Optimal Determination of Dig Limits For Improved Grade Control

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ABSTRACT

Ore and waste cannot always be visually separated. The distinction between ore and waste may need to be made on the basis of blasthole sampling. Hand drawn dig limits or computer-generated contour lines do not account for uncertainty, economics or equipment. Geometric methods for interpreting grade information such as the polygonal or triangulation methods share the same limitations as contouring. Geostatistical techniques exist for the realistic modeling of grade variations and uncertainty. Decision analysis techniques exist for the binary classification of ore and waste blocks in the presence of this uncertainty. Block-by-block classification is not enough; mining equipment cannot exercise free selection of blocks. There is a need for procedures to determine optimal dig limits that account for (1) the uncertainty in the grades, (2) the economics of risk-qualified decision-making, and (3) the ability of the mining equipment to dig to specific limits. This paper presents a procedure for optimal determination of dig limits that account for these constraints. Simulated annealing (SA) is used with modest computational requirements because the number of variables is relatively small. The solution of the optimization problem is dig limits that maximize profit under multiple constraints. The method is illustrated with different constraints and tested against hand drawn dig limits in a "Dig Limit Challenge," which illustrates the efficacy of the new method.

INTRODUCTION

Material falling below the cutoff grade is waste, or perhaps classified as low grade and stockpiled for possible future processing. Grade control is the procedures for classification of ore and waste. There are four important scenarios associated with this classification: (1) waste is correctly identified as waste, (2) ore is correctly identified as ore, (3) ore is incorrectly identified as waste and shipped to the waste dump incurring a lost opportunity cost, and (4) waste is incorrectly identified as ore and shipped to the mill for processing and incurs a treatment cost that is not paid for by the sale of the recovered mineral. Grade control attempts to avoid the third and fourth scenarios.

Typical grade control practice is to (1) obtain sample information using drill, blast hole, and geologic data, (2) assay and interpret the data for grade information, (3) map the grade information, and (4) assign ore zones based on the mapped information. The engineer or geologist defines ore zones or dig limits using polygons that enclose regions consisting of either ore or waste. Figure 1 shows a map of grade information and waste dig limits. The sample information is shown as circles. Dark shades of gray indicate high grade and lighter shades indicate low grade. The proposed dig limits are provided to a surveyor, and the limits are staked out in the mine. The stakes delineate the ore or waste boundary and convey the decision information to the shovel operator. Modern GPS instrumentation will ultimately replace the need to stake limits in the pit.

Lost profit due to misclassification is a consequence of limited information. The mineral deposit is sparsely sampled and the grades at un-sampled locations must be estimated. All estimates have associated uncertainty. Uncertainty is most important for material near the cutoff

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grade. Figure 2 shows the distribution of waste grades on the left and ore grades on the right. These represent material that can be easily identified. The marginal material is represented by the distribution in the middle of the figure. This material has a high probability for misclassification because it is not clearly ore or waste.

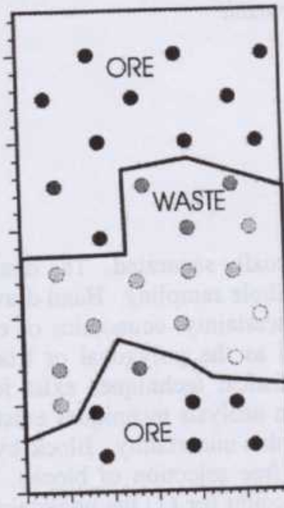

Figure 1 Hypothetical dig limits.

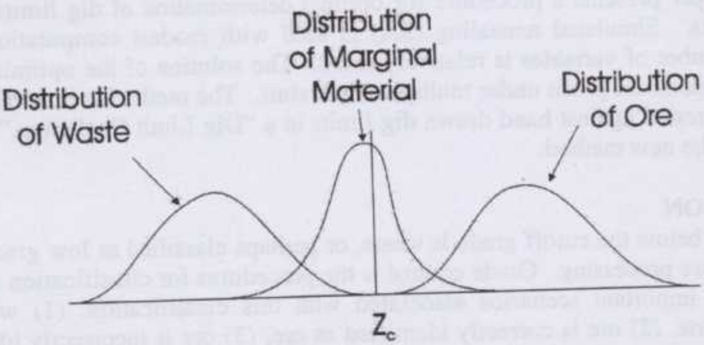

Figure 2 The distribution on the left is the distribution of waste grades, and the distribution to the right is the distribution of ore grades. The distribution in the middle is the distribution of grades for marginal material, that is, material with grades near the cutoff grade.

Mining equipment cannot extract dig limits that are too tortuous or have low "digability". Digability is a measure of how difficulty an ore or waste dig limit is to extract. Figure 3 illustrates the notion of digability. A large polygon with no sharp boundaries would have high digability. A small tortuous polygon would have low digability. Clearly, digability depends on both geometry and the mining equipment. The same polygon would have different measures of digability for a large cable shovel and a small hydraulic loader.

There are many methods for interpreting and mapping spatial information. Many of these have been adapted for grade control. A common method for interpreting spatial information is contouring (A. Jones et. al. 1986). For the purpose of grade control, the grade information is contoured and the dig limits are set at the contour of the cut off grade (L. D. Kornze, et. al. 1985). The contour maps are either hand drawn or drawn by computer. Hand drawn maps subjectively assess the spatial continuity of the deposit. Other historical methods such as the polygon method and triangulation do not account for the spatial continuity of data. Inverse distance weighting can be adapted to account for spatial continuity (A. Noble. 1992). These methods do not account for uncertainty in grade, systematically account for the limitations of the mining equipment, nor do they provide maps that are realistically representative of the heterogeneity of the deposit.

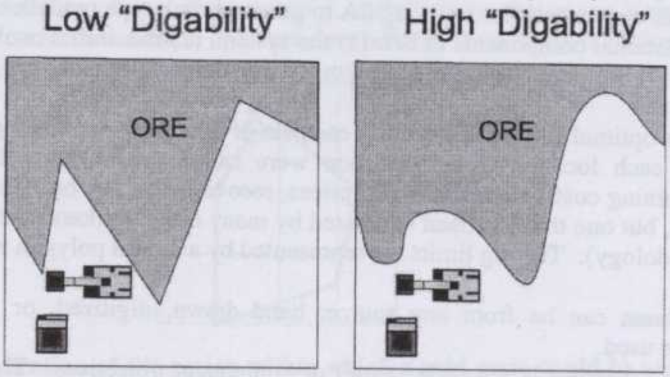

Figure 2 The map on the left shows dig limits that have high digability. The map on the right shows dig limits that have low digability.

Geostatistical methods for mapping are scientific, account for uncertainty, and realistically represent heterogeneity (A. G. Journel, and C. J. Huijbregts. 1978, C. V. Deutsch and A. G. Journel. 1998., P. Goovearts. 1997). There are methods for optimal identification of ore and waste blocks (I. M. Glacken, 1996, P. Goovearts. 1996, R. M. Srivastava. 1987, R. M. Srivastava et. al. 1992, C. V. Deutsch et. al. 2000). These methods discretize the deposit into regular blocks and identify individual blocks as ore or waste. Individual selection of blocks is not enough; there is a need for a complete dig limit.

We develop a method for optimal dig limit determination. The proposed dig limits account for uncertainty in grade and the limitations of the mining equipment. We use synthetic and real examples to illustrate the behavior and flexibility of the method. The efficacy of the method is illustrated using with a dig limit challenge where mining professionals are pitted against automatic dig limit selection.

METHODOLOGY

We pose the problem of optimal dig limit selection as an optimization problem to be solved using the simulated annealing (SA) optimization procedure. Simulated annealing is a general optimization technique that has been successfully applied to many difficult optimization problems.

Two reasons for using SA are: (1) the objective function for dig limits is difficult, and (2) the solution space for selecting optimal dig limits is large. The objective function is difficult to optimize because the grade map is not a differentiable function. Thus, gradient type optimization techniques cannot be used. SA does not require derivatives, and can be set up to account for the limitations of the mining equipment. The solution space is large because the vertices of the dig limit polygon could be located anywhere on the grade map. SA samples a large portion of the sample space and efficiently avoids sub-optimal solutions.

SA was first developed as a technique for statistical mechanics problems. Metropolis et. al. came up with basic formulation of SA to study the molecular properties of materials in 1953. The Metropolis algorithm was extended into a more general optimization algorithm by Kirkpatrick, Gelatt, and Vechhi and independently by Cerny to address combinatorial optimization problems. These combinatorial problems are typified by the famous traveling salesman problem, that is, "what is the shortest path through *n* cities returning to the starting city and visiting each city only once?" The simulated annealing (SA) algorithm starts with an initial path through all of the cities. Random changes or perturbations to the path are proposed. Random changes that lead to a shorter path are accepted. Changes that result in longer paths are sometimes accepted. The path is perturbed until the path length has stopped decreasing. Conditional acceptance of perturbations that increase the path length is the key to the technique; these changes are sometimes accepted because they make it possible to avoid local extremes. Later, Geman and Geman used SA in the restoration of degraded images. Rothman made one of the first attempts to bring SA to the geosciences by using it for nonlinear inversion and the estimation of residual statistics. Farmer brought the technique to geostatistics by using SA to generate rock type models.

There are six essential components in SA: (1) the system, (2) the initial configuration, (3) the objective function, (4) the perturbation mechanism, (5) the decision making program, and (6) the annealing schedule.

The System in optimal dig limit selection consists of a map of expected profit, that is, the expected profit at each location if that location were called ore. This calculation requires knowledge of the mining costs, treatment costs, prices, recoveries and so on. This is an important part of the problem, but one that has been addressed by many other workers (see references above – just before methodology). The dig limits are represented by a closed polygon with some number of vertices.

The Initial Guess can be from any source: hand drawn, digitized, or simply the block boundaries could be used.

The Objective Function is a weighted sum of component objectives. These represent any feature that can be summarized by a mathematical function. In our case:

$$O_{profit} = \sum_{i=1}^{N} w_i \cdot O_i \quad (1)$$

$$O_{profit} = profit + \lambda \cdot penalty_{digability}$$

where the component objectives are the profit earned by the dig limits minus the penalty due to the digability of the dig limits converted to profit by the weight λ. Profit is calculated by summing the fractional area weighted profit of blocks falling within a dig limit polygon:

$$P_i = \sum_{ix=1}^{nx} \sum_{iy=1}^{ny} frac^i_{(ix,iy)} \cdot P_{(ix,iy)} \quad (2)$$

where $frac^i_{(ix,iy)}$ is the fractional area of the block indexed at location (ix,iy) within polygon i and $P_{(ix,iy)}$ is the profit for location (ix,iy).

We introduce the notion of a penalty function to summarize the "digability" of a polygon. An example penalty function is shown in Figure 3. The horizontal axis is the angle between three consecutive vertices on the dig limit polygon, and the vertical axis is the penalty associated with the angle. More acute angles are penalized more. In this example, angles less than 40° are penalized significantly. Penalty is converted to profit using the weight λ in the objective function. One could also use swing radius or some other measure of dig limit tortuousity. The global penalty is the sum of all the vertex penalties around the dig limit.

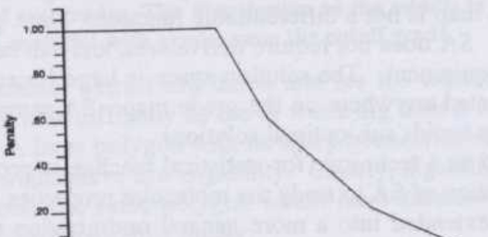

Figure 3: The horizontal axis is the angle between three consecutive vertices in the dig limit polygon. The vertical axis is the associated penalty.

The Perturbation Mechanism randomly changes to the dig limits. The perturbations must not be too drastic or too small or excessive perturbations will be required for the dig limits to be optimized. The mechanism chosen here randomly selects a vertex in the dig limit polygon and proposes a random change to the x and y coordinates as shown in Figure 4. The implementation of this perturbation mechanism requires a few rules: (1) vertex movements are not permitted to cross existing dig limits, perturbations that move a vertex within a specified distance to another vertex cause the deletion of one of the vertices, perturbations that move a vertex farther than a specified distance cause the addition of a vertex equidistant from the two vertices.

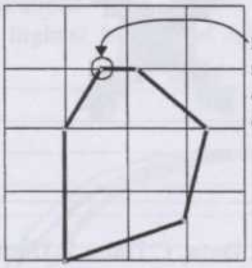

Figure 4: An illustration of the region within which a grid node could be moved for a candidate perturbation.

The Decision Making Program is the key feature of the SA algorithm. All changes that increase the profit are accepted and changes that decrease the objective function are accepted with an exponential distribution (the Boltzmann distribution) with a T parameter specifying the variance. Conditional acceptance is controlled by the magnitude of ΔO and a "T" parameter. The magnitude of ΔO is a consequence of the perturbation and the T parameter is controlled by the annealing schedule (see Numerical Recipes). If a perturbation is accepted the new configuration updates the current configuration. If a perturbation is rejected SA reverts to the configuration before the perturbation. Starting with a high value for T in the beginning of the algorithm amounts to accepting all perturbations. As the algorithm proceeds T is reduced and the probability for accepting downhill perturbations is reduced. If the T parameter is reduced too quickly the SA algorithm may find only suboptimal solutions. If the T parameter is reduced too slowly excessive CPU time is required for convergence.

The dig limit algorithm is summarized as follows: (1) construct an expected profit map using the grade information and propose an initial dig limit polygon, (2) calculate the value of the global objective function using the dig limits, (3) randomly select a vertex in the polygon and randomly select a new location, (4) calculate the value of the perturbed dig limits, (5) invoke the decision making program, (6) if the perturbation is accepted then update the old dig limits with the perturbed dig limits, check to see if the T parameter needs to be reduced, and if the stopping criteria are not met return to step 3, (7) if the perturbation is rejected disregard the perturbation, and check to see if the T parameter needs to be reduced or if the stopping criteria are not met then return to step 3, and (8) if the stopping criteria are met then stop.

EXAMPLES

We first look at synthetic data to illustrate some important features of the method. Convergence is examined by using the synthetic data and different random number seeds. The effect of different annealing schedules is examined using fast, moderate and slow annealing schedules. A real example is also used to show how the method can be used in an operating mine. Then, the performance of the method will compared to hand drawn dig limits.

Synthetic Example

The synthetic data set is shown in Figure 5. The example is on a 50x50 grid. The mapped attribute is expected profit. The values quoted in the results are presented as fractions of the maximum profit that could be earned with perfect selection. Different penalty function weights were used to illustrate the effect of increasing equipment size. This was achieved by using varying λ in the objective function and using the same penalty function throughout the experiment. The resulting dig limits are shown as heavy outlines are shown in Figure 6. The fractional profit for each map is shown in Table 1. Figure 7 shows that the dig limits approach optimality asymptotically.

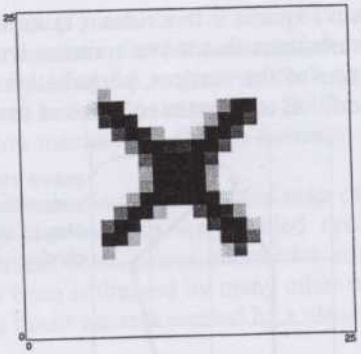

Figure 5: Three synthetic examples: X Data, C Data, Z Data.

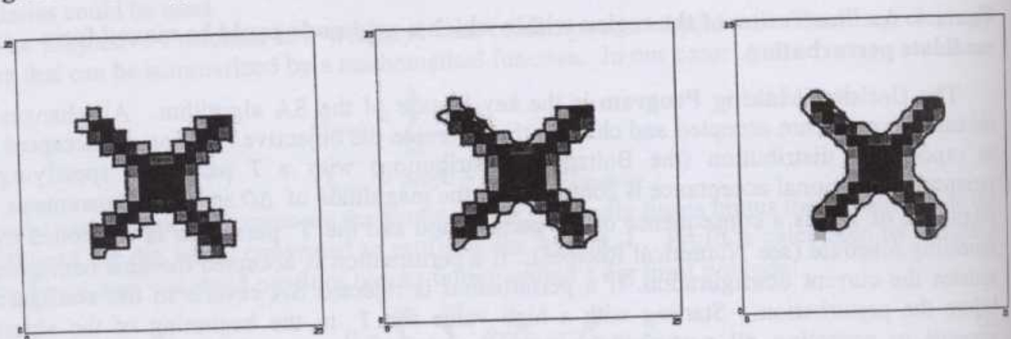

Figure 6: The affect of increasing penalty function weight for X Data. The left map has no penalty function weight, the left map has strict weight.

	Fraction of profit	Start Penalty	End Penalty
None	1.00	655	3530.02
Moderate	0.99	655	1740.36
Strict	0.89	655	32.42

Table 1 Tabulated results for three different penalty function weights.

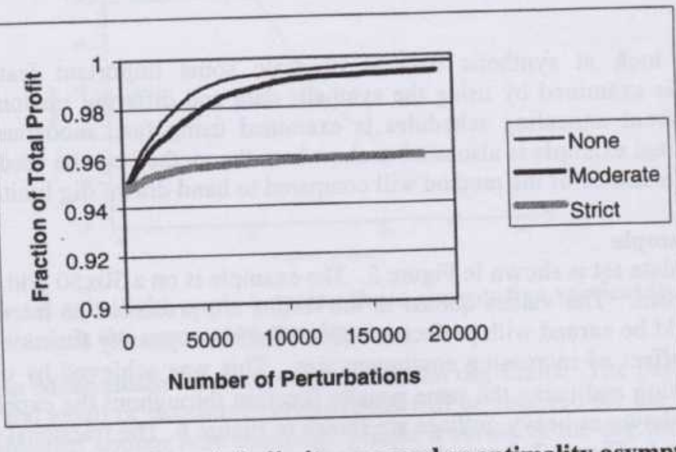

Figure 7 The automatic selection of dig limits approaches optimality asymptotically.

Increasing the importance of the penalty for digability results in smoother dig limits and less profit. Selecting the weight requires calibration for each mine as it reflects the selectivity capacity of the equipment. As we mentioned previously, poor selection of the annealing schedule parameters results in excessive CPU time. Figure 8 plots the value of the global objective function versus the number of perturbations. The "Fast" annealing schedule requires the fewest perturbations and achieves the highest fraction of total profit as shown by the numeric results presented in Table 2.

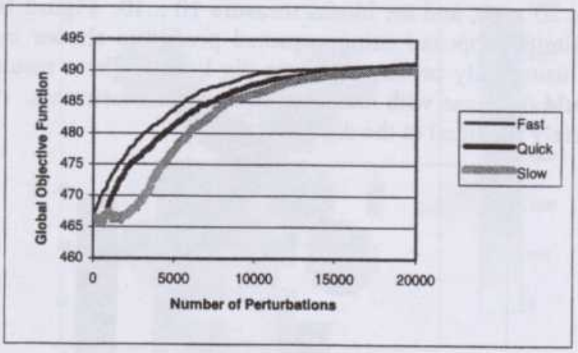

Figure 8 Three different annealing schedules were used.

	Fraction of Total Profit	Start Penalty	Finish Penalty
Fast	0.994	655	484.27
Quick	0.993	655	484.14
Slow	0.915	655	414.02

Table 2 The final results for using three different annealing schedules.

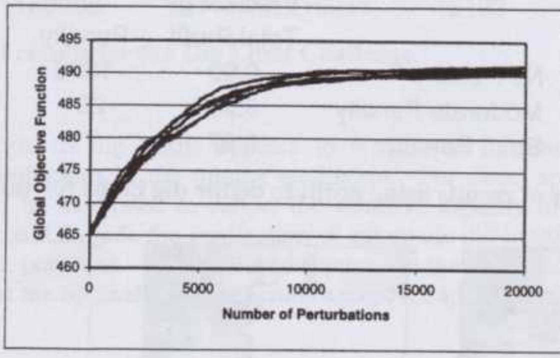

Figure 9 Experimental results showing the repeatability of the algorithm.

Random Number	*Fraction of Total Profit*	*Final Penalty*	*Final Global Objective*
1	0.993	1514.52	490.45
2	0.992	1493.82	489.95
3	0.994	1653.53	491.34
4	0.994	1580.99	491.12
5	0.995	1538.84	491.20

Table 3 The numeric results for the repeatability experiment. Five different random paths lead to the same final profit and penalty.

Repeatability is a scientific requirement. We show repeatability by selecting 5 different random paths and keeping all other parameters the same. The results are shown graphically in Figure 9 and numeric results are shown in Table 3. For the five different random seeds the optimal solutions are the same.

A Real Example

In this section the dig limit selection method is used on a real data set. A single small bench is considered. The grid is 20 x 20, and the blocks measure 10 x 10. Figure 10 shows the expected profit map. The dig limits proposed using expected profit are shown in Figure 11. Table 4 summarizes the results using only profit to propose dig limits. These results are expected; profit and global penalty should decrease with increased equipment constraints. Notice that blocks low in profit are not completely included in the dig limits.

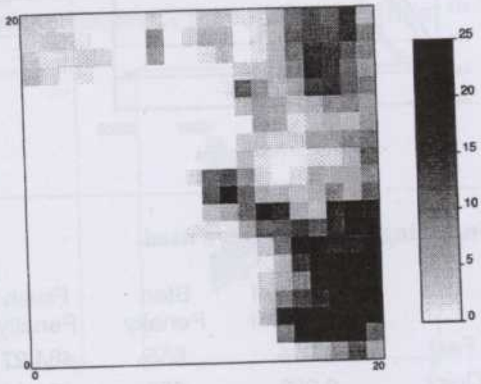

Figure 10: The map on the left is an expected profit map created using the MPS procedure. The map on the right is the associated ore/waste map.

	Fraction of Total Profit	Penalty
No Penalty	0.99	1898
Moderate Penalty	0.98	261
Strict Penalty	0.97	71

Table 4: The summary of results using profit to define dig limits for the real example.

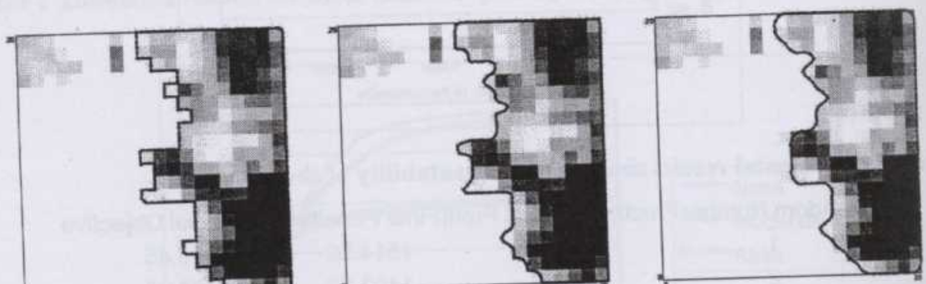

Figure 11: The dig limits using a map of expected profit and no, moderate and strict penalty function weights.

The Dig Limit Challenge

The Dig Limit Challenge was constructed to compare automatic selection of dig limits to hand drawn dig limits. Four mining professionals were given a map of expected profit, shown in Figure 12, and asked to propose their best dig limits. The dig limits were digitized. From the digitized coordinates the minimum, maximum and average line segments were determined and a penalty curve was constructed from the cumulative distribution function of the angles. The penalty function is calculated as $penalty_i = 1 - q_i$. To make the comparison "fair" the automatic dig limit was forced to honor the penalty for each contestant. The results in Table 5 show that automatic dig limits improve profit. The results are compared graphically in Figure 13.

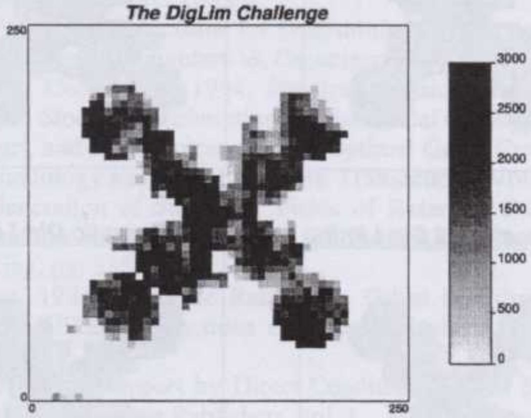

Figure 12: The map of expected profit used in the Dig Limit Challenge.

	Proposed Limits					
	By Hand		Automatic		Difference	
	Profit	Penalty	Profit	Penalty	Profit	Penalty
Candidate 1	1196888	14529	1214297	14556	17409	27
Candidate 2	1204202	16347	1216690	16342	12488	5
Candidate 3	1197635	11487	1216118	14494	19383	7
Candidate 4	1168998	12111	1210911	12123	41913	13

Table 5: Summary of results for the Dig Limit Challenge.

DISCUSSION

Automatic selection of dig limits appears to work well to simultaneously account for uncertainty and the limitations of the mining equipment. For direct application to a mine, the penalty function must be calibrated to reflect the selective capacity of the mining equipment. Further applications could include the application of automatic dig limits to underground mining and multiple ore/waste polygons. The results are optimal in the context of the problem. There is no practical way to test the optimality of the results except for application.

CONCLUSIONS

We show a method for proposing automatic dig limits using simulated annealing. We introduce the idea of a penalty function and its application to establishing dig limits that account for the limitations of mining equipment. The method converges to the same solution despite the random path. The method favorably compares to that of hand drawn dig limits but has the advantage of being repeatable.

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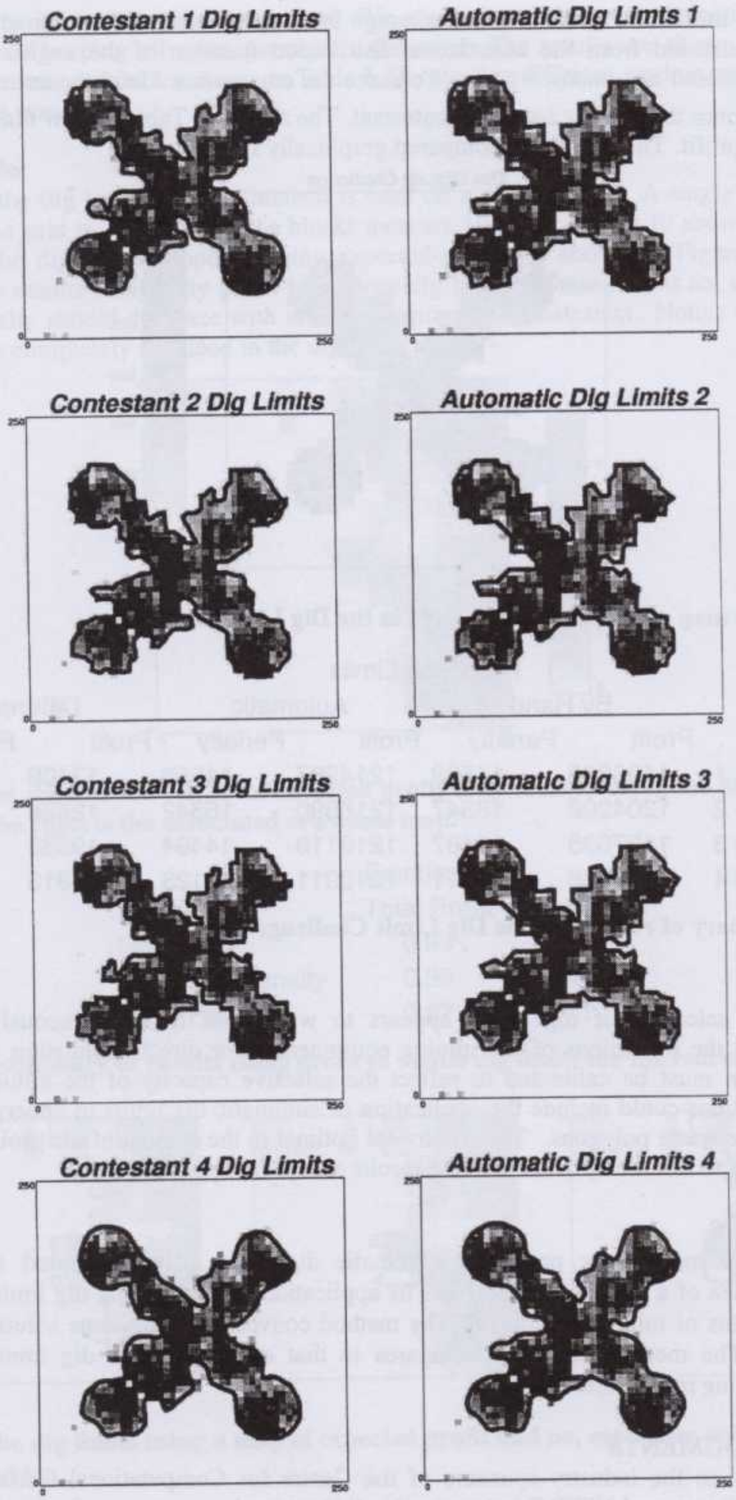

Figure 13: The Dig Limit Challenge results. The contestants are on the left, and the automatic dig limits are on the right.

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