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Automatic Determination of Dig Limits Subject to Geostatistical, Economic, and Equipment Constraints

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ABSTRACT

Grade control in open pit mining requires the specification of dig limits that account for mineral grades, economic costs, and selectivity of mining equipment. Visual identification of ore rock types is ideal, but this is not always possible in lower grade disseminated deposits where ore/waste contacts are not visually discernible. In this case, conventional grade control practice consists of a two-step procedure (1) create a map of mineral grades at some selective mining unit scale, and (2) determine practical ore-waste boundaries or dig limits on the basis of the gridded block grades or assay information. This procedure is laborious, depends on a subjective assessment of where the boundary should be, and may be economically sub-optimal.

We pose the determination of dig limits as an optimization problem and solve that problem with the technique of simulated annealing. Simulated annealing has the unique advantage of being able to combine multiple non-linear constraints into a single objective function. We use maximum profitability and the ability of the equipment to mine the proposed dig limits as constraints in the determination of optimal dig limits.

We require a map of expected profit for eah block. Geostatistical techniques are recommended, for mapping expected profit but not necessary. Geostatistics will provide a quantification of the uncertainty in the grades within rock types using all available blasthole samples and exploration drilling. Some variant of L-optimal estimation or kriging can be used to determine the block-by-block classification that is economically optimum. We also need the expected profit for each block. It is unrealistic to assume free selection, that is, each block cannot be extracted independently of its neighbors. The optimal balance of "accepting dilution" and "wasting ore" is achieved to maximize profit subject to equipment constraints.

Mining equipment cannot mine isolated ore or waste blocks. The concept of an equipment curve is proposed as a means to quantify the selectivity and physical limitations of different mining equipment. Economic profitability and mining "digability" are simultaneously considered by the simulated annealing optimization algorithm. These two considerations are balanced by dynamic weighting of these two component objective functions; this weighting requires a subjective calibration.

We illustrate determination of optimal dig limits with simulated annealing. The optimal dig limits are presented and these results compared over a variety of different equipment curves and mining scenarios. Limitations, future work and other areas of application are identified.

INTRODUCTION

Surface mining requires quantification of ore and waste zones. These zones must be realistic for the mining equipment. The limits should also minimize the amount of waste sent to the mill and the amount of ore sent to the waste dump. Grade control starts with geological mapping and blast hole sampling. A traditional method is to hand contour the dig limits using the rock types and cutoff grade.

There are some shortcomings to hand contouring: (1) the uncertainty and variability of the grades is difficult to account for in a quantitative manner, that is, no provision is made for assessing the impact of uncertainty and errors of classification, (2) grade information from previously mined benches and exploration drilling is not easy to consider, (3) mining equipment limitations are not systematically accounted for, that is, the limits may be unrealistically complex of overly simplistic, and (4) hand contoured dig limits are subjective, that is, there is neither an objective measure of optimallity nor a reproducable procedure. The first progression beyond hand-countouring is to consider geostatistical tools to quantify the variability and uncertainty in the grades.

Kriging is a key algorithm in geostatistics; it is an estimation technique that minimizes estimation variance given a prior variogram or covariance model. Kriging estimates should not be plotted on a map, however, since the values were not calculated to have the correct "joint" variability. A map of kriging estimates will be too smooth and will not carry a measure of joint uncertainty in the grade estimates.

Simulation is an algorithm that extends kriging to provide a set of realizations have the correct joint variability and, taken altogether, characterize spatial uncertianty. Early practitioners did not know how to directly use simulated realizations for decision making; it was easier to make decisions with just one answer rather than a set of realizations. Decision analysis tools were then customized to geostatistical applications (Srivastava, 1987; Glacken, 1996). These tools permit optimal ore/waste classification on a block-by-block basis.

A number of geostatisticians have developed variants of optimal classification schemes considering geostatistical models and decision analysis (Glacken, 1996; Deutsch, Norrena, and Magri, 1998; Dimitrakopolous; Isaaks; Srivastava; and Verly). These workers systematically consider free selection at a fixed block size. The choice of a block size, however, is inadequate to capture the fact that mining equipment (1) can dig to limits that do not correspond to arbitrary block boundaries, and (2) cannot freely select a lone ore block in waste or waste block in ore. The assumption of block-by-block free selection is the most consequential limitation of existing grade control procedures.

Figure 1 shows three maps derived from a simulated deposit. These black and white maps show the classification of each block individually. The ore and waste regions must be "smoothed" into practical dig limits before staking them in the pit or transmitting them to the GPS-equipped loading equipment.

The key idea of this paper is to take the next step forward from geostatistical modeling of grades and block-by-block decision making. We want to determine polygonal dig limits that simultaneously account for optimal decision making and the mining equipment. This problem is posed as an optimization problem. We solve that optimization problem and show some examples. The optimization technique for dig limit determination could be applied to the results of kriging, simulation, or any other mapping technique. Limitations and areas of future work are identified.

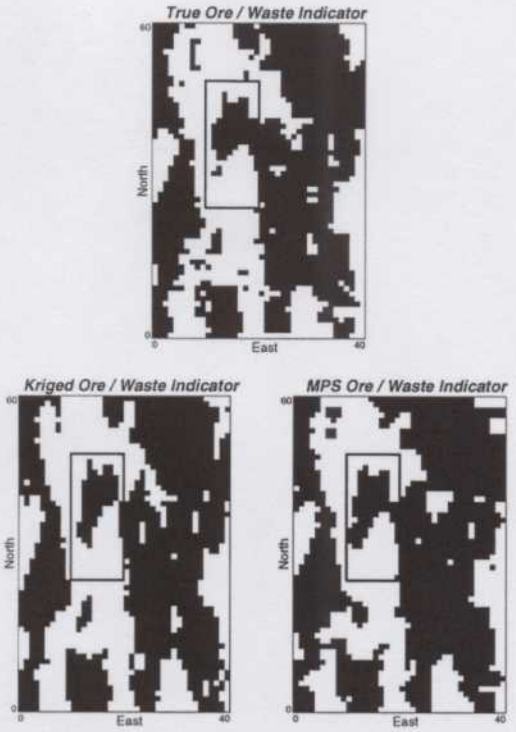

Figure 1: Ore / waste classification based on the true grade, the kriged grade, and maximum profit selection (MPS) using geostatistical simulation and economics.

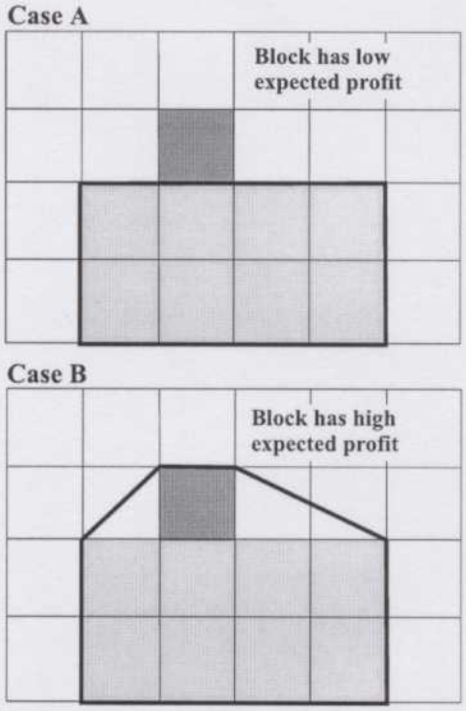

Figure 2: Case A - the top ore block is marginal and should be left because dilution makes it uneconomic; Case B - the top ore block is high grade ore and dilution is acceptable because the total of the dilution and ore is still economic.

The Danger of Image Analysis: Some ideas from image analysis could be used for dig limit determination. Dig limits could be considered as binary ore and waste (ore could be further subdivided into low-grade, mid-, and high-grade), which is particularly well suited to image analysis methods. Successive application of erosion and dilation is one approach to "smooth" a binary image. This is not suitable for dig limit determination because the value of the ore is not accounted for. Figure 2 shows two cases (1) Case A where the top ore block is marginal and should be left because dilution makes it uneconomic, and (2) Case B where the top ore block is high grade ore and the dilution is acceptable the value of the ore outweighs the total dilution. These two cases are indistinguishable from a binary image cleaning perspective. Moreover, image cleaning typically works with pixels and not polygons.

METHODOLOGY

Our problem is to determine practical mining limits that minimize the amount of waste sent to the mill and ore sent to the waste dump, that is, maximize profit. Dig limits are represented by two dimensional polygons. The bench height is assumed constant for the purpose of grade control; the problem of split-benching could be handled as a separate problem. The polygons may enclose areas of waste or ore. In practice, there are both ore and waste polygons on any particular bench. High grade areas will consist of waste polygons within a "matrix" or ore; low grade areas will consist of ore polygons within a "matrix" of waste.

The number of polygons and an initial guess at the polygon geometry can be made manually or automatically; an automatic procedure is used below. The optimization problem is to modify the polygon to maximize an objective function that consists of two parts: (1) profit, and (2) digability. Each ore and waste polygon may be modified by changing the number of vertex points and by changing the vertex coordinates. Profit is defined from prior geostatistical modeling of the grades.

Digability may not be a word in the English language, but most geologists and mining engineers will understand our meaning. Digability is a measure of the difficulty with which an ore or waste dig limit may be extracted. A large polygon with no sharp boundaries would have high digability. A small tortuous polygon would have low digability. Clearly, digability depends on both geometry and the mining equipment. The same polygon would have different measures of digability for a large cable shovel and a small hydraulic loader. We show one mehthod to quantify digability.

Details of this optimization problem will be developed below; however, we note that this problem is not a classical optimization problem. There is no evident way to calculate gradients, that is, derivatives of the objective function with respect to the variables (number of vertices and vertex coordinates). The combination of profit and digability will involve subjective weighting that is not handled by classical optimization techniques. The solution space is combinatorially large with many local maxima. Genetic algorithms and simulated annealing are two optimization techniques that have gained popularity for dealing with these types of optimization problems.

Simulated annealing is used in this paper. There are a number of reasons for this choice: (1) genetic algorithms require a "population" of solutions to be maintained, which can become CPU demanding with a large number of variables, (2) simulated annealing is simpler to code, and (3) recent developments in simulated annealing have made it extremely fast and robust.

Metropolis and coworkers published a paper in 1953 outlining a numerical technique to determine molecular structure of alloys. The Metropolis algorithm was extended by Kirkpatrick and coworkers in 1983 to address combinatorial problems in computer design; they called their solution method simulated annealing or SA. These combinatorial problems are typified by the famous traveling salesman problem, that is, "what is the shortest path through n cities returning to the starting city and visiting each city only once?" The simulated annealing (SA) algorithm starts with an initial path through all of the cities. Random changes or perturbations to the path are proposed. Random changes that lead to a shorter path are accepted. Changes that result in longer paths are sometimes accepted. The path is perturbed until the path length has stopped decreasing. Conditional acceptance of perturbations that increase the path length is the key to the technique; these changes are sometimes accepted because they make it possible to avoid local minima and find the global minima.

One can easily imagine application of the SA algorithm to the problem of dig limit determination: initial dig limits are iteratively perturbed until convergence to optimality, that is, maximum profitability and digability. Two issues need to be addressed: (1) the objective function that simultaneously accounts for profitability and digability, and (2) implementation details of SA such as the perturbation mechanism and annealing schedule.

The Starting Point for dig limit determination is a regular 2-D grid of expected profit. This block model of profit could come from kriging or expected profit calculation using a set of simulated realizations (Deutsch, Magri, Norrena, 1998). The expected profit depends on the mineral commodities present, prices (p), recoveries (r), ore mining costs (c_o), waste mining costs (c_w), and treatment costs (c_t). For simplicity, we show examples with a single metal and constant recovery; however, it is no problem whatsoever to consider multiple metals, recovery curves as a function or grade, and confounding factors such as variable work index and contaminants.

The expected profit in a barren or low grade area is negative and constant at the waste mining cost $-c_w$, expressed in dollars per tonne. In high grade areas, the expected profit is positive and variable depending on grade, e.g., $profit = p \cdot r \cdot Z - c_o - c_t$. The grades, or Z-values, may be modeled by a set of realizations $\{z^{(l)}(\mathbf{u}), l = 1, \ldots, L, \mathbf{u} \in A\}$, where L is the number of realizations and $\mathbf{u}$ is a location vector in the area A. The expected value of profit would be an average over the uncertainty in grades, which is quantified by geostatistical simulation.

The expected profit is modeled by a 2-D block model for a particular region of a particular bench. The resolution of this block model should be about 1/2 to 1/3 of the blasthole spacing. A larger resolution would make it difficult to capture irregular-spaced information from the bench above and rapid changes in the grade. A smaller resolution could not be justified from the available data. The resolution of this block model does *not* have to reflect any particular "selective mining unit" or SMU volume since the dig limits define the mining unit and the dig limits will reflect both profitability and digability.

A dig limit is a closed polygon that encloses ore or waste. Each polygon is defined by a number of vertices and vertex coordinates. Computing the fractional area of grid blocks that fall within such a polygon is straightforward (see published code in Deutsch, 1990).

The Initial Polygons could be determined in a number of ways. The geologist or engineer in charge of grade control could digitize initial polygons on the computer or on manually. An automatic algorithm could be used to outline the ore and waste zones. We use an ad-hoc automatic algorithm for initial polygon determination: (1) a set of possible vertices is determined as the grid line intersections where there is an ore to waste transition, and (2) a rule-based algorithm is used to trace around polygons. The initial polygon lines are not allowed to cross or be too far apart, which means a particular bench is initially divided into a number of ore and/or waste polygons. Subject to digability constraints, polygons may ultimately merge with each other.

The Perturbation Mechanism is important in SA optimization problems. The perturbations must not be too drastic or most perturbations will not be accepted and convergence will be slow. The perturbations must not be too minor or many perturbations will be required to achieve convergence. Standard practice is to choose a reasonable mechanism and any inefficiencies will be revealed in slow convergence. The algorithm coded here only rarely takes more than one minute on a PC for convergence; thus, the algorithm is efficient or the inefficiencies translate to acceptable CPU time.

The mechanism chosen here is to (1) randomly pick a polygon and vertex, and (2) choose to move that vertex with uniform probability within a specified distance (about 20% of the grid block dimension). Figure 3 shows the region for perturbation for one vertex of a polygon. This simple perturbation mechanism must be supplemented by a series of rules including (1) an additional vertex is added at the midpoint between the distant vertices if the vertices get too far apart, (2) vertices are merged if they get too close, (3) a candidate perturbation is rejected if the polygon lines cross, and (4) polygons are merged if they get close or split if they become narrow in a particular region. The rules related to polygon merging and splitting are particularly sen-

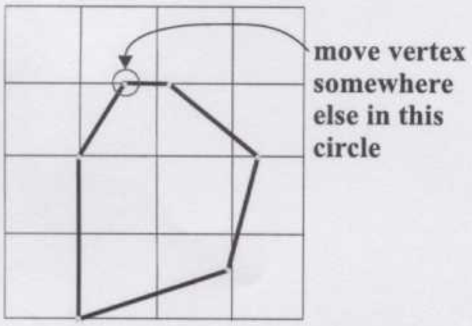

Figure 3: Illustration of the region within which a grid node could be moved for a candidate perturbation.

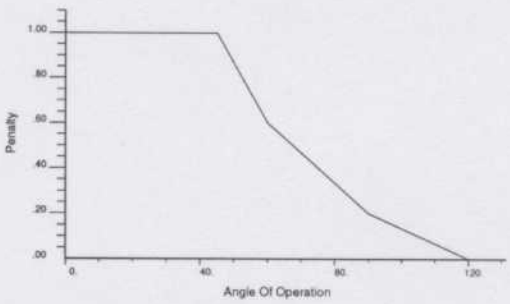

Figure 4: An example equipment curve: the ordinate axis is the penalty and the abscissa is the angle of operation.

sitive; ideally, the number of polygons is determined by the grade control expert in advance. The goal of the optimization is to refine the exact location of the boundaries.

Profitability is straightforward to calculate. Ore and waste polygons must be identified and handled differently. The profit of an ore polygon is the sum of all fractional blocks within the polygon. The profit for ore polygon i is calculated:

$$P^i = \sum_{ix=1}^{nx} \sum_{ny=1}^{ny} frac^i_{(ix,iy)} \cdot P_{(ix,iy)} \qquad (1)$$

where $frac^i_{ix,iy}$ is the fractional area of the block indexed at location (ix, iy) within polygon i and $P_{(ix,iy)}$ is the profit for location (ix, iy). The "profit" of waste polygons multiplied by -1 to ensure that the units and the sign are the same as for ore polygons, e.g., the profit of waste polygon j is calculated:

$$P^j = -\sum_{ix=1}^{nx} \sum_{ny=1}^{ny} frac^i_{(ix,iy)} \cdot P_{(ix,iy)} \qquad (2)$$

The objective is to maximize profitability, that is, to ensure that no profitable material is assigned to the waste polygons unless the digability of the polygon is adversely affected. A highly profitable block can not be included in waste because it will have a large adverse affect on profitability.

Profitability is defined as the sum over all np polygons of the profitability of each:

$$P_{profitability} = \sum_{ip=1}^{np} P^i$$

where the profit of each polygon is defined depending on the polygonal classification of ore and waste.

The fractional area routines of Deutsch, 1990 are implemented in the code and used for input to equations (1) and (2). An alternative is to use some kind of fast point-in-polygon routines, but that is less exact and there is no need for such approximations since the CPU speed is acceptable.

Digability is an intuitive concept, but more ambiguous to calculate. The concept of a penalty function is introduced as a method to measure digability of *tortuous* and *smooth* polygons. An example penalty function is shown in

Figure 4. The penalty curve is for a hypothetical cable shovel. The ordinate axis is the normalized penalty and the abscissa axis is the angle defined by three consecutive vertices. In this example, angles less than 40^o are penalized significantly. Digability is defined as -1 multiplied by the sum over all polygons and all vertices of the angle penalty coming from the equipment curve:

$$p_{digability} = -\sum_{ip=1}^{np} \sum_{iv=1}^{nv(ip)} pen_{iv_{ip}} \qquad (3)$$

where $pen_{iv_{ip}}$ is the penatlty at vertex iv of polygon ip. There are np polygons and $nv(ip)$ vertices for polygon $ip, ip = 1, \ldots, np$.

The examples presented later in this paper will attest to the efficacy of this definition of *digability*; however, we admit that experience is needed to accurately define the equipment curve for different equipment. It is our expectation that experts from a particular mine could calibrate the equipment curve to the equipment, the operators, the operating conditions, and visual geological control.

The Combined Objective Function is a weighted sum of profitability and digability:

$$O = \lambda \cdot P_{profitability} + (1 - \lambda)\, P_{digability} \qquad (4)$$

where $\lambda \in [0, 1]$ is a weight that balances profitability and digability and serves as a "tuning" parameter. As λ approaches 0, the emphasis is on mining equipment constraints. As λ approaches 1, the emphasis is on profitability. This parameter cannot be chosen arbitrarily. If set to one maximum profitability would be assured, but the equipment constraints would be ignored. Equipment constraints are "real" and must be considered. In practice, λ can be determined automatically to ensure that both profitability and digability play an equally important role (see Deutsch and Cockerham, 1994).

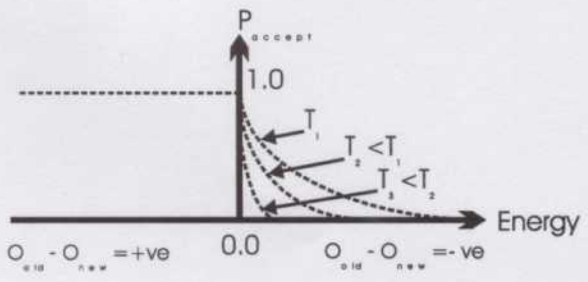

Figure 5: An illustration of the probability of accepting perturbations in simulated annealing (SA). The probability of accepting unfavorable changes is very small at low temperature.

The Acceptance Rule of SA is the key to the success of the algorithm. There are many interesting papers on the subject, see reference list, but we recall the essence of the algorithm. All perturbations that decrease the objective function $O - O_{new} = \Delta O \leq 0$ are accepted; however, some perturbations that increase the objective function $\Delta O > 0$ are accepted. Conditional acceptance of perturbations that increase O should theoretically follow the Boltmann distribution. The Boltzmann distribution summarizes the notion that sometimes molecules move to higher energy states, but less often at low temperature. The Boltzmann distribution:

$$p = e^{\frac{-\Delta O}{T}}$$

where p is the probability of acceptance, ΔO is the positive increase in objective function, and T is the "temperature," which must be determined by well established empirical rules. The annealing schedule is shown on Figure 5. There is a small probability of accepting unfavorable changes at low temperature. The idea is to start the "temperature" parameter quite high and reduce it to zero (see the literature on the well established rules of how to reduce the temperature parameter).

As mentioned, the T parameter controls the decision mechanism. Initially the t parameter starts ay a high value thus there is a high probability for accepting perturbations that increase the objective funciton; virtually all perturbations are accepted. As the algorithm proceeds the T parameter is reduced and the probability for accepting unfavorable perturbations is reduced. At the limit, only perturbations lowering the objective function are accepted. Large scale changes are made at high temperature and fine-tuning of the limits takes place at low temperatures.

METHOD EVALUATION

The vertices of the initial dig limit polygon in Figure 6 are interatively perturbed to conform to optimal dig limits that yield maximum profit. The initial profit is calculated by summing the profit earned by the fraction of ore blocks falling within the dig limits.

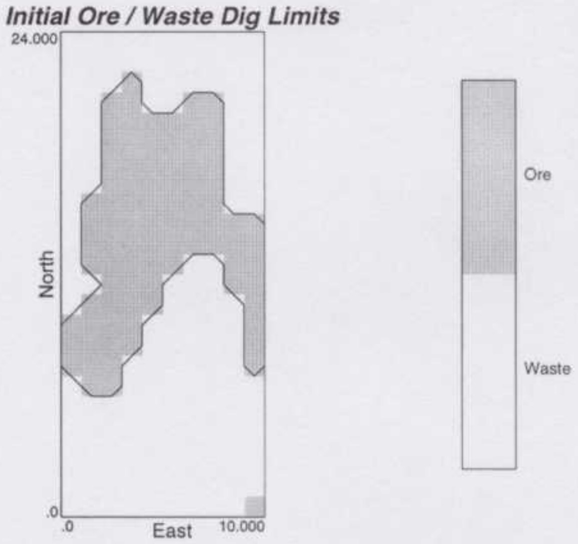

Figure 6: A map showing an initial dig limit polygon for the map of true ore blocks.

This example is taken from Deutsch, Magri, and Norrena, 1998. The true grades were generated by geostatistical simulation. We acknowledge that great care must be exercised in the use of numerical simulation to demonstrate that a method is "optimal;" however, we are not comparing different estimation methods. Our goal is simply to show different dig limits.

First, a fine scale model was generated of realistic complexity. The fine scale true grade model was sampled at some realistic spacing. Ramdom sample errors proportional to the grades were added. The samples were then used with kriging and the maximum profit selection procedure (MPS), at a coarser scale, to identify blocks of ore and waste. For comparitive purposes, the true fine scale model was block averaged up to the same scale as the kriged and MPS model. The truth model is intended to mimic a disseminated gold mine, and the following economic parameters have been selected: milling cost, $c_m = \$12.00/t$, ore mining cost $c_o = \$1.00/t$, waste mining cost $c_w = \$1.00/t$, recovery $r = 0.8$, price $p = \$12/g$. These parameters give a marginal cutoff grade of $1.25g/t$. The mining units were 5m x 5m x 10m. $

The top map in Figure 1 shows the map of ore and waste if the truth were known, a convenience of having created the reality. The cutoff grade was used to identify the ore and waste. The bottom left map shows ore and waste blocks that were distinguished using kriging with the blasthole samples and the cutoff grade. The bottom right map shows the result of the MPS procedure with the sample data to discern between blocks. We will zoom in on the areas blocked out with rectangles.

The blocked out area measure 10 blocks by 24 blocks each. There are 83 true blocks of ore and 157 blocks of waste. The kriged ore waste map shows 79 blocks of ore and 161 blocks of waste. The MPS map shows 88 blocks of ore and 152 blocks of waste. Kriging misclassified 20 ore blocks and 16 waste blocks. MPS misclassified 9 ore blocks and 14 waste blocks. The maximum available true profit is \$ 28450.15. For easy comparison the profits in all cases have been scaled to \$30000 making the maximum achievabe profit \$30000.

The blocked out areas shown in Figure 1 were used to evaluate the proposed dig limit determination method. We first establish dig limits to the MPS profit data. Dig limits are shown for varying degrees of relation between profit and digability; the λ parameter in the objective function was altered. The results are shown on the following table:

	Profit
No Penalty	\$ 18454
Moderate Penalty	\$ 18027
Strict Penalty	\$ 16977

These profit numbers are calculated from the underlying true grades and not the profit derived from kriged grades or the expected grade taken from the MPS results. Figure 7 shows the dig limits for the first three cases (1) no equipment constraints, (2) moderate equipment constraints, and (3) strict equipment constraints. As expected, the profit is highest when no equipment constraints are used, and lowest when strict dig limits are used.

Expected profit can be derived directly from blasthole kriging instead of MPS. The aim is not just to show that MPS outperforms kriging in classification on average, but to show that the proposed method for automatically fitting diglimits is useful for different methods to establish block-by-block estimates. The maps in Figure 8 show moderate equipment constraints. The map on the left shows dig limits knowing the true underlying grade distribution, the middle map shows the ore / waste blocks selected by kriging, and the right hand map shows the ore waste blocks selected using MPS. Indeed, the MPS procedure outperforms kriging as shown in the table below:

	Profit: Free	Profit: Dig Limits
True	\$ 30000	\$ 25648
Kriged	\$ 14919	\$ 14669
MPS	\$ 18449	\$ 18027

Equipment selection can be aided by this procedure for automatic dig limit determination. The proposed method is used with different equipment penalty curves to consider different mining equipment. The capital cost of the mining equipment, the operating cost, and the different ore grade and tonnes are then used in an economic calculator. These results can be used to support other decision making considerations. Figure 9 shows dig limits for two hypothetical mining equipment scenarios. The equipment curve on the left is for large equipment, and the equipment curve on the right is for small equipment.

FUTURE WORK

A few academic examples are shown here. There are many areas of outstanding work that require more complete development. The most critical outstanding work is to apply

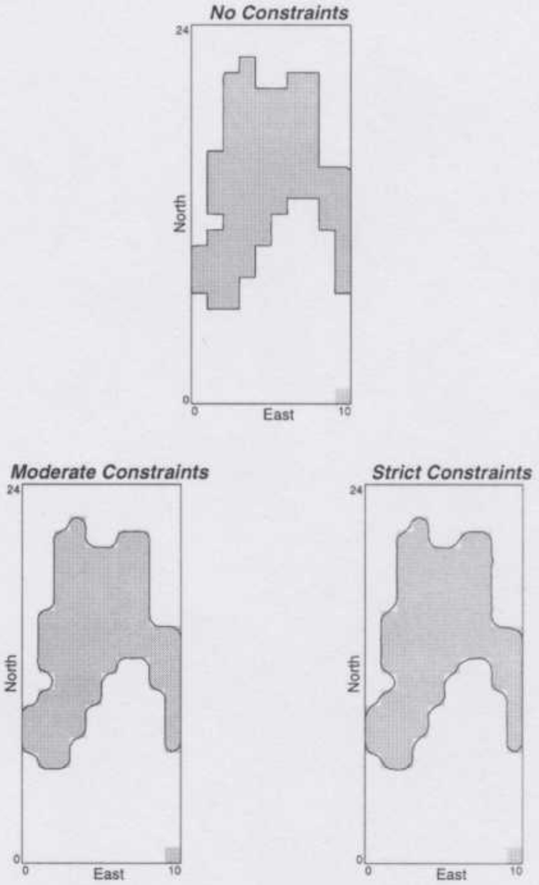

Figure 7: Dig limits for the cases of (1) no equipment constraints, (2) moderate equipment constraints, and (3) strict equipment constraints.

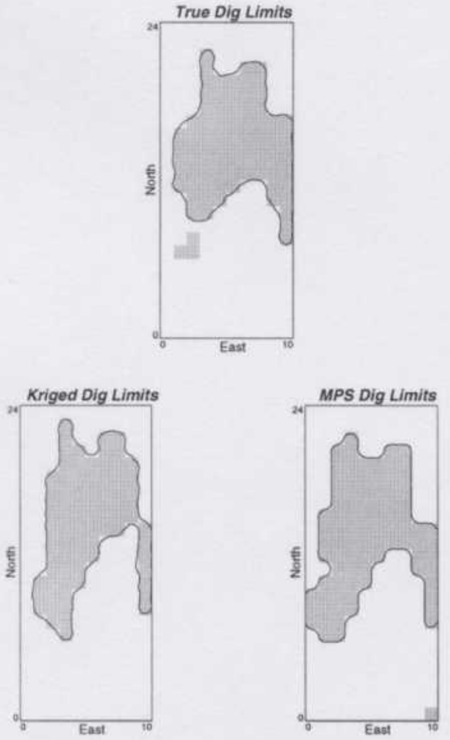

Figure 8: The converged dig limits for the true, kriged, and MPS ore - waste maps.

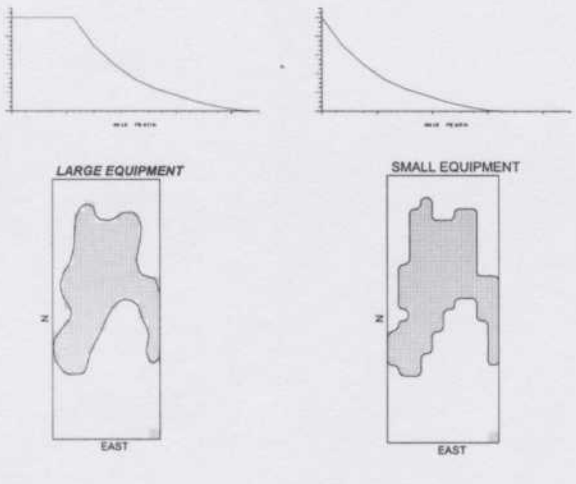

Figure 9: Dig limits for two hypothetical mining equipment scenarios.

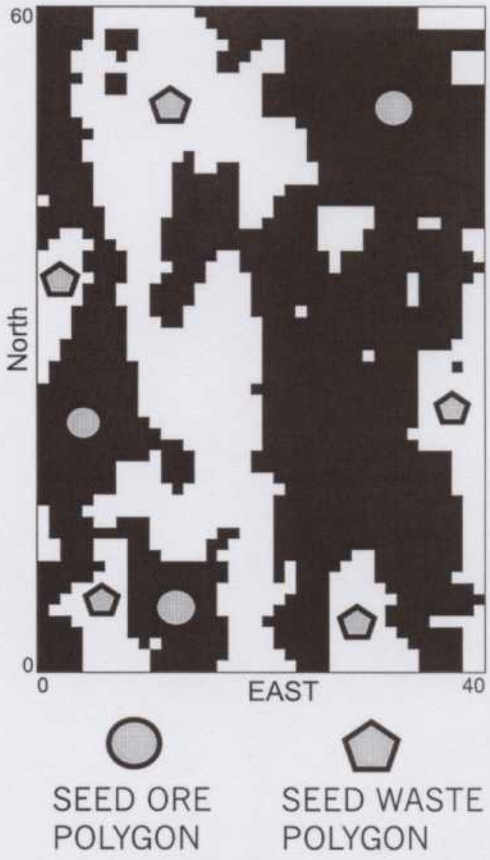

Figure 10: Future work includes planting seed poly gons in both ore and waste.

the method at an operating mine and see if it is possible to (1) calibrate a reasonable equipment curve, (2) compare the results to existing grade control, and (3) refine the procedure for practical considerations that have not been used in this academic exercise.

Handling multiple ore and waste polygons has not been tested. There is no theoretical problem; however, there are a number of ad-hoc programming considerations to simultaneously handle waste polygons in ore and and ore polygons in waste. There are polygons inside other polygons, there is a need to consider splitting and merging of polygons, and the optimization must consider all polygons.

The problem of classification is not limited to the mining industry. There are applications in the environmental industry where areas to remidiate must be identified and those areas cannot be flagged independently of surrounding areas. There are applications in the medical industry where images and zones must be classified and this classification cannot proceed pixel-by-pixel; there is a larger scale structure that must be observed.

CONCLUSION

Free selection has been the single most important limiting assumption of geostatistics-based grade control. Optimal mapping of block grades and classification of blocks is well established. This paper presents an important extension to those well known grade-control procedures: a technique to determine optimal grade control polygons that account for maximum profitability and digability.

Profitability is defined from the expected profit within ore polygons and outside waste polygons. A geostatistical model of grades provides the basis to calculate the expected profit. The fractional area of each block inside the limits is calculated analytically using public code. The polygon vertices are constrained so that the boundaries do not cross. Digability is defined as the ease of mining a particular polygon. Sharp angles over short distances lead to a penalty. The magnitude of the penalties comes from an equipment curve that is calibrated for each piece of mining equipment.

There are many areas of future work required to sort out all of the implementation details. Nevertheless, this automatic procedure for optimal determination of dig limits accounts for many considerations that are awkward to account for by hand-smoothing of block-by-block values. It is easy to imagine an interactive software that would allow the grade control geologist or engineer to semi-automatically map dig limits with intervention in areas of great complexity or unusual mining limitations.

ACKNOWLEDGEMENTS

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