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Direct Geostatistical Simulation With Multiscale Well, Seismic, and Production Data

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Abstract

Multiple realizations of facies, porosity, and permeability are used for better representation of reservoir heterogeneity for more accurate performance forecasting and uncertainty assessment. These geostatistical realizations must reproduce all available data to be reliable. The available well, seismic, and production data are at different scales and must be linked to the reservoir modeling scale. The most common geostatistical simulation algorithms call for a Gaussian or normal transformation. It is not possible to merge the data types after this non-linear transformation. This has led to development of "Direct" approaches for simulation and data integration.

Considering the variables without transformation ensures reproduction of the different data and the prescribed covariance or variogram model; however, until now, the resulting global histogram of the simulated realizations is not reproduced. The problem is that there is no theory that specifies the shape of the conditional distributions. The mean and variance are determined from the well-known normal or simple kriging equations; however, theory has not existed to specify the shape of the conditional distributions. A number of ad-hoc solutions have been proposed, but they violate the data and artificially reduce the modeled space of uncertainty.

This paper develops a theory for determination of the required distribution shapes and reproduction of the global histogram. The results have significant theoretical and practical consequences. Data from multiple sources and scales can be directly reproduced in reservoir models with no need for transformation. There is no need for ad-hoc post-processing transformation or correction schemes. Several applications with synthetic data are shown to illustrate the technique.

Introduction

The sequential paradigm to simulation has become increasingly popular. It has advantages over classical geostatistical techniques such as matrix methods, spectral methods, and moving average methods. These classical methods only work with multivariate Gaussian distributions and require a kriging step to make the simulated realizations honor local data. The sequential simulation approach is somewhat more flexible for continuous and categorical variables and accomplishes the simulation in one step.

Sequential simulation methods have historically been applied to transformed variables, that is, a Gaussian transform of continuous variables and an indicator transform for categorical variables[1]. The indicator transform could also be used for continuous variables; however, this approach is more demanding for inference – we do not concern ourselves with indicator methods in this paper. The application of Monte Carlo simulation from a series of conditional distribution is a classical statistical procedure that is well-grounded in Bayesian statistics. Sequential simulation can be seen as Monte Carlo simulation from a multivariate distribution by decomposing that multivariate distribution into a succession of conditional distributions by recursive application of Bayes Law. The sequential paradigm is not approximate; however, care must be taken to avoid artifacts by poor implementation decisions such as using too few previously simulated values.

The Gaussian transformation makes implementation of sequential simulation remarkably straightforward. A decision is made to model the full multivariate distribution with a multivariate Gaussian distribution after univariate transformation to a normal or Gaussian distribution. Then, the conditional distributions at each step of the sequential simulation are Gaussian in shape with mean and variance given by simple (co)kriging. The original Z variable is transformed to a Y Gaussian variable, simulation is done in Y Gaussian space, and the simulated y values are back transformed to z values. The covariance or variogram of the Y random variable is correct.

Variogram reproduction is guaranteed by use of all data and previously simulated grid blocks and by application of simple kriging. In practice, a limited search neighborhood is used, but variogram reproduction can be checked and more data used if variogram reproduction is deemed unacceptable. Secondary data such as seismic data can also be used after transformation to a Gaussian distribution and assuming that both variables are jointly multivariate Gaussian. Sequential Gaussian simulation (SGS) is arguably the most powerful and commonly used geostatistical simulation technique at the present time.

The histogram of any particular SGS realization does not match the input histogram exactly. The back transformation in SGS would only impose the histogram exactly if the Gaussian or normal values were exactly normal with a mean of 0, variance of 1, and correct shape. Simulated realizations show statistical or ergodic fluctuations between realizations. These variations are an important part of uncertainty; we expect variability in the sample statistics over any study area of finite size. It is wrong to transform the results of SGS to impose the histogram exactly.

Working in Gaussian *space* makes calculations straightforward; however, it was shown early in the development of sequential techniques that the variogram structure could be reproduced without transformation to Gaussian space[2,3]. Direct sequential simulation (DSS), applied directly with the original Z data values, would lead to simulated values that follow the correct variogram. The Monte Carlo simulation at each step must consider probability distributions with the mean and variance given by simple (co)kriging, but the shape of the conditional distribution does not affect the global mean and variogram. Until now, there has been no good way to decide what shape of distribution to use in DSS. In general, regardless of the shape chosen for the conditional distributions, the global histogram of the final values taken altogether is *not* reproduced. The histogram is important; it is a first order statistic that has a first order effect on calculations made with the simulated realizations. The inability of DSS to honor the input histogram has been a significant problem.

Notwithstanding this significant problem with DSS, interest in a *direct* method has grown. The main reason is that we must use a direct method to simultaneously account for data of different volumetric scales. Transforming data of different scales to Gaussian space is problematic: the transform to a Gaussian distribution is non-linear and yet most averaging is linear (porosity) or very particular (permeability). A direct method would avoid the need for this problematic transformation. There are other reasons such as the integration of secondary variables at the correct scale and with the correct level of precision. The problem of global histogram reproduction must be addressed for successful application of DSS.

The same quantile-transformation procedure used to transform original Z values to Gaussian Y values can be used to transform the output (simulated values from direct simulation) to the correct input histogram[4]. The problem with this back transformation is that the final global histogram has no uncertainty (ergodic fluctuations) and, more importantly, large-scale data is not reproduced. The transformation can be modified so that local hard data are reproduced (the values before and after transformation can be averaged together with a special weighting function); however, the problems of block data statistical fluctuations are important.

Caers[5] proposed to reproduce the global histogram by formulating an objective function as a measure of difference between the input global histogram and the histogram of the simulated values. This objective function can be used to selectively accept or reject certain simulated values to ensure that the final realization reproduces the global histogram. This approach also removes most ergodic fluctuations and could introduce artifacts. Soares[6] proposed a different approach to reproduce the histogram in DSS. The central idea of his proposal was also to draw values selectively based on the kriged mean and variance. The procedure does not seem to work well except when the variogram is nearly pure nugget effect.

We propose another method for histogram reproduction. The challenge has always been to determine the shape of the conditional distribution. The true beauty of the Gaussian approach is that the shape is always Gaussian or normal. The key ideas behind our method is to (1) work in original Z space, that is, a true DSS application, and (2) work out the shape of the conditional distributions as a function of their mean and variance using the normal-score or Gaussian transformation. We can have the best of both worlds, that is, no data transformation and guaranteed reproduction of the input histogram within statistical fluctuations.

In petroleum reservoir characterization, any geostatistical simulation algorithm is only as good as its ability to integrate constraints from seismic and production data. We can use the block kriging approach[7,8] to extend the DSS algorithm to account for additional "soft" constraints from seismic and production data. Incorporating seismic and production data into a fine-scale geologic model presents two major problems. First, as previously mentioned, these additional data measure earth properties at different volumetric scales; appropriate care must be taken to account for the spatial correlation between data of different scales. Additionally, the information brought forth by the various data types may be in conflict with each other, e.g. the locations of low and high values may disagree, the histograms of porosity and permeability implied by the soft data may be inconsistent with those of the well data. In this paper, we propose to use block kriging to address the volume support issue and direct simulation to reconcile the differences in global histograms of various data types.

Direct Sequential Simulation and Methodology for Histogram Reproduction

Sequential simulation is described in many sources including Deutsch and Journel[1] and Goovaerts[9]; the details will not be repeated here. An important result of the theory of sequential simulation states that the prescribed covariance model is reproduced as long as each simulated value is drawn from a local conditional probability distribution whose mean and variance are the simple kriging mean and variance[3]. These probability distributions need not be Gaussian and their shapes can vary from one location to the next along the simulation random path. We propose to use a family of distribution shapes that we infer from the multivariate Gaussian transform procedure.

Consider an original Z variable with a stationary histogram $F_Z(z)$. In the Gaussian approach, this variable is transformed to a Y variable with stationary standard normal distribution $G(y)$. The quantile or normal-score transformation is widely used to transform any z-value to a corresponding y-value:

$$y = G^{-1}(F_Z(z)) \tag{1}$$

This transformation can be reversed to get back to the original variable:

$$z = F_Z^{-1}(G(y)) \tag{2}$$

The cumulative distribution functions, $F(z)$ and $G(y)$, are known and their inverse (quantile) functions, $F_Z^{-1}(p)$ and $G^{-1}(p)$, are also known. Thus, we have a direct link between Z and Y space. This transformation is unique, reversible, and non-linear.

A fantastic property of the multivariate Gaussian model is that the shape of every conditional distribution is known to be univariate Gaussian with the mean and variance given by simple kriging. The distribution of uncertainty in Z space can be determined from the non-standard univariate Gaussian distribution by Monte-Carlo simulation (drawing L random values) or straightforward back-transform of L regularly spaced quantiles:

$$z^l = F_Z^{-1}\left(G\left(\sigma_y \times G^{-1}(p^l) + y^*\right)\right), \quad l = 1,\ldots,L \tag{3}$$

where y^* and σ_y^2 are the mean and variance of the non-standard Gaussian distribution of uncertainty, and the p^l values are uniformly spaced between 0 and 1. The distribution of uncertainty in Z space is assembled from the z^l values. This distribution, denoted $F_{Z,y^*,\sigma_y}(z)$, is completely defined by Equation 3. Note that we use the Gaussian parameters as subscripts to denote a conditional distribution relating to a particular conditional distribution in Gaussian space. The shape, mean, and variance of this distribution depend on the original Z distribution, but are not the same as the original Z distribution. In this way, the shape of every z-conditional distribution is explicitly known. We propose to use those known shapes in DSS.

The DSS kriging system is

$$\sum_{j=1}^{n} \lambda_j C(u_j,u_i) = C(u,u_i), \quad i = 1,\ldots,n \tag{4}$$

where

- n is the number of nearest data retained for kriging,
- u_i's are the locations of the nearest data,
- u is the location of the current grid cell whose value is to be simulated,
- λ_i's are the kriging weights, the unknowns to be solved for, associated with data values at locations u_i's, and
- $C(u_i,u_j)$ is the covariance between two data locations u_i and u_j.

The mean and variance of the local distribution are given by

$$m_{DSS} = m_Z + \sum_{j=1}^{n} \lambda_j [z(u_j) - m_Z] \tag{5}$$

$$\text{and } \sigma_{DSS}^2 = C(0) - \sum_{j=1}^{n} \lambda_j C(u,u_j) \tag{6}$$

where m_Z and $C(0)$ are the global mean and variance of the data, respectively.

The kriging equations are often expressed in terms of the correlogram $\rho(u_i,u_j)$:

$$\sum_{j=1}^{n} \lambda_j \rho(u_j,u_i) = \rho(u,u_i), \quad i = 1,\ldots,n \tag{7}$$

$$\text{with } m_{DSS} = m_Z + \sum_{j=1}^{n} \lambda_j [z(u_j) - m_Z] \tag{8}$$

$$\text{and } \sigma_{DSS}^2 = C(0)\left\{1 - \sum_{j=1}^{n} \lambda_j \rho(u,u_j)\right\}. \tag{9}$$

We will express the kriging systems in terms of the correlogram; however, the resulting kriging variance is systematically scaled by the global variance so that the correct variability is retained. This is equivalent to using the covariance throughout the kriging equations.

The conventional sequential simulation algorithm can be modified slightly to implement DSS:

1. Determine the appropriate mean and variance, m_{DSS} and σ_{DSS}^2, in Z units by kriging, see Equations 7-9, using all

relevant original data and previously simulated grid nodes.

2. Find the corresponding Gaussian mean and variance, y^* and σ_y^2, that would yield a z-conditional distribution with the z mean and variance (m_{DSS} and σ_{DSS}^2) from step 1.
3. Draw a simulated z value from this conditional distribution, that is, $z = F^{-1}_{Z,y^*,\sigma_y}(p)$ where p is a random number uniformly distributed between 0 and 1.

Step 2 in this procedure could potentially require significant computing effort. However, for practical implementation, we build a database of local distributions with various combinations of y mean and variance. The corresponding z mean and variance are then calculated for each distribution in the database. Determining the correct local distribution shape amounts to a fast table look-up based on the z kriging mean and variance.

Integration of Large-Scale Constraints using Block Kriging

Direct Sequential Simulation can be extended to handle additional constraints from different data sources. An example would be to use seismic data to constrain a porosity model. Suppose that, in addition to porosity from well data, we also have seismic-derived porosity generated from an attribute-analysis study. The seismic-derived porosity is vertically coarser than the well data because of the inherent limited vertical resolution of seismic acquisition. Moreover, a seismic datum informs a much larger volume of reservoir rock than a well log measurement. Therefore, each well log datum is assumed to be of "quasi-point" support whereas each seismic datum can be considered to be of "block" support. Denote

- $\bar{\rho}(V,u_j)$ as the block-to-point correlogram between the coarse block V and the data point u_j; it is calculated by numerical integration of the point-to-point correlogram between u_j and all points $v \in V$, i.e., $\bar{\rho}(V,u_j) = \frac{1}{V}\int_{v\in V} \rho(v,u_j)dv$,
- $\bar{\bar{\rho}}(V,V)$ as the block-to-block correlogram between block V with itself, namely $\bar{\bar{\rho}}(V,V) = \frac{1}{V^2}\iint_{v\in V, v'\in V} \rho(v,v')dvdv'$; again, $\bar{\bar{\rho}}(V,V)$ is calculated by numerical integration, and
- λ_V as the kriging weight assigned to the large-scale block-support value z_V,

then the DSS kriging system is modified to account for such soft coarse-scale datum as:

$$\begin{aligned}
&\lambda_V \bar{\rho}(V,u_i) + \sum_{j=1}^{n} \lambda_j \rho(u_j,u_i) = \rho(u,u_i), \\
&\qquad\qquad i = 1,\ldots,n \qquad (10)\\
&\lambda_V \bar{\bar{\rho}}(V,V) + \sum_{j=1}^{n} \lambda_j \bar{\rho}(V,u_j) = \bar{\rho}(u,V)
\end{aligned}$$

with the kriging mean and variance given by

$$m_{DSS} = m_Z + \lambda_V (z_V - m_Z) + \sum_{j=1}^{n} \lambda_j [z(u_j) - m_Z] \qquad (11)$$

and $$\sigma_{DSS}^2 = C(0)\left\{1 - \lambda_V \bar{\rho}(V,u) - \sum_{j=1}^{n} \lambda_j \rho(u,u_j)\right\}. \qquad (12)$$

The above could be generalized to handle multiple coarse-scale constraints. One example of such an application may be to construct a porosity model subject to vertically coarse seismic data and a history matched scaled-up model. In this case, we have well data of "quasi-point" support, seismic data of block support V_1 and production-derived data of block support V_2. The DSS system can be modified as follows:

$$\begin{aligned}
&\lambda_{V_1} \bar{\rho}(V_1,u_i) + \lambda_{V_2} \bar{\rho}(V_2,u_i) + \sum_{j=1}^{n} \lambda_j \rho(u_j,u_i) = \rho(u,u_i), \\
&\qquad\qquad i = 1,\ldots,n \\
&\lambda_{V_1} \bar{\bar{\rho}}(V_1,V_1) + \lambda_{V_2} \bar{\bar{\rho}}(V_1,V_2) + \sum_{j=1}^{n} \lambda_j \bar{\rho}(V_1,u_j) = \bar{\rho}(u,V_1) \\
&\lambda_{V_1} \bar{\bar{\rho}}(V_2,V_1) + \lambda_{V_2} \bar{\bar{\rho}}(V_2,V_2) + \sum_{j=1}^{n} \lambda_j \bar{\rho}(V_2,u_j) = \bar{\rho}(u,V_2)
\end{aligned} \qquad (13)$$

Solving for the kriging weights allows for the calculation of the mean and variance of the local histogram:

$$\begin{aligned}
m_{DSS} = {} & m_Z + \lambda_{V_1}(z_{V_1} - m_Z) + \lambda_{V_2}(z_{V_2} - m_Z) \\
& + \sum_{j=1}^{n} \lambda_j [z(u_j) - m_Z]
\end{aligned} \qquad (14)$$

and

$$\sigma_{DSS}^2 = C(0)\left\{1 - \lambda_{V_1} \bar{\rho}(V_1,u) - \lambda_{V_2} \bar{\rho}(V_2,u) - \sum_{j=1}^{n} \lambda_j \rho(u,u_j)\right\} \qquad (15)$$

where

- $\bar{\bar{\rho}}(V_1,V_2)$ is the block-to-block correlogram between blocks V_1 and V_2, i.e., $\bar{\bar{\rho}}(V_1,V_2)=\frac{1}{V_1V_2}\iint_{v_1\in V_1,v_2\in V_2}\rho(v_1,v_2)dv_1dv_2$,
- λ_{V_1} and λ_{V_2} are the weights given to block-support data Z_{V_1} and Z_{V_2}.

We will show an application of each of the above simulation algorithms next.

Application of DSS on a Synthetic Data Set

A synthetic data set was generated to illustrate the proposed algorithms. Although the data is synthetic, the statistics are representative of an actual reservoir and the modeling procedure is typical of a reservoir modeling study. We have a total of 5 wells with porosity and permeability logs sampled at half-foot intervals. The prescribed variogram model for porosity is areally isotropic with range of 2 km and has a vertical range of 15 ft. The prescribed permeability variogram has the same vertical range as the porosity variogram but is slightly areally anisotropic with a long range of 2.5 km in the SW-NE direction and a short range of 1.5 km in the SE-NW direction. The reservoir covers a 10-km by 10-km by 50-ft volume which is discretized by a 100x100x100 geocellular model, see Figure 1. We would like to populate this model with porosity and permeability subject to well, variogram, seismic and production data constraints.

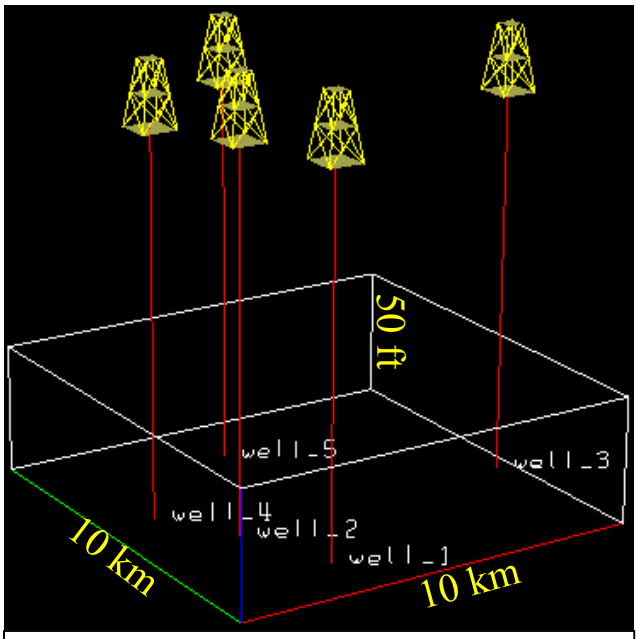

Figure 1: Location of five wells and dimension of the reservoir.

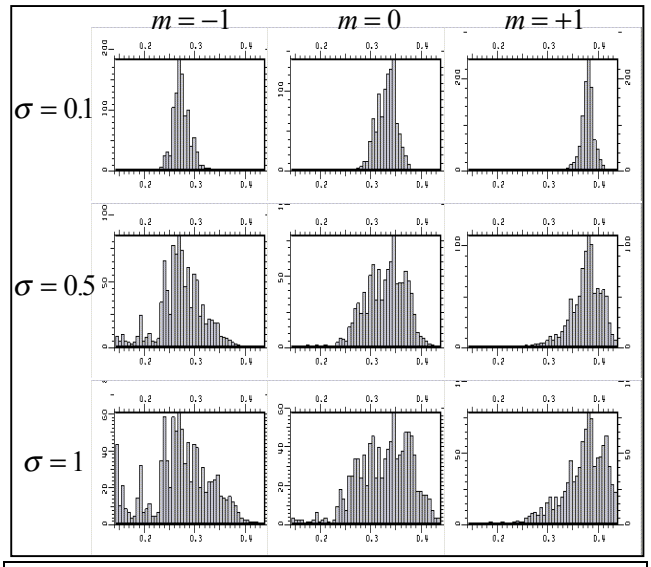

Figure 2: Local distributions inferred from histogram of well porosity. From left to right: mean in normal space of -1, 0, and +1. From top to bottom: standard deviation in normal space of 0.1, 0.5, and 1. Note that the distribution corresponding to normal mean of 0 and standard deviation of 1 (middle histogram of the bottom row) is the marginal histogram.

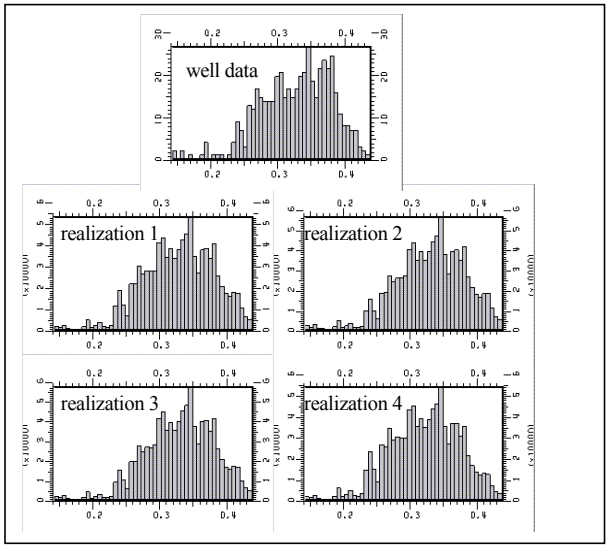

Figure 3: Histogram reproduction for four DSS realizations.

First, we perform DSS to generate four realizations of porosity to illustrate DSS histogram reproduction. Figure 2 shows a small subset of the "look-up" table of local distributions. The entry corresponding to Gaussian mean of 0 and Gaussian variance of 1 is the marginal well porosity

histogram. Figure 3 shows the histograms of the well porosity and of the four porosity realizations. The input marginal histogram is reproduced within ergodic fluctuations.

A seismic attribute study can often provide a porosity map that informs the interval average porosity of the reservoir, see Figure 4b. Given the sparse well control and the high areal resolution of the seismic data, this porosity map is very important to further constrain the 3D porosity model. The seismic map could be thought of as a downscaling constraint where the scale ratio is 100 to 1 in the vertical direction. The average of the porosity values of the cells in any one vertical column of grid cells is constrained by the value of the seismic map over that column. There is no downscaling areally since the seismic map has the same resolution as the simulation grid. Applying DSS with the seismic map as the large-scale constraint, see Equations 10-12, we get a porosity realization as shown in Figure 4c.

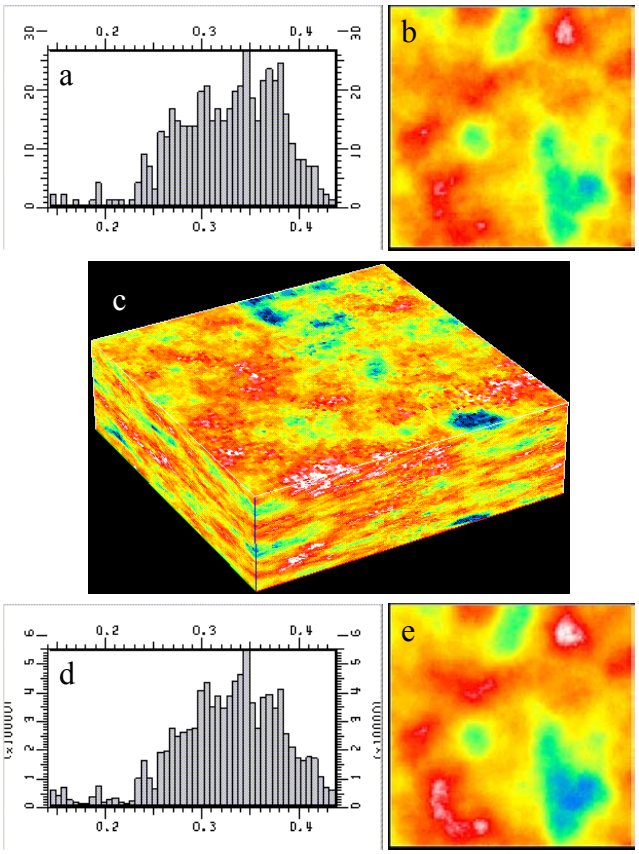

Figure 4: a) histogram of well data, b) seismic-derived porosity map to be used as "soft" constraint, c) a realization of porosity, d) histogram of the porosity realization, and e) average porosity map of the realization obtained by taking the vertical average of simulated values along columns of grid cells.

In addition to tying the wells and honoring the prescribed vertical variogram model, this porosity realization reproduces the seismic-derived porosity map, that is, the vertical average of the simulated values correlates to the seismic-derived input porosity map, see Figure 4b and 4e.

The seismic-derived porosity map may not be consistent with the well data. The wells may be preferentially located in better parts of the reservoir or the seismic gives us new information in a portion of the reservoir not sampled by a well. For example, consider the case where the seismic attribute analysis gives a much lower estimate of porosity in the southeast corner of the reservoir as shown in Figure 5b. The porosity model can be updated by rerunning the algorithm with the new porosity map. Figure 5c shows the porosity realization constrained to the new map and Figure 5d shows the new histogram reproduction. Unlike the previous example, the well porosity histogram is no longer consistent with the porosity histogram implicit to the revised seismic porosity map. DSS reconciles this difference and the histogram of the simulated porosity is a compromise of the two input histograms.

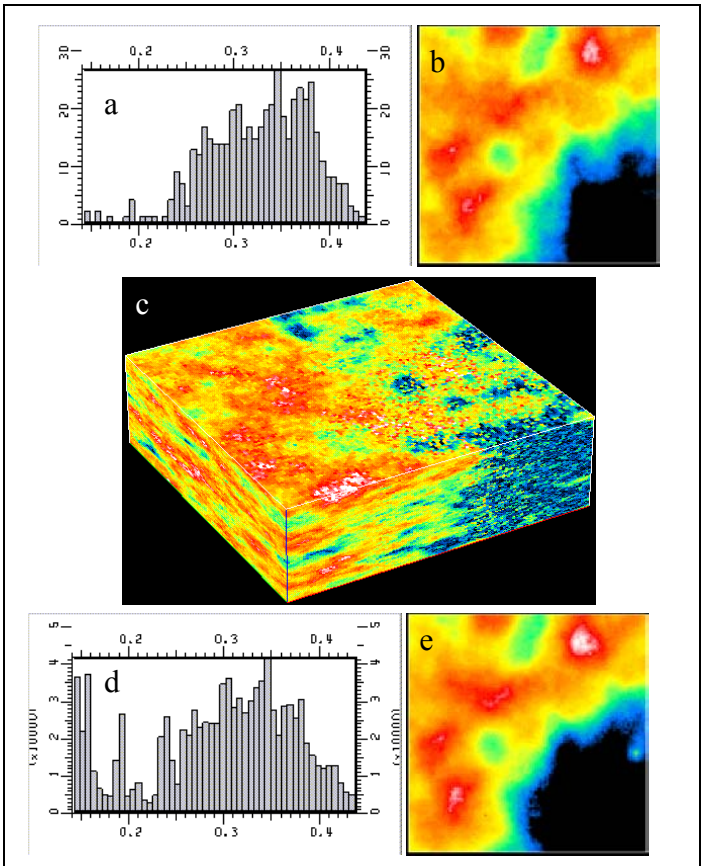

Figure 5: a) histogram of well data, b) revised seismic porosity map with lower average porosity in southeast corner, c) a realization of porosity, d) histogram of the porosity realization, and e) average porosity map of the realization obtained by vertical averaging of simulated porosity values. Note the differences between 5a and 5d.

Permeability realizations are often generated from porosity realizations using either collocated cokriging, cloud transform (a p-field approach), or simulated annealing simulation. These techniques allow for the reproduction of the porosity-permeability relationship observed on core data. The reservoir model is then scaled up and taken to the history-matching phase. Typically, the reservoir engineers have to assign permeability multipliers to the scaled-up model in order to get the simulated flow responses to match the observed production data. These permeability multipliers can vary from one reservoir region to another and can differ drastically in terms of magnitude. Very often, the modeling process stops with a satisfactory history match and the fine-scale geologic porosity-permeability model is rarely updated to reflect the changes made to the scaled-up model by the reservoir engineers. We could perform this fine-scale "model updating" by applying DSS with downscaling.

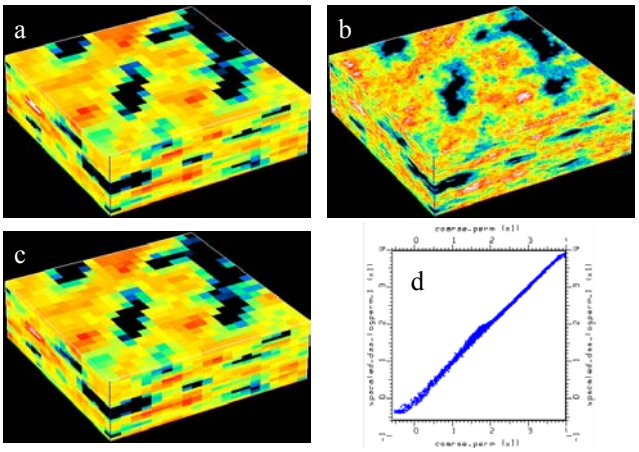

Figure 6: a) large-scale constraint from 20x20x20 history-matched permeability model, b) a fine-scale 100x100x100 realization of permeability, c) scaled-up permeability of the realization, and d) cross-plot of constraint shown in (a) vs. scaled-up result shown in (c).

In this synthetic case study, the final history-matched coarse-scale permeability model is shown on Figure 6a. It has 20x20x20 blocks so the scale ratio between the (fine) geologic and the (coarse) flow simulation models is 5x5x5 to 1x1x1.

The objective is to constrain the fine-scale permeability model such that the average of the simulated permeability values of the fine grid cells that comprise a given coarse grid block identifies the permeability of that coarse grid block. We can sidestep the complex issue of non-linear permeability averaging by applying geostatistical downscaling on a power average of permeability[10]. In this case, we are linearly downscaling the logarithm of permeability, which amounts to geometric averaging of permeability (or power averaging with exponent of zero). Figure 6b shows a realization of log-permeability together with the input coarse permeability constraint and the resulting upscaled realization. It can be seen that the fine-scale model reproduces the large-scale features of the input coarse grid and the fine-scale features implied by the variogram model.

Since porosity and permeability are correlated, we would like to update the fine-scale porosity model to reflect changes made to the fine-scale permeability model. At the same time, we want to preserve the correlation with the seismic-derived average porosity map described earlier. We propose to use DSS with two downscaling constraints, see Equations 13-15, to accomplish this goal. Figure 7c shows such a porosity realization that honors both scaling constraints reasonably well. It is, of course, possible for the constraints to be conflicting in which case the simulation algorithm will attempt to compromise. Figure 7h shows that the updated model of

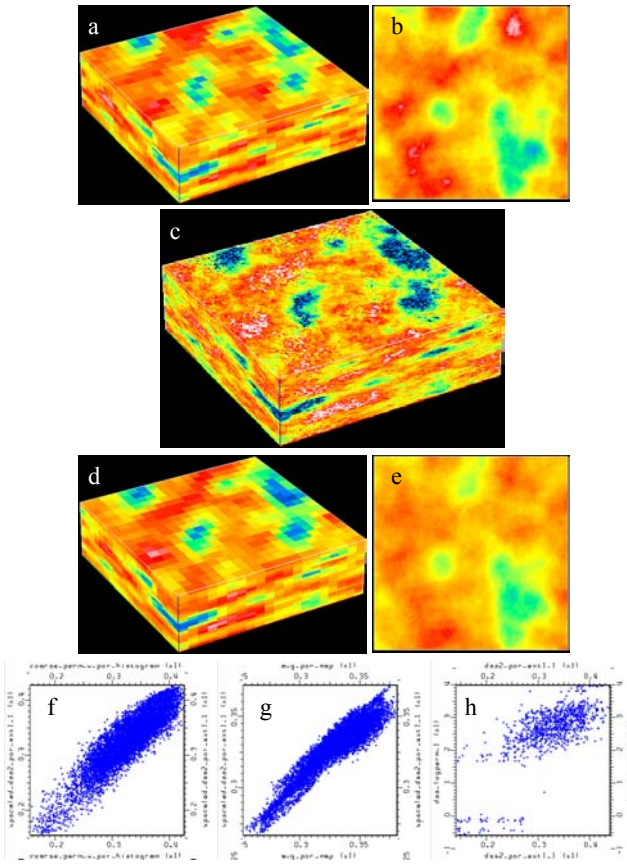

Figure 7: a) large-scale constraint from 20x20x20 history-matched model, b) seismic-derived average porosity map, c) a fine-scale 100x100x100 realization of porosity, d) scaled-up porosity model, e) average porosity map of the realization, f) cross-plot between constraint shown in (a) and result shown in (d), g) cross-plot between constraint shown in (b) and result shown in (e), and h) cross-plot between porosity (c) and permeability (Fig 6b) realization.

porosity and permeability maintains a 0.6 correlation coefficient.

Conclusions

We have shown an approach to simultaneously use DSS and reproduce the global histogram without ad-hoc post-processing or selective sampling. The procedure amounts to pre-calculate the shapes of the conditional distributions that will be needed. These shapes are calculated by back transforming the theoretically correct shapes from Gaussian space using theoretically correct back transformation procedure.

DSS, as is SGS, can be easily extended to account for additional constraints using techniques such as collocated cokriging or block kriging. The advantage of using DSS is that it does not call for the non-linear Gaussian transform; thus, DSS allows for more direct integration of linear constraints such as those from seismic or production data whereas SGS only allows reproduction of the trend or ranks of these soft data. A potentially important application of DSS is that of populating unstructured grids with petrophysical properties accounting for the various scales of hard and soft data and the differences in grid block volumes.

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