the higher the initial rate, the longer the production time before the minimum BHP is reached and the greater the quality value (total N_p). Also the smaller the transmissibility between the aquifer and the well, the smaller will be the water production and the greater the total N_p for the same final BHP.

Fig. 1 shows, as an example, some of the parameters that affect the oil production and presents the quality map that integrates all the parameters. The smaller the top depth the greater the final cumulative oil production because the thicker the oil column. The higher the oil volume the better. The higher the horizontal permeability in the upper layers where most of the oil production occurs the better. The lower the vertical permeability between the aquifer and the production layers the better. Several other parameters also affect the flow of fluids inside the reservoir and only a flow simulator is capable of accounting for all the interactions between the parameters. The quality map (shown in the lower right corner), built with the results of flow simulations integrates all the parameters in a single value for each cell.

In the example of **Fig. 1**, the quality map was built using a specific realization (Realization 1) but that realization may not be the best model of the true reservoir. The uncertainty in the geological model due to the sparse sampling of the reservoir by wells is unavoidable but it can be modeled through multiple equiprobable realizations.

Ideally one quality map ought to be built for each realization with the well in each cell of the horizontal grid but that is too CPU demanding. The alternative is to obtain only some points for every realization and then to interpolate by kriging a quality map for each realization. The number of points necessary depends on the reservoir heterogeneity but 10% of the total number of cells has been found to be sufficient, provided the points are well distributed over the entire grid. A mix of deterministic and random selection of the positions for the well is appropriate to sample each realization. The sampling positions should change for each realization in order to guarantee that each cell is sampled at least once.

Using the quality maps for all the realizations, two new maps can be produced: the mean quality map and the map of quality uncertainty. The mean quality value of each cell is obtained by taking the mean over all realizations of that cell's quality values. The average value of the mean quality map is a measure of the production potential of the reservoir. The quality uncertainty of each cell is obtained by calculating the standard deviation over all realizations of that cell's quality values. The average value of the standard deviation quality map is a measure of the uncertainty in the reservoir flow responses.

For decision making purposes, such as choosing the location of vertical producer wells, the quality uncertainty must be considered. A good well location has high mean quality and low uncertainty.

An L-optimal quality map can be generated if a loss function is defined. A company may have translated its objectives with respect to minimizing risk or maximizing possible production into a loss function. However, if no loss function is available, an appropriate estimate for this kind of decision is the lower quartile value of the distribution. The lower quartile is the theoretical result of L-optimal estimates when the loss of overestimating the quality value (estimation greater than real value) is three times greater than the loss of underestimating the quality value. Using the lower quartile values to decide between two cells of equal mean quality where to locate a well, the cell with smaller uncertainty will be preferable.

Fig. 2 presents the quality maps of two realizations (with the positions of the data points that were used in the kriging), the mean quality, the quality uncertainty and the lower quartile quality maps.

The kriged quality map of Realization 1 shows very little difference from the quality map of the same realization in **Fig. 1**, which was built not by a limited number of samples and then kriging but exhaustively with a quality value obtained for every cell of the horizontal grid.

The positions of wells that provided data for the generation of the realizations are shown in the mean quality, quality uncertainty and lower quartile quality maps. It is evident that the uncertainty is small near to the data points and increases as the distance to the wells increases. The greater the uncertainty (quality standard deviation) the greater the relative difference between the mean quality value and the lower quartile quality value for the same cell.

Using the Quality Maps

In order to demonstrate the applications of the quality map and to determine its benefit, a very large case study with fifty different reservoirs was undertaken. Several reservoirs are needed to provide reliable results and conclusions.

The reservoirs were generated using geostatistical tools and their volumes, productivity and lithology mimic medium size offshore sandstone/shale oil reservoirs with strong bottom aquifers. Each reservoir was sampled by five wells whose positions are shown in the three last maps of **Fig. 2**. With only the data from the wells plus a smooth image of the true top, twenty stochastic realizations were generated for each reservoir. The smooth image of the true top represents the contribution of seismic data.

It is beyond the scope of this paper to present all of the details needed to reconstruct the fifty reservoirs and twenty stochastic realizations of each. However, a number of comments may be made: different geostatistical techniques were used to create the realizations than were used for the truth; a realistic amount of data was used in each case; and uncertainty in the geostatistical parameters was considered.

The quality maps were used to address five different reservoir engineering problems: (1) Finding the best locations for wells; (2) Including uncertainty in decision making; (3) Determining a representative realization; (4) Ranking realizations; and (5) Comparing reservoirs.

1. Locating Wells Since the quality map accounts for the interactions between the reservoir heterogeneity and the flow of the fluids, it should be a good map for locating wells. This use was evaluated by comparing the results of locating the wells with the quality map and locating the wells with the oil volume map.

For this demonstration no uncertainty was considered. Only the first realization (Realization 1) of each reservoir, taken as a deterministic model, was used for both methods (quality and oil volume). The methods were compared with respect to the profit obtained with production of the wells located with each map. Profit was evaluated as the discounted oil production (discounted rate $= 7.5\%$) minus the cost of the wells (cost of one well $= 150,000 m^3$ of oil) minus a percentage of the water production (3%) as operational costs.

Eleven different numbers of wells were located with both maps for each reservoir and the comparison between the two methods was made with the mean profit over the eleven results. The five sampling wells were always used for production too. Hence for a total number of fifteen wells for example, only ten wells needed to be located.

An optimization program was developed to locate the wells based on the maximization of the total quality allocated to the wells. For a given configuration of a particular number of wells, the program first allocates each cell to the closest well. Then the program evaluates the quality of each well (Q_w) by adding all the quality values of the cells (Q_c) that belong to that well, weighting the quality of each cell by an inverse distance weight (w_c). The total quality (Q_t) is the sum of all the well qualities and that is what the program seeks to maximize.

$$w_c = \frac{1}{a \cdot d_{w-c}^b} \text{ and } w_c = 1 \text{ for } d_{w-c} = 0$$

$$Q_w = \sum_{c=1}^{nc_w} Q_c \cdot w_c$$

$$Q_t = \sum_{w=1}^{nw} Q_w$$

where: nc_w = number of cells that belong the the well w
nw = total number of wells

The initial configuration for each additional well is obtained sequentially by searching the entire grid for the best position (maximum Q_t) of that well given the location of the previous wells. The optimization of the configurations is made taking two wells at a time and trying all the possible combinations for the positions of both wells inside an grid area defined by one cell to each side of the previous well location (total of nine possible locations for each well). All the combinations of wells taken two at a time are tried. Every time a change in a well location occurs, the combination of two wells is revisited because for two wells that were tried before without changing, a better location may be found for at least one of the wells after a change in a third well location. A configuration is final when no further improvement in Q_t is possible after trying all the combinations of two wells at a time.

The weights w_c affect the well location and the resulting profit. The higher the exponent b, the more concentrated the wells in the high quality area, but wells too concentrated in the same area are not optimal. In this case study, after some sensitivity analysis, the coefficients a and b were defined as $a = 2$ and $b = 2$ and used for all reservoirs.

The same optimization program was used to locate wells with the oil volume map too, just replacing quality by oil volume.

Fig. 3 shows the locations of three different numbers of wells obtained with the quality map and with the oil volume map of Realization 1. The profits evaluated for this realization and with each one of the scenarios are also presented, showing the benefits of the quality map over the oil volume map for locating wells.

The quality map provided better well locations than the oil volume map for 84% of the reservoirs. Over the fifty reservoirs, the average gain per reservoir of locating the wells with the quality map instead of with the oil volume map was $267 Mm^3$ of oil. This gain is equal to 1.8 times the cost of one offshore well and it represents an increment of 4.5% in the reserves.

The fact that for 16% of the reservoirs the oil volume map worked better than the quality map for locating wells in this case study is explained by the setting of the same well controls *(rate limit = none, BHP limit* $= 50\frac{Kgf}{cm^2}$ and *water cut limit* $= 97\%$*)* and total time of production *(20 years)* in the quality map generation and the same a and b values *(*$a = 2$ *and* $b = 2$*)* in the optimization program for every reservoir. Through sensitivity analysis, though, it is always possible to find the appropriate parameters to use in the quality map generation and in the optimization program in order to get better locations with the quality map than with the oil volume map for any reservoir.

2. Accounting for Uncertainty in Decision Making Decision making with a specific realization (like Realization 1 in the previous section) does not account for any uncertainty. The goodness of the decision depends on the luck of choosing a realization that is close to the true reservoir.

One approach to make the decisions robust to the geological uncertainty is to model this uncertainty through several equiprobable realizations and use all the realizations in the decision making. The steps of this "full approach", using the quality map, to decide the best number of vertical producer wells are;

1. Generate several realizations of the reservoir structure and petrophysical properties: $l = 1, ..., L$. Each reservoir model l is a complete specification of all static properties such as geometry, porosity and absolute permeability.

2. Define the range of necessary number of wells (nw) for each reservoir. This range, from nwi to nwf, should be wide enough to guarantee that the best economic number of wells (nw^*) is included. A few sensitivity flow simulation runs may be necessary.

3. For each number of wells, optimize the location of the wells using the lower quartile quality map and maximizing the sum of the qualities associated with the wells ($Q_t = \sum Q_w$).

4. Calculate the profit for each number of wells and each realization: $P_{nw,l}, nw = nwi, ..., nwf; l = 1, ..., L$. The fluid production and injection curves are obtained by running a flow simulator and the defined quantitative measure of profit is applied over the scenario specifications and curves for each situation (nw and l).

5. Determine the optimal number of wells nw^* by some type of L-optimal profit. In the simplest case this will be based on expected values $E\{P_{nw}\} = \frac{1}{L}\sum_{l=1}^{L} P_{nw,l}$. The optimal number of wells nw^* is such that $E\{P_{nw^*}\}$ is maximum.

The purpose of the case study here is to compare the results of the decision making with and without accounting for uncertainty. The true profit of the scenario defined using the full approach is compared to the true profit of the scenario defined using just one realization (Realization 1).

The same range of eleven different numbers of wells for each reservoir that was used in the previous comparison (quality map versus oil volume map for Realization 1) was used here for both approaches. For the one realization approach, the best configuration for each number of wells using the quality map of Realization 1 was already defined and also the profits were already evaluated. Thus for this approach, the best number of wells was defined simply as the one with maximum profit.

For the full approach, the best configuration of each number of wells was defined using the optimization program and the lower quartile quality map. A flow simulator was run for each number of wells and each realization and the profit was evaluated for each situation. A mean profit over all the realizations was calculated for each number of wells and the best number of wells was defined as the one with maximum mean profit.

The best scenario defined with each of the two approaches was applied in the true reservoir and the true profits were used to compare the approaches. The access to the true results of the decisions was possible in this case study because the "true" (synthetic) reservoirs are known.

Accounting for uncertainty in the decision making using the full approach provided better location of wells for 58% of the reservoirs. The average gain, among the fifty reservoirs, of the full approach over the one realization approach was $193 Mm^3$ of oil. This gain is equal to 1.3 times the cost of one offshore well and represents an increment of 3.7% in the reserves.

The apparent contradiction between the significant gain ($193 Mm^3$ of oil) and not so significant number of reservoirs (58%) for which the full approach was better than just one realization is explained by the fact that the average loss ($158 Mm^3$) of using the full approach in the cases where the Realization 1 led to better decisions was much smaller than the average gain ($447 Mm^3$) of the full approach for the other cases. This suggests that one realization may be, by luck, closer to the true reservoir than the mean of the realizations for some situations but the probability of very bad decisions using just one realization, taken at random, is also high.

3. Determining a Representative Realization For many situations the number of possible production scenarios and/or the number of realizations and/or the complexity of flow model may be so large that the step number 4 of the full approach would be too CPU demanding. An option to reduce the computational effort of this step is to identify a representative realization and to use only this realization to run the flow simulator for each scenario.

The representative realization is the realization that gives a flow response closer to the mean response (over all realizations) than any other of the realizations for a specific scenario. Ideally the best scenario defined running the flow simulator only with the representative realization for each scenario would be the same as the one defined with the full approach.

The quality map is the right "measure" of each realization to be used for identifying the representative realization. The idea is to select the realization whose quality map is closer to the mean quality map than any other of the realizations. But this closeness is more important near the well locations. Thus, the wells must be located prior to the identification of the representative realization and the procedure is the same as with the full approach, i.e. using the lower quartile quality map.

The same weighting formula that was defined for locating the wells is used here to weight the quadratic difference between the realization quality value and the mean quality value for each cell. The representative realization is the one that has the minimum total weighted quadratic difference.

$$D^l = \sum_{c=1}^{nc_t} (Q_c^l - \bar{Q}_c)^2 . w_c, \;\; l = 1, \dots , L$$

$l =$ representative when D^l is minimum.

For each of the fifty reservoirs, one representative realization was identified for each scenario and then the best scenario was defined as the one with maximum profit. The true results of the scenarios defined with the representative realization were compared to the true results of the scenarios defined with the full approach.

The use of the representative realization led to the same decision (same best scenario) as the full approach for 44% of the reservoirs. The decision with the representative realization was better (higher true profit) than the decision with the full approach for 20% of the reservoirs by luck. The loss of using the representative realization compared to using the full approach was $45 Mm^3$ of oil but the representative realization still had a gain of $147 Mm^3$ of oil if compared to using one realization at random.

4. Ranking Realizations The number of realizations necessary to model the geological uncertainty may be large depending on the reservoir heterogeneity and available data. A methodology for ranking realizations is useful for the purposes of visualization or decision making or evaluation of the uncertainty in the flow responses, such as the production curves of a chosen scenario.

Like the representative realization identification, any realization rank is scenario dependent. Imagine a simple case with only two realizations, the first with the best production area in the North and the second with the best area in the South. For just one producer well, a scenario with the well in the North would rank the first realization as the best, while a scenario with the well in the South would give a different rank with the second realization as the best.

The quality maps of the realizations along with the weighting system of the cell quality values for a specific scenario may be used to rank realizations. Given the scenario, the sum of the qualities (Q_t) associated with the wells is evaluated for each realization and the realizations are ranked by Q_t.

Ideally a ranking methodology would lead to the same rank obtained with the flow response of interest. Normally, there is good correlation between different types of flow responses and the profit is a good summary of all of them.

A ranking of the twenty realizations was done with the quality map for each of the eleven scenarios in each of the fifty reservoirs. Another ranking of the twenty realizations was obtained with the profits and the correlation coefficient between the two ranks was evaluated for each case. To check the goodness of the ranking with quality map, the same exercise was repeated using the oil volume map for each realization.

Fig. 4 shows the distribution of the correlation coefficients between the rank with quality map and the rank with profits. **Fig. 5** shows the correlation coefficients between the rank with oil volume map and the rank with profits.

The mean correlation coefficient when using the quality map was 0.532 and in more than 60% of the cases the coefficient was between 0.5 and the maximum (0.932). In Ref. 6, it was shown that correlations higher than 0.5 are good enough to choose low-side, expected and high-side for the realizations.

When the oil volume map was used to rank realizations, the mean correlation coefficient was just 0.285, showing that static parameters work poorly to rank realizations by their flow responses.

5. Comparing Reservoirs The mean and the uncertainty quality maps may help in comparing reservoirs as long as the same well controls and time of production are used when generating the maps.

Normally the reservoirs are compared by their original oil in place (OOIP) and reserves as well by the present value of the profit due to the production of the reserves. The volumes are classified into different categories according to the uncertainty in their existence.

There are two ways to calculate reserves. When the development plan is defined or already implemented, the reserves are the expected additional cumulative production of the current or planned production scenario. When the development plan is not defined yet (new reservoirs), the reserves are evaluated based on the OOIP and on a borrowed (guessed) recovery factor from some analogous reservoir.

A correlation between reserves and the average value of the mean quality map could be derived from reservoirs that are similar and for which the value of the reserves are known with small uncertainty (exhausted reservoirs, for example). This correlation could be a better way to estimate the reserves than a guessed recovery factor. This correlation could also be used for identifying reservoirs where the production potential (mean quality) is high but the expectation of production based on the current production scenario (reserves) is low. Such reservoirs would be candidates for a development plan review.

The goodness of the correlation between the mean value of the mean quality map and the reserves was checked using the fifty reservoirs. The reserves were defined as the mean over all realizations of the cumulative oil production after twenty years for the best scenario chosen with the full approach. **Fig. 6** shows the points for the fifty reservoirs and the correlation coefficient between them which was 0.831.

Fig. 7 presents the comparison between reserves and OOIP just to show that quality is better correlated with reserves than OOIP. The correlation coefficient between reserves and OOIP was just 0.591.

Whichever the way to define the reserves, normally no uncertainty is associated with them or with the expected profit of their production. But between two reservoirs with similar reserves (or profit), the one with smaller uncertainty should be more valuable.

The average value of the uncertainty (standard deviation) quality map may be used as a quantification of the flow response uncertainty in general and of the reserves in particular.

The correlation between the uncertainty estimated with the quality maps and the uncertainty in reserves was checked for the fifty reservoirs. The uncertainty in reserves was defined as the standard deviation over the realization reserves. **Fig. 8** presents the comparison between these two evaluations of uncertainty, showing that the correlation coefficient was high (0.764).

Fig. 9 shows that if the standard deviation of OOIP was used to estimated the uncertainty in reserves, the correlation would be much smaller (coefficient $= 0.456$).

The correlation between the uncertainty estimated with the quality maps and the uncertainty in general flow responses was also calculated and the correlation coefficient was 0.748, very similar to the result with reserves. The uncertainty in general flow responses was evaluated by the mean of the standard deviations over the realization profits for each scenario.

Conclusions

1. The quality map permits a simple two-dimensional visualization of "how good the area is for production" values and of the uncertainty in those values.

2. The quality map along with a simple optimization algorithm may be used to determine good locations for vertical producer wells.

3. The lower quartile quality map allows the incorporation of geological uncertainty into reservoir management decision making. An application of this map for the definition of the best production scenario, through the "full approach", was done using fifty realistic reservoirs, and the expected profit of the decisions presented substantial increment when accounting for uncertainty compared with the use of one deterministic geological model.

4. One representative realization can be identified for each production scenario using the quality maps, allowing scenario comparisons with similar results to the ones using the expected value over all realizations but with much less CPU time expense. There is a loss in using the results of just one realization; however, this loss is less with the representative realization than with one realization chosen at random.

5. The realizations may be ranked using the sum of total quality (Q_t) associated with the wells. This ranking permits low-side, expected and high-side realizations to be identified.

6. The average value of the mean quality map has good correlation with the production potential of a reservoir and the average value of the uncertainty quality map has good correlation with the uncertainty in flow responses. These two average values may be used to compare reservoirs.

Future Work

The only decision-making problem for which the quality map, as defined in this work, applies directly is the location of vertical producer wells. Different ways to build the quality map and/or different quality units should be analyzed for different problems.

As one example, for the problem of horizontal well location, two or three quality maps may be necessary, fixing the layer for the single well completion when building each map.

The problem of locating injector wells after the definition of the producer well configuration may be solved with the help of a quality map built with all the producer wells and one single injector well and by varying the position of the injector well.

The quality unit may be a measure of profit, instead of cumulative oil production, to incorporate different costs of wells in different areas of the reservoir or to compare reservoirs with different well costs.

Nomenclature

a, b	= coefficients in the cell quality weighting formula
BHP	= bottom hole pressure
d_{w-c}	= distance from the well to the cell
D^l	= weighted quadratic difference between the quality of Realization l and the mean quality over all realizations
L	= total number of realizations
N_p	= cumulative oil production
nc_w	= number of cells that belong to the well w
nc_t	= total number of cells in the grid
nw	= number of wells
nwi	= minimum number of wells in the range of nw
nwf	= maximum number of wells in the range of nw
nw^*	= best economic number of wells
OOIP	= original oil in place
P	= profit
Q_c	= quality of cell c
Q_w	= quality of the well w
Q_t	= total quality
Q_c^l	= quality of the cell c in Realization l
$\bar{Q}_c$	= average quality over all realizations of the cell c
w_c	= quality weight of cell c

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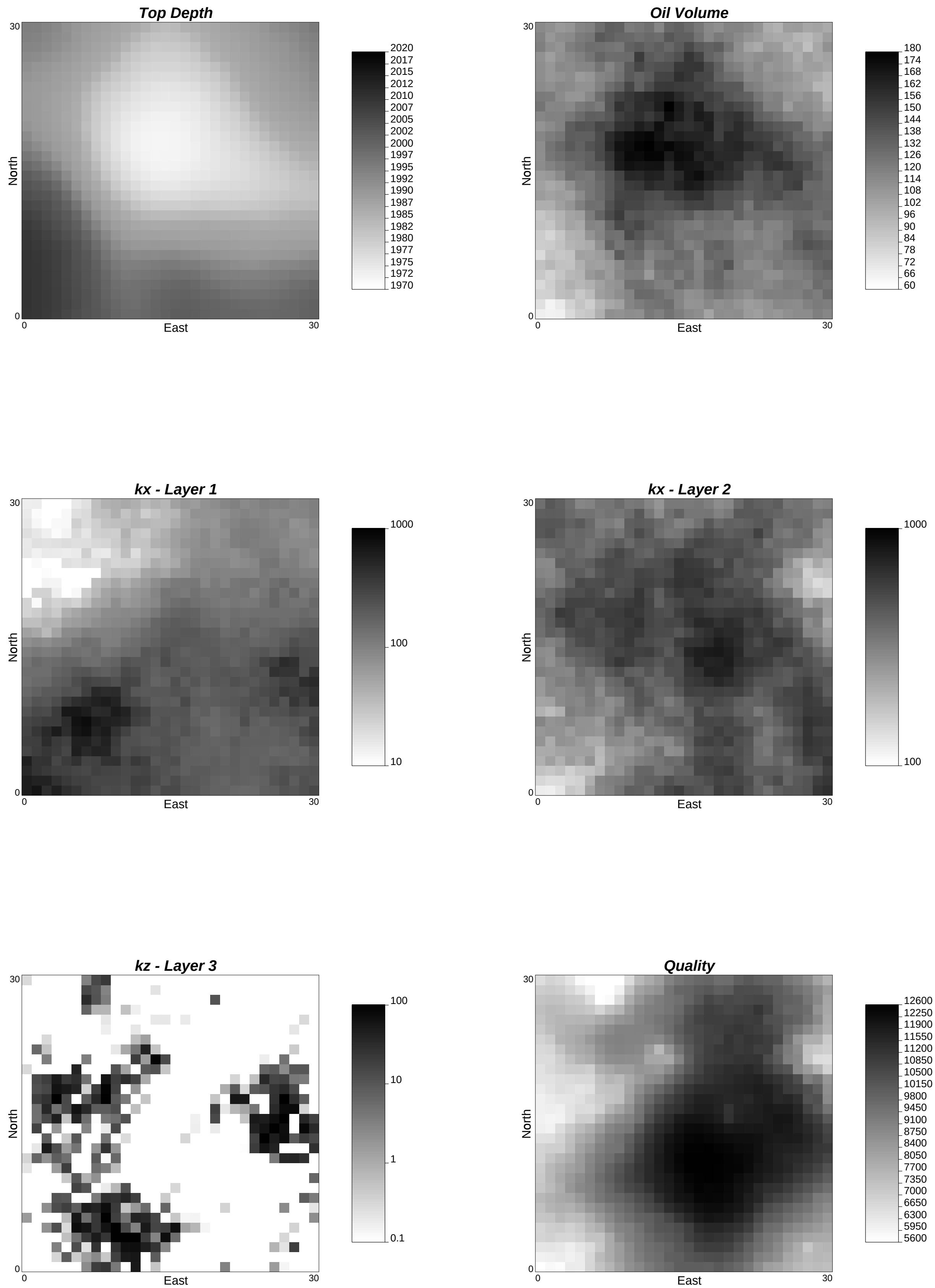

Figure 1: Presentation of the quality map. Several data, including top depth, oil volume, and vertical and horizontal permeability, are integrated into the quality map (lower right corner).

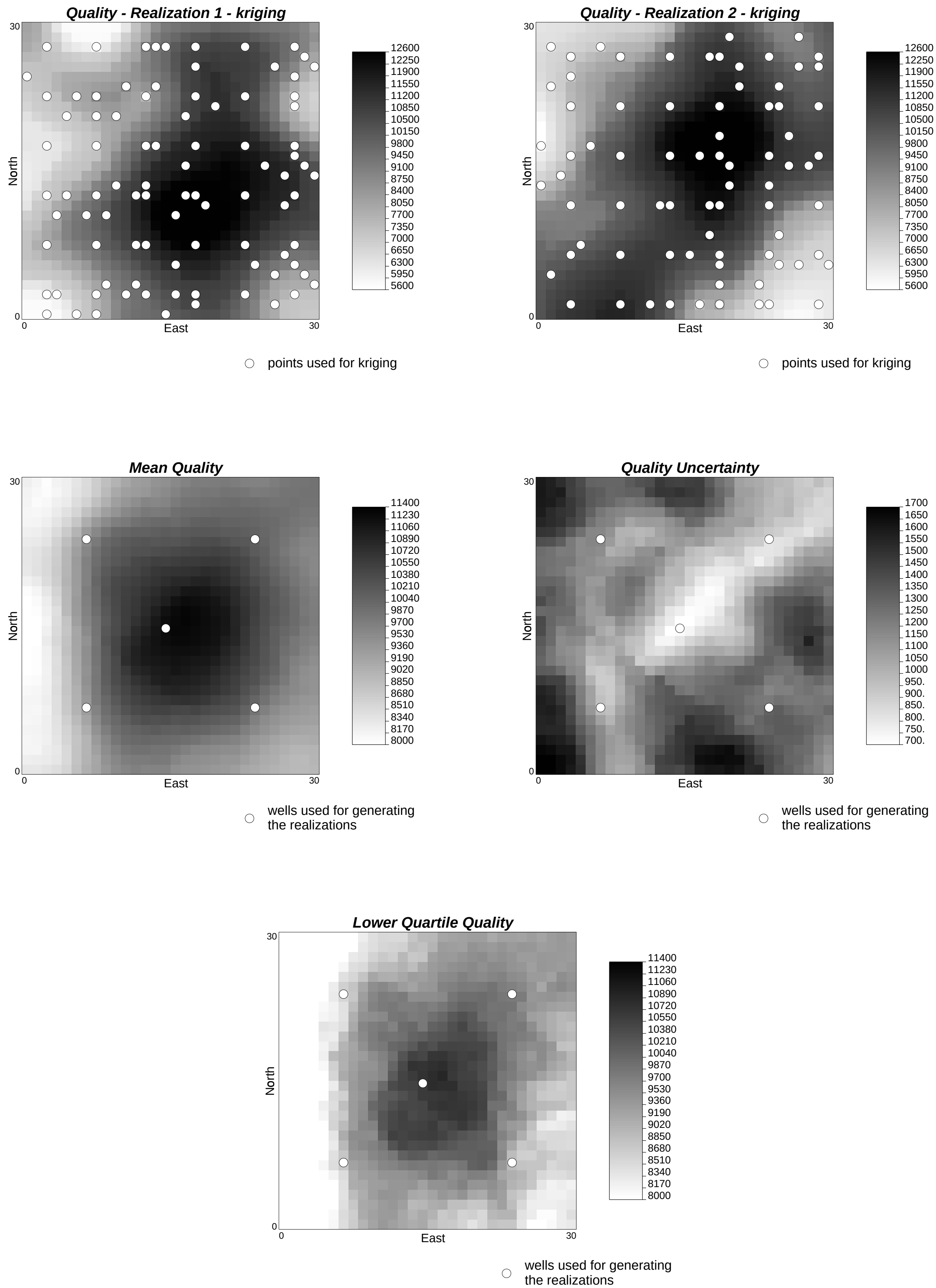

Figure 2: Different types of quality map: quality map of the first two realizations and the mean, uncertainty and lower quartile quality maps that were generated based on a set of twenty realizations.

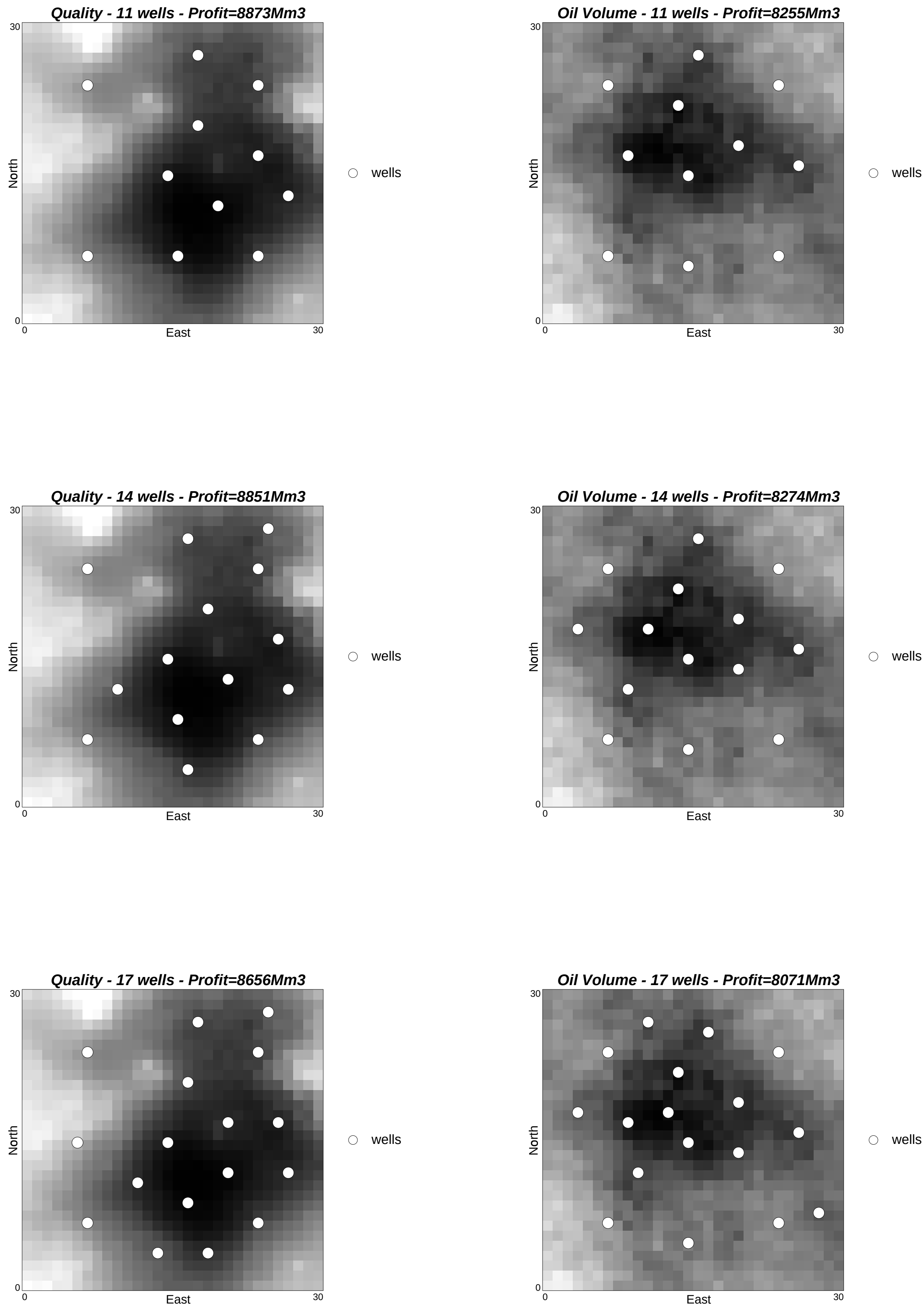

Figure 3: Comparison between location of wells with quality map and with oil volume map and respective profits.

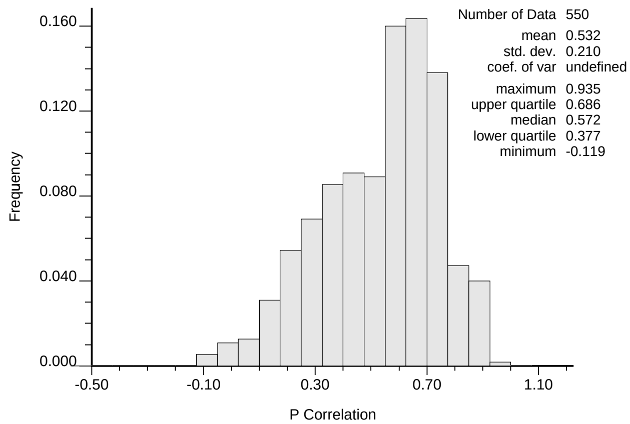

Figure 4: Correlation Coefficient between the rank of 20 realizations obtained with Profit and with Total Quality Qt.

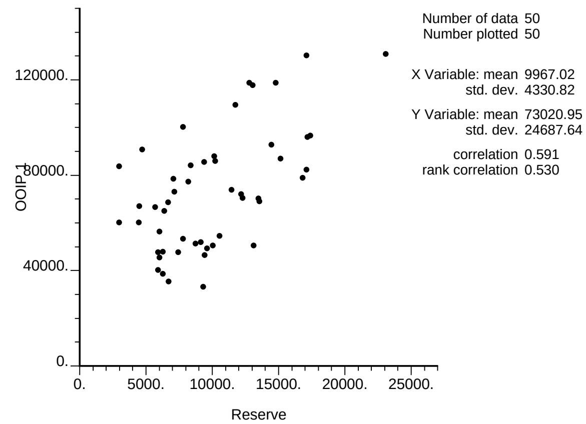

Figure 7: Reserve versus OOIP.

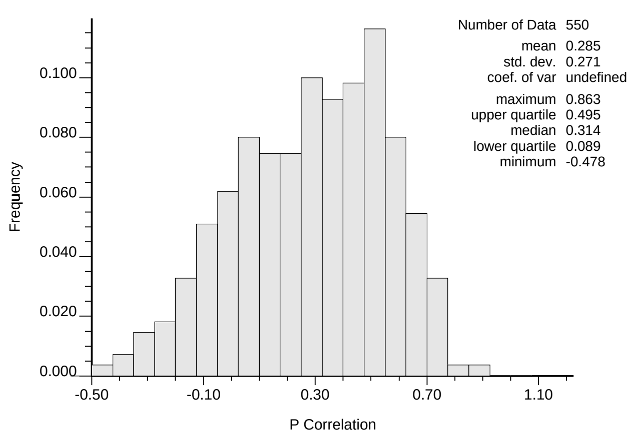

Figure 5: Correlation Coeficient between the rank of 20 realizations obtained with Profit and with Oil Volume.

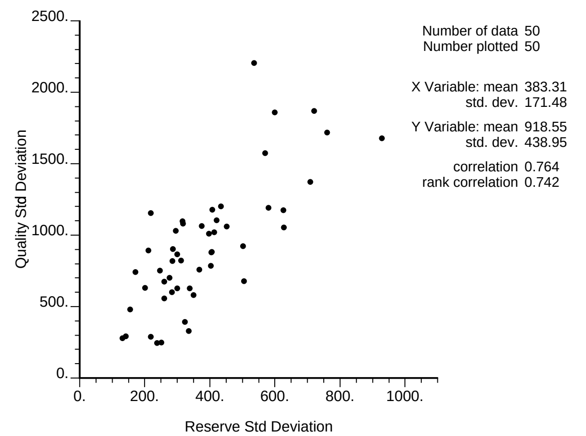

Figure 8: Reserve Uncertainty versus Quality Uncertainty.

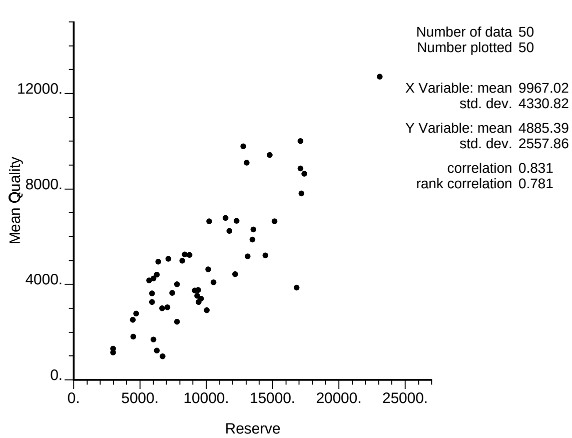

Figure 6: Reserve versus Quality.

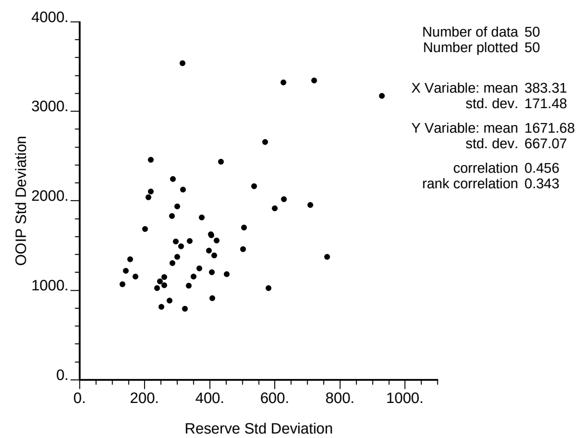

Figure 9: Reserve Uncertainty versus OOIP Uncertainty.