Multivariate Geostatistical Simulation of a Nickel Laterite Deposit

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ABSTRACT

There are many variables to consider in the evaluation of nickel laterite deposits including the concentrations of nickel, iron, silica, and magnesia. These variables show complex relationships with each other such as mineralogical constraints, non-linear correlation, and heteroscedasticity. The complex multivariate relationship between the variables cannot be reproduced by conventional Gaussian or indicator geostatistical techniques. We introduce the stepwise conditional transformation method as a pre- and post-processing multivariate transform for the stochastic simulation of a nickel laterite deposit. Transformed data are multiGaussian and independent at $\mathbf{h} = 0$; thus, cosimulation can be avoided after verification that variables are independent at all lag distances. Back transformation restores the features of the input multivariate distribution to the simulated realizations. The complex multivariate correlations are reproduced and sensitivity issues are addressed.

INTRODUCTION

The increasing demand for realistic geologic models has brought greater attention to the field of geostatistics. For their simplicity, Gaussian techniques are commonly applied to create numerical models of continuous variables. Implicit to these techniques is the requirement for multivariate Gaussianity; however, geologic data rarely conform to such well-behaved Gaussian distributions.

The use of Gaussian techniques, such as sequential Gaussian simulation (Isaaks, 1990) or turning bands simulation (Journel, 1974), to simulate regionalized variables is dependent on the characteristics of a Gaussian variable. In the presence of two or more variables, the conventional procedure is to transform each variable to a Gaussian distribution one at a time. This ensures that each variable is univariate Gaussian; however, the multivariate distributions (of two or more variables at a time) are not explicitly transformed to be multivariate Gaussian. An important assumption inherent in these techniques is that the multivariate distribution is also Gaussian. Under this assumption of multivariate Gaussianity, the only statistics needed to quantify the relationship between multiple variables are the correlation coefficients. The variogram and cross variograms are modeled with the linear model of coregionalization (LMC) or a Markov model of coregionalization, from which the required correlation coefficients are derived. This common practice is limiting and does not address the case when the multiGaussian assumption is violated.

Important characteristics of the multivariate Gaussian distribution are homoscedasticity (constant variance independent of conditioning values) and linearity. Real multivariate distributions show non-linearity, heteroscedasticity, and mineralogical constraints. Figure 1 shows a schematic illustration of these common non-Gaussian behaviors.

Correlations between multiple variables are affected by physical constraints. Some variables show negative correlation because they compete for space within the constant mass. In a similar way, constraints are present due to mineralogical relations, geochemical transformations and

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stoichiometric constraints. These complex multivariate features are common to nickel laterites, which are already complicated by spatially varying trends due to depositional processes.

For complex multivariate data, we propose the application of the stepwise conditional transformation as a multivariate transformation technique that ensures the transformed variables, taken together, are multivariate Gaussian with zero correlation. Thus, conventional Gaussian simulation techniques can be applied with no requirement for cokriging or to fit a model of coregionalization. The correlation between the variables is accounted for in the transformation and back transformation.

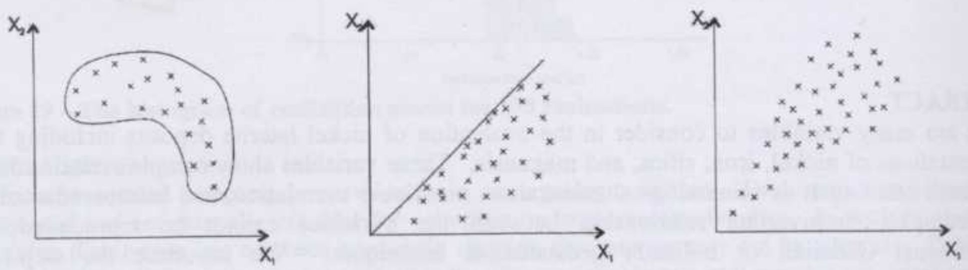

Figure 1 Examples of Problematic Bivariate Distributions for Geostatistical Simulation: non-linear relations (left), mineralogical constraints (centre), and heteroscedasticity (right).

STEPWISE CONDITIONAL TRANSFORMATION

This technique, first introduced by Rosenblatt in 1952, is identical to the normal score transform in the univariate case. For bivariate problems, the normal transformation of the second variable is conditional to the probability class of the first variable. Correspondingly, for k-variate problems, the k^{th} variable is conditionally transformed based on the first k-1 variables.

$$
\begin{aligned}
Y_1' &= G^{-1}\left[F_1(z_1)\right] \\
Y_2' &= G^{-1}\left[F_{2|1}(z_2 \mid z_1)\right] \\
&\vdots \\
Y_n' &= G^{-1}\left[F_{n|1,\ldots,n-1}(z_n \mid z_1,\ldots,z_{n-1})\right]
\end{aligned}
\qquad (1)
$$

Figure 2 shows the steps to accomplish this conditional transformation for a bivariate case. Once the data are separated into classes based on their conditional probabilities, each group of data is normal score transformed. Simulation is then performed on the normal score values and the normal values are back transformed. For example, Z_1 can be determined from Y_1; Z_2 can be calculated from the simulated value of Y_2 with the correct conditional distribution based on Z_1.

Conditional transformation of the data leads to transformed secondary variables that are a combination of both the primary and the secondary variable. Also, the multivariate spatial relationship of the original model variable is not transformed for **h** > 0, that is, there is no modification of bivariate spatial distributions Y(**u**) and Y(**u**+**h**), or trivariate distributions Y(**u**), Y(**u**+**h**$_1$) and Y(**u**+**h**$_2$), and so on.

This transformation results in independence of the transformed variables at **h** = 0, that is $C'_{ij}(\mathbf{0}) = 0,\ i \neq j$. Consequently, the simulation of multivariate problems may not require cosimulation; provided the independence at **h** = 0 is approximately preserved for **h** > 0.

The transformed variables do not show complex multivariate distributions, but are well-behaved Gaussian distributions. For example, non-linear, heteroscedastic and constraint features (see Figure 1) are automatically accounted for in the transformation and model of coregionalization.

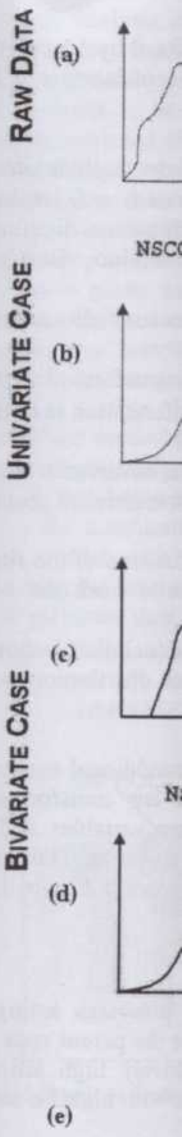

Figure 2 Schematic Ill
data variables and the
data, Y_1, and categor
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$Z_2|Y_1$ class independent
showing no correlation

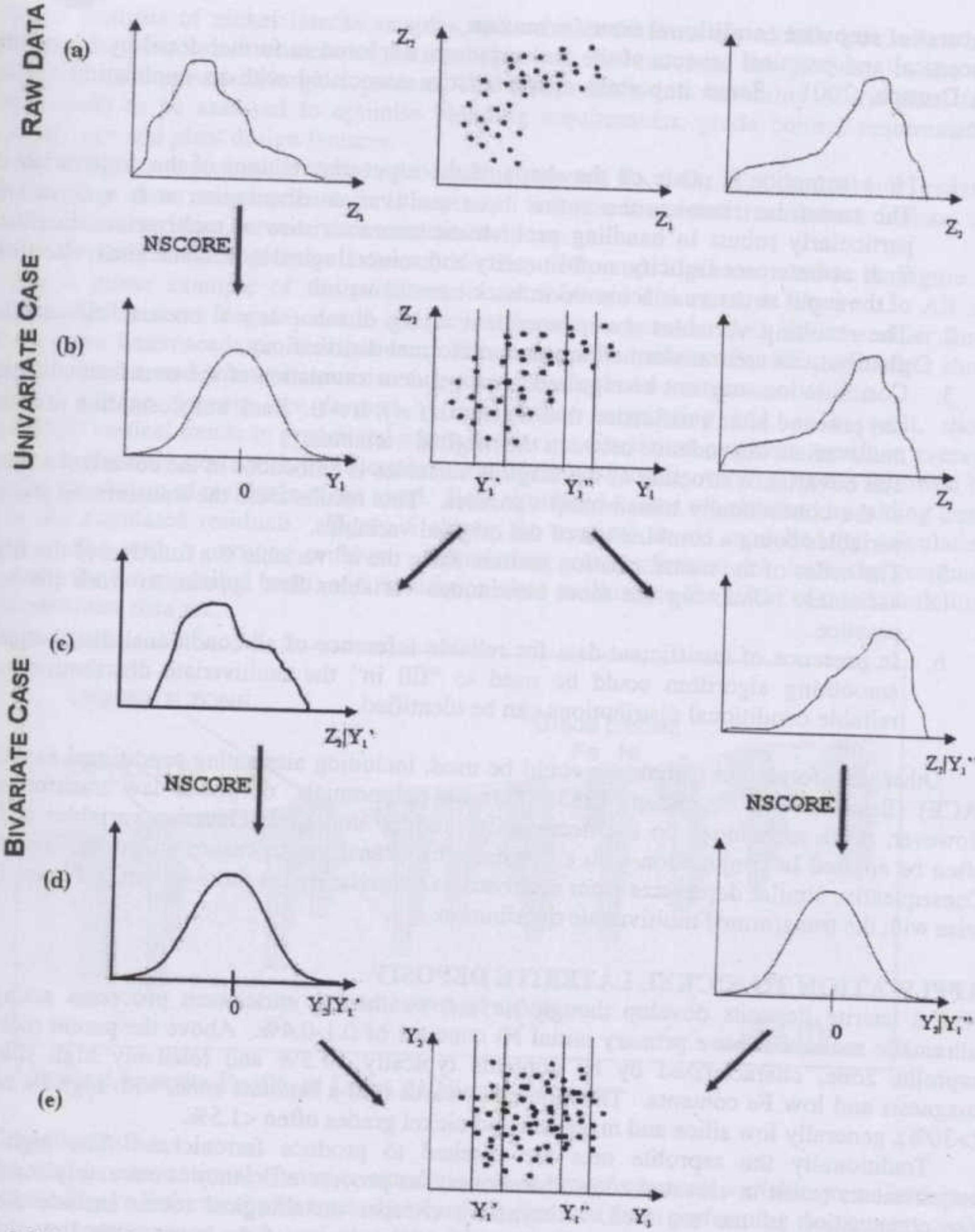

Figure 2 Schematic Illustration of Stepwise Conditional Transformation: (a) Distribution of data variables and the scatterplot of Z_1 and Z_2; (b) Transform variable Z_1 into normal score data, Y_1, and categorize Z_2 data conditional to probability classes of Y_1 ; (c) Determine conditional distributions for each Z_2 class conditional to Y_1 data; d) Normally transform each $Z_2|Y_1$ class independently; and (e) the scatterplot between the transformed Y_1 and Y_2 variables showing no correlation.

Features of stepwise conditional transformation

Theoretical and practical aspects of the technique are explored in further detail by Leuangthong and Deutsch, 2001. Some important characteristics associated with its application are listed below:

1. No assumption is made on the shape of the input distributions of the multivariate data. The transform removes the entire input multivariate distribution at $\mathbf{h} = 0$, making it particularly robust in handling problematic characteristics of multivariate distributions such as heteroscedasticity, non-linearity and mineralogical-type constraints. Restoration of the input structure is achieved in back transformation.
2. The resulting variables are independent at lag distance $\mathbf{h} = 0$ because all conditional distributions are transformed to standard normal distributions.
3. Cosimulation may not be required. Independent simulation of the transformed variable can proceed after verification that $C_{ij}(\mathbf{h}) \equiv 0,\ i \neq j,\ \mathbf{h} > 0$. Back transformation restores th multivariate dependence between the original variables.
4. The covariance structure of the original variables is embedded in the covariance structur of the conditionally transformed variables. This results from the transformed secondar variables being a combination of the original variables.
5. The order of the transformation matters since the n^{th} variable is a function of the first n variables. Choosing the most continuous variables first appears to work the best practice.
6. In presence of insufficient data for reliable inference of all conditional distributions. smoothing algorithm could be used to "fill in" the multivariate distribution so t reliable conditional distributions can be identified.

Other transformation techniques could be used, including alternating conditional expectat (ACE) (Brieman and Friedman, 1985), hermite polynomials, or power law transformatio However, these techniques do not necessarily produce univariate Gaussian variables and n often be applied in conjunction with a normal score transform to ensure univariate Gaussiar Consequently, similar departures from multivariate Gaussianity (as those shown in Figure 1) arise with the transformed multivariate distribution.

APPLICATION TO NICKEL LATERITE DEPOSIT

Nickel laterite deposits develop though surface weathering enrichment processes acting ultramafic rocks that have primary initial Ni contents of 0.1-0.4%. Above the parent rock i saprolite zone, characterized by Ni contents typically >1.5% and relatively high silica magnesia and low Fe contents. This grades upwards into a limonite zone, with high Fe cor (>30%), generally low silica and magnesia and nickel grades often <1.5%.

Traditionally the saprolite ores are smelted to produce ferronickel. The high-e requirements result in elevated costs. Moreover, the process efficiency is extremely sensit the composition of the ore feed. Alternative cheaper metallurgical routes include the pressure acid leach process which is well suited for treatment of the lower grade limonitic however, is not appropriate for saprolite ores due to their high magnesia contents leading t acid consumption.

The data used in this exercise was taken from the Loma de Niquel deposit, a recently ir mine in Venezuela that treats saprolite ores through the pyrometallurgical route. Only dat the most recent drilling campaigns has been used as this data contains the complete s variables required for the study. This information comprises close spaced grade control dril 25 by 12.5 metre grids in areas under production and wider space drilling in the periphera The deposit is located on an elevated ridge extending for over 8 kms.

Figure 3 shows a typical vertical cross section. The limonitic overburden generally thicknesses of 5 m and is underlain by the saprolite ore zone with thickness averaging 10 to

In the analysis of nickel laterite samples, the critical elements are nickel, iron, silica and magnesia. The permissible content of various elements and ratios of the elements in the ore directly affect the pyrometallurgical process. The composition and variability of the plant feed material needs to be assessed to optimise blending requirements, grade control requirements, mining strategy and plant design features.

Simulating the spatial multivariate relationships between Fe, SiO_2, MgO and Ni is a first step in determining these requirements. It is essential that the simulated realisations honour the correlation characteristics between these various elements.

The correlation characteristics between Ni, Fe, SiO_2 and MgO are shown in Figure 5, providing a prime example of the problematic distributions illustrated in Figure 1. All six crossplots exhibit some degree of non-linearity. Crossplots between SiO_2 and the other three variables show heteroscedastic distributions. The scatterplots between Fe, SiO_2 and MgO show strong correlations, and characteristics that suggest mineralogical constraints.

Nickel laterite deposits are formed by sub-surface weathering and, as a result, show characteristic vertical trends in grades and mineralogy. For simulation purposes, trends in average grades are commonly dealt with by deterministic modeling of locally varying trends followed by stochastic simulation of residuals of the trend. Real simulated values are obtained by adding trend back to the simulated residuals. Figure 6 shows the vertical trends modelled by calculating average grades with a moving window. Simulation was performed for both the residuals (accounting for the modelled trend) and the original data to illustrate the effect of trend modelling for this particular data set.

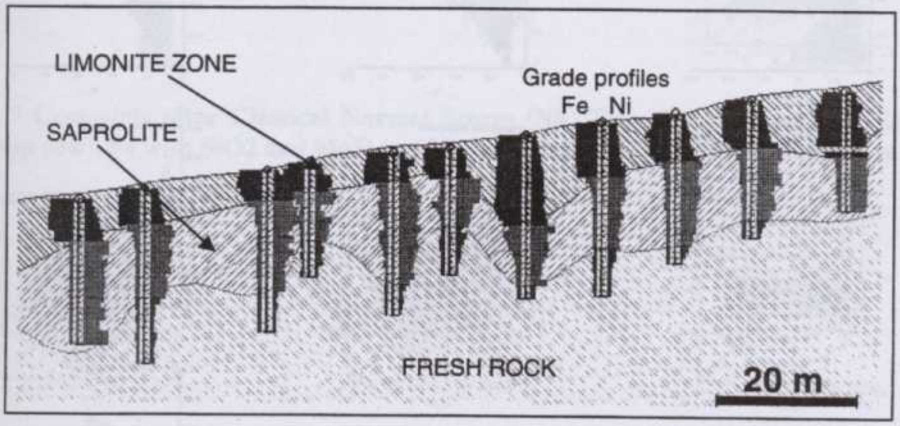

Figure 3 Typical Laterite Profile at Loma de Niquel Deposit.

Data Transformation

The first step in conventional Gaussian simulation is to transform each variable to a Gaussian distribution. Figure 7 shows the bivariate distributions obtained from performing the conventional normal score transform on each variable. The bivariate Gaussian distribution is well behaved, exhibiting linear, homoscedastic correlations and is characterized by elliptical probability contours. The crossplots in Figure 7 shows the exact opposite features with non-linear and heteroscedastic behaviour, and is further complicated by mineralogical constraints. Probability contours of the bivariate Gaussian distribution, corresponding to the shown correlation, is superimposed to illustrate the differences between expected multiGaussian behaviour and the characteristics of the transformed true data correlations.

Stepwise conditional transformation results in the crossplots shown in Figure 8. Contours of the bivariate Gaussian distribution with zero correlation are also shown on the crossplots. The transformation sequence is Ni, Fe, SiO_2 and finally MgO. For this particular exercise, there are nearly 10000 data values available. Ten classes were specified, resulting in 10 classes of Fe conditioned to Ni, 100 classes of SiO_2 simultaneously conditioned to Fe and Ni, and 1000 classes of MgO conditioned to SiO_2, Fe and Ni. As the transformation proceeds and we progress to the fourth variable, only a nominal 10 MgO data are found per class. Since every class contains approximately the same number of data; thus, normal score transform of the class will always result in the same normal scores. This results in a "banding effect" that is easily visible in the crossplots of the subsequently transformed variables. The conditional distributions are Gaussian; however, the limited data within each class does not allow transformation to exhibit the full range of the N(0,1) distribution. This "banding" effect is a visual artifact; it does not affect the simulation results.

Variography

The correlations between the stepwise conditionally transformed (SC) variables are nearly zero. The direct variograms for each SC variable were calculated and modeled. These are shown in Figure 9, with the modeling of the SC original data given on the left and the variograms corresponding to the SC residuals on the right. In particular, a vertical trend is clearly visible in the direct variogram of SC FE (left). The cross-variograms between the SC variables should be checked for independence at all lags, **h**. Figure 10 shows the cross variograms of the SC residuals; cross variograms of the SC original variable appear similar but with greater correlation in the vertical direction as a result of the vertical trend (not shown here).

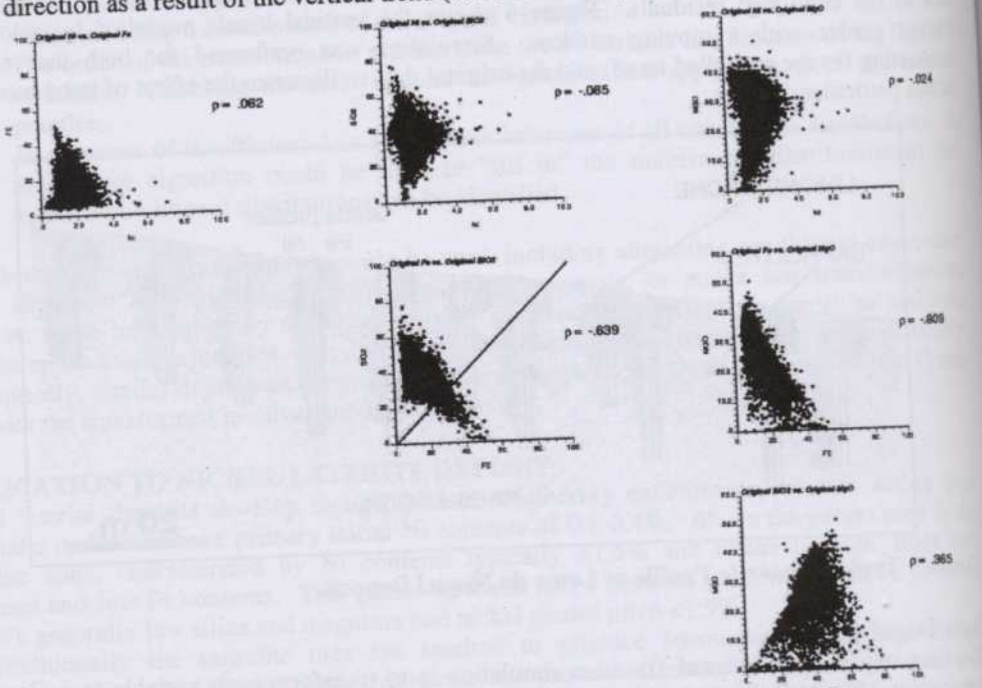

Figure 5 Crossplots of Original Data: Ni with Fe, SiO2 and MgO (top row), Fe with SiO2 and MgO (middle row), and SiO2 and MgO (bottom row).

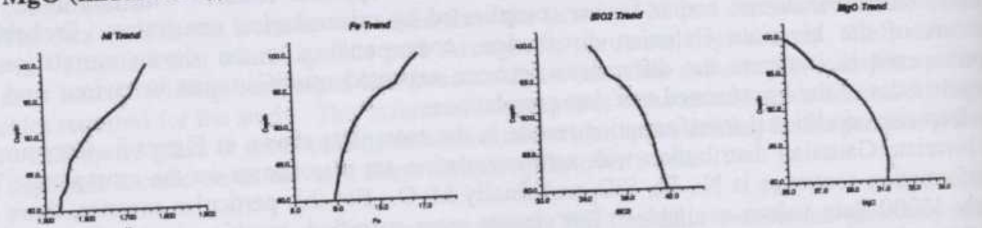

Figure 6 Vertical Trend Models obtained from Moving Window Averages. Vertical distance measures correspond to stratigraphic depth coordinates.

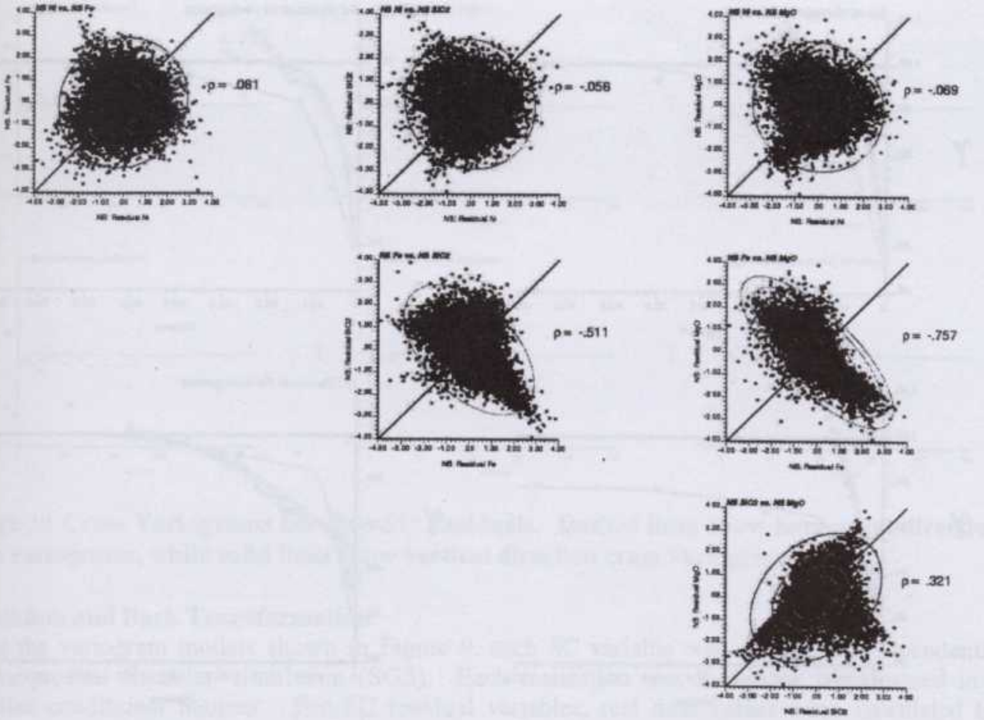

Figure 7 Crossplots after Classical Normal Scores (NS) Transform: Ni with Fe, SiO2 and MgO (top row), Fe with SiO2 and MgO (middle row), and SiO2 and MgO (bottom row).

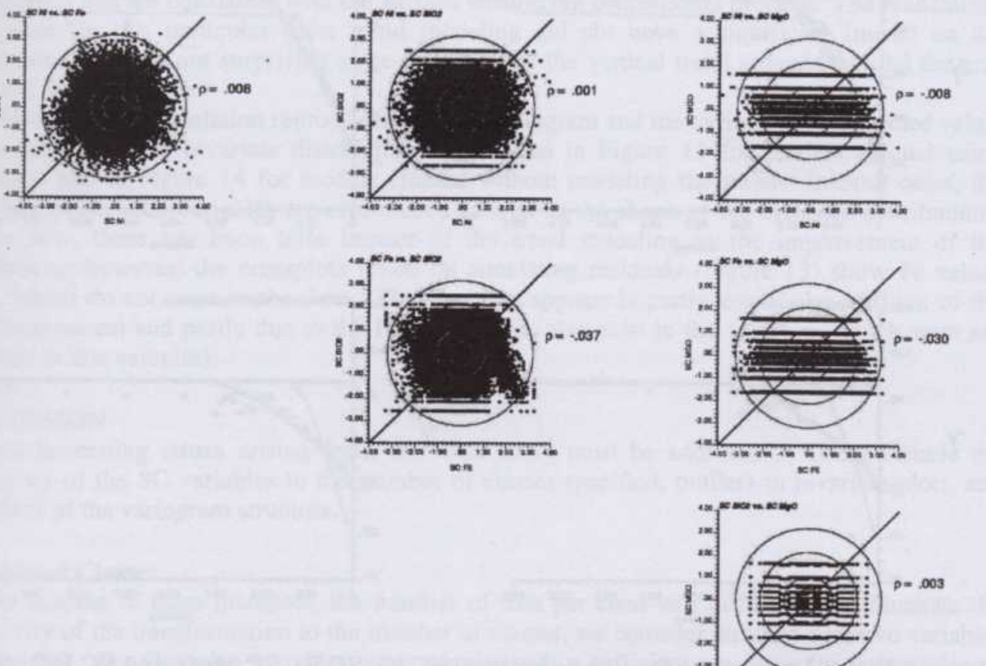

Figure 8 Crossplots after Stepwise Conditional (SC) Transform: Ni with Fe, SiO2 and MgO (top row), Fe with SiO2 and MgO (middle row), and SiO2 and MgO (bottom row).

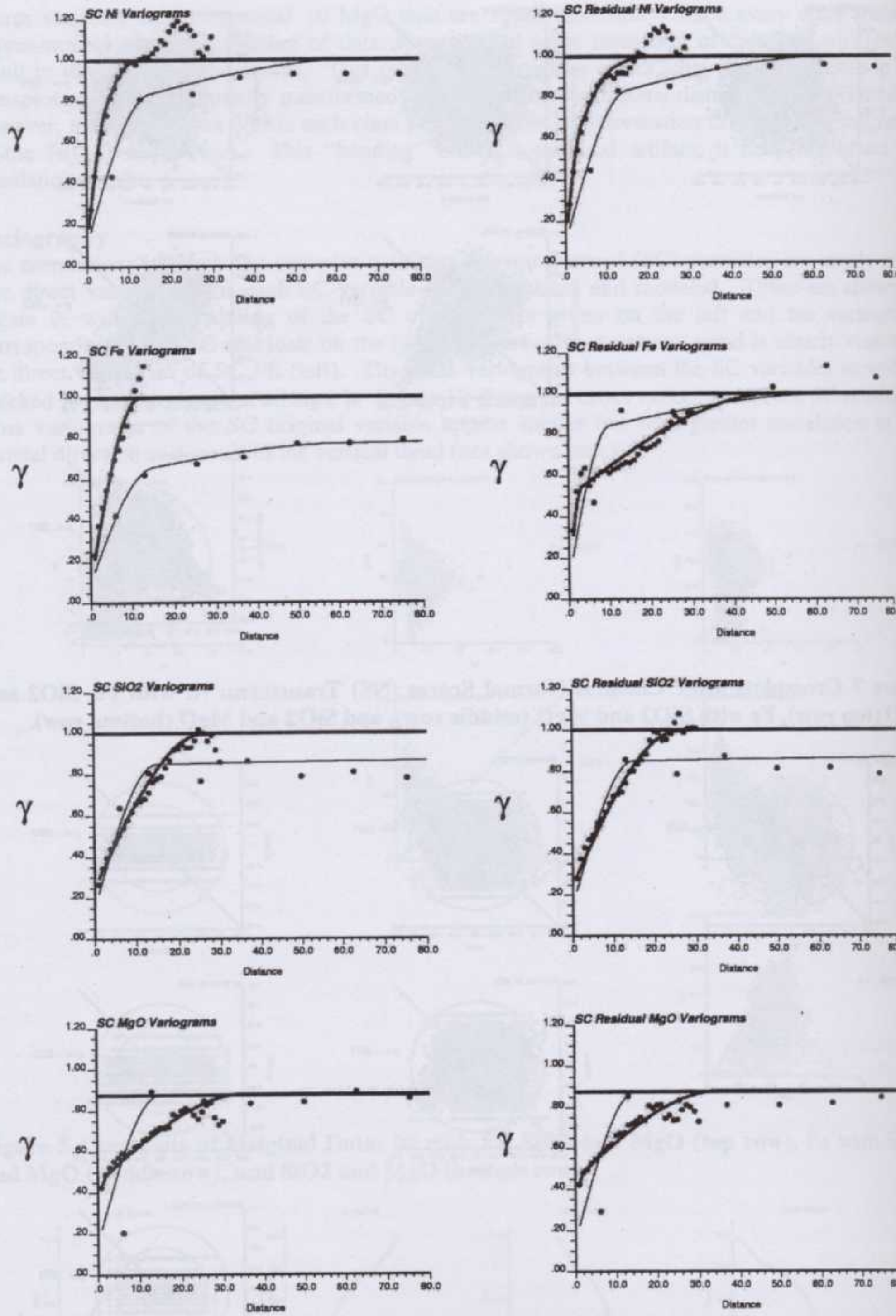

Figure 9 Direct Variograms for (top to bottom) SC Ni, SC Fe, SC MgO, and SC SiO_2 (based on original data (left) and residuals (right)). Horizontal variograms are shown as thinner lines than vertical variograms, and experimental variograms are shown as points while models are solid lines.

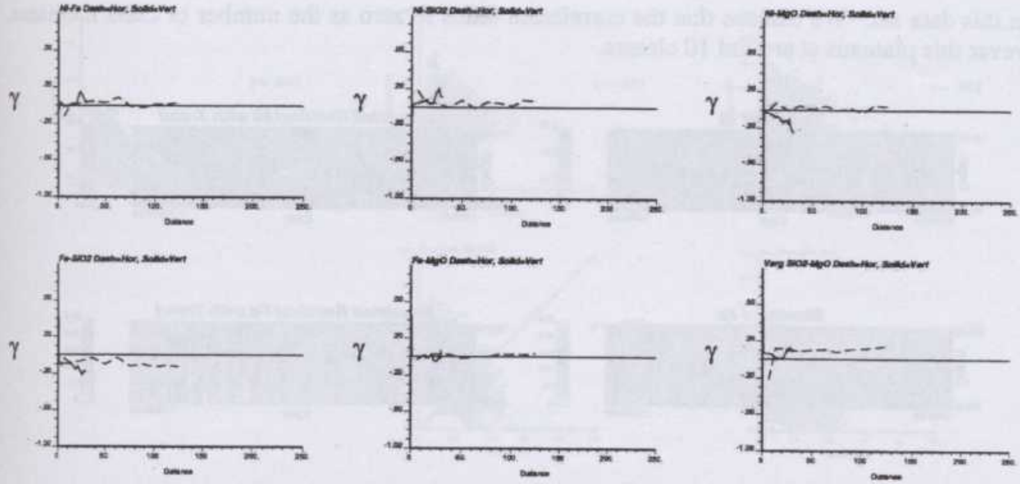

Figure 10 Cross Variograms between SC Residuals. Dashed lines show horizontal direction cross variograms, while solid lines show vertical direction cross variograms.

Simulation and Back Transformation

Using the variogram models shown in Figure 9, each SC variable was simulated independently with sequential Gaussian simulation (SGS). Each realization was then back transformed in a stepwise conditional manner. For SC residual variables, real data values were calculated by adding back spatially varying trend components to the simulated residuals. Figure 12 shows a cross section of one realization for each variable after back transformation. The realizations show an apparent vertical continuity on the order of 10-20 m (consistent with the range of the vertical variograms) and are consistent with the surface weathering depositional process. The realizations show that for this particular case, trend modeling did not have a significant impact on the simulations. This is not surprising since the range of the vertical trend extends beyond the grid limits of the simulation.

We know that simulation reproduces both the histogram and the variogram in expected value. Reproduction of the bivariate distributions is verified in Figure 13 for models created using residuals, and in Figure 14 for models created without modeling the trend. In both cases, the correlations between variables are reproduced as well as the shape of the bivariate distributions. Up to now, there has been little impact of the trend modeling to the improvement of the simulations; however, the crossplots based on simulating residuals (Figure 13) show Fe values <5%, which do not exist in the data. This problem appears is partly due to the addition of the trend component and partly due to the fact that trends also exist in the variance (which were not modeled in this exercise).

DISCUSSION

Several interesting issues arising from this case study must be addressed. These include the sensitivity of the SC variables to the number of classes specified, outliers in bivariate plots, and the effect of the variogram structure.

Number of Classes

As the number of class increases, the number of data per class will decrease. To illustrate the sensitivity of the transformation to the number of classes, we consider only the first two variables (Ni and Fe). For Gaussian variables, zero correlation is a sufficient condition for independence. Depending on the available data and the number of classes specified, the tendency of the correlation coefficient towards zero will be examined. Correlation coefficients for number of classes from 1 to 10 and 15 to 50 in increments of 5 were calculated. Figure 15 shows the relation between the number of classes and the correlation coefficient using only the first two variables from this data set. We can see that the correlation tends to zero as the number of class increases, however this plateaus at around 10 classes.

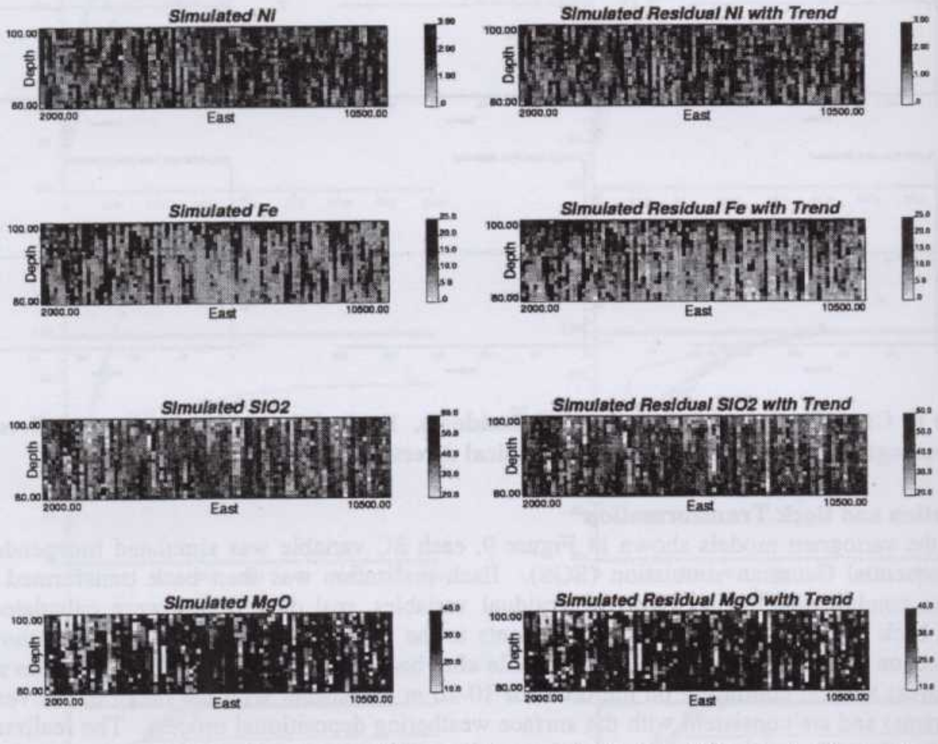

Figure 12 Simulated Realizations of (top to bottom) Ni, Fe, SiO_2 and MgO after Back Transformation. Realizations without trend modeling (left) and with trend modeling (right).

Sensitivity to Outliers

The data used in the case study was minimally cleaned by the removal of three outliers. In the back transformation stage of the workflow, simulated values are interpolated in the transformation table to return the simulated value in the units of the original data. Outliers have the effect of extending the domain in which this interpolation occurs. As a result, the crossplots of back transformed values show a bivariate distribution that reproduces these outlier values, which may be inappropriate. Figure 16 shows the comparison of the original data distribution (uncleaned and cleaned) and the resulting back transformed simulated values for Ni and Fe. Outliers should be removed prior to transformation to avoid simulating features that are not truly part of the mineralization.

Effect of Variogram Structure

Although the simulations within the case study are dependent on horizontal variograms with high nugget effect, it should be noted that the back transformation process is not affected by the variogram continuity. To validate this, the four-variable data set was simulated using a Gaussian shaped more continuous variogram in both vertical and horizontal directions. Simulation was performed using this new variogram model. Results were backtransformed and the trends were added. Cross plots of these simulated values are virtually identical to those shown in Figure 13, with similarly shaped distributions and comparable correlations.

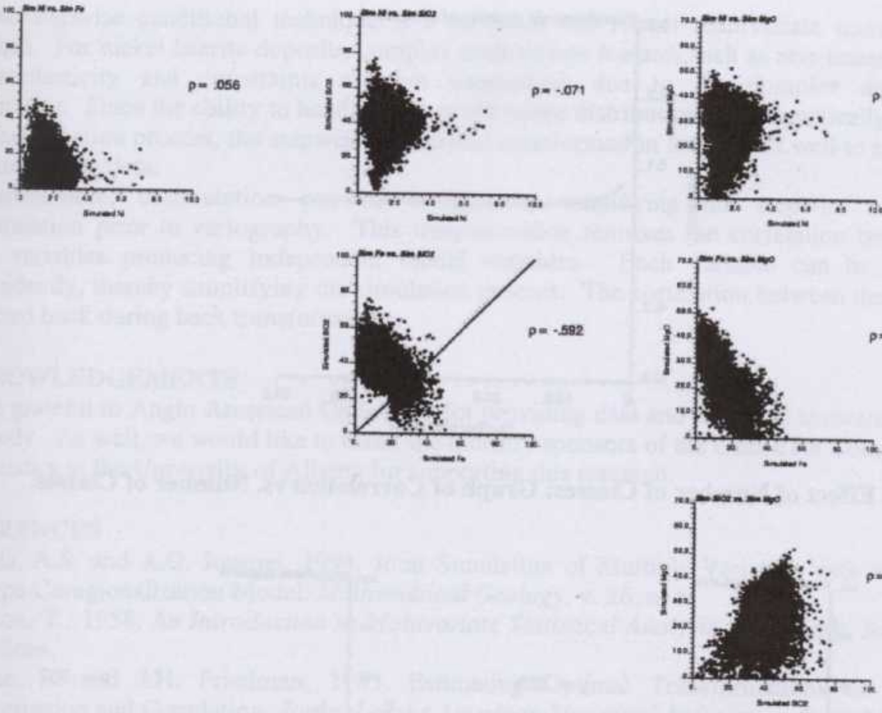

Figure 13 Crossplots of Simulated Residuals after Back Transformation and Incorporation of Trend: Ni with Fe, SiO2 and MgO (top row), Fe with SiO2 and MgO (middle row), and SiO2 and MgO (bottom row).

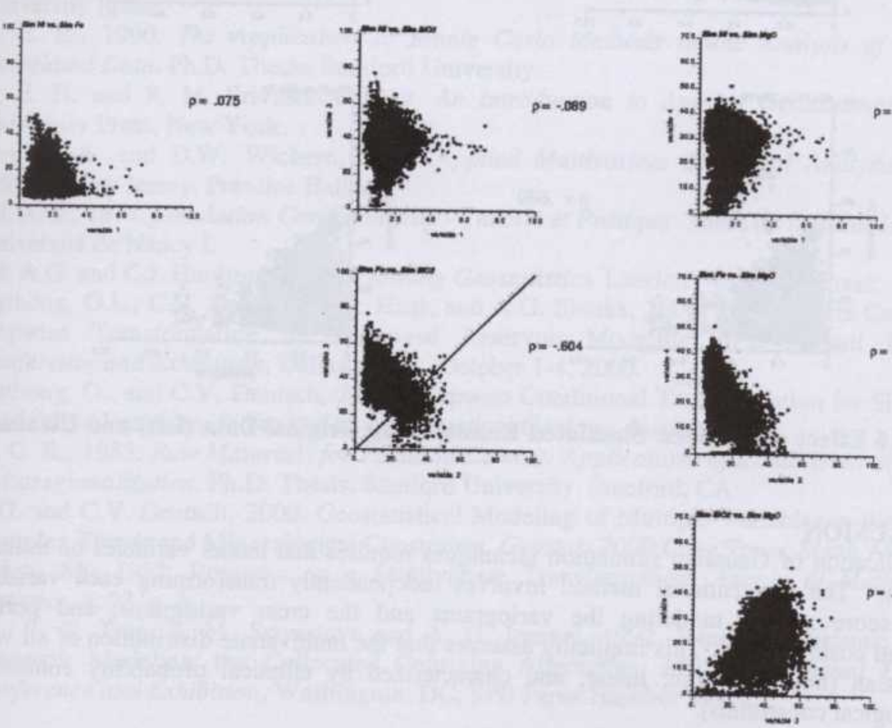

Figure 14 Crossplots of Simulated Original Variables: Ni with Fe, SiO2 and MgO (top row), Fe with SiO2 and MgO (middle row), and SiO2 and MgO (bottom row).

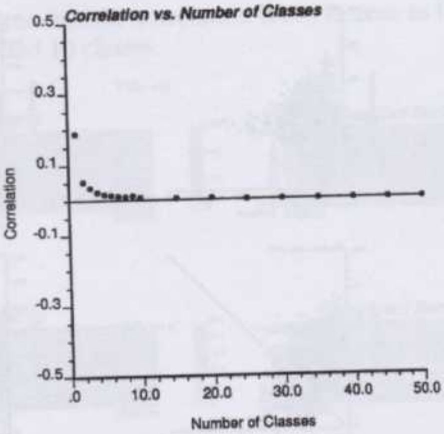

Figure 15 Effect of Number of Classes: Graph of Correlation vs. Number of Classes.

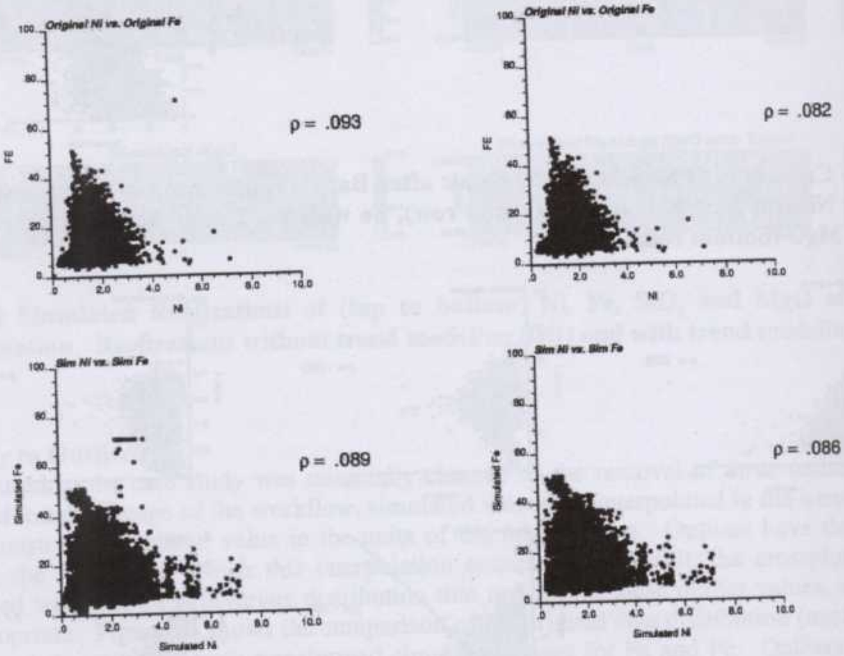

Figure 16 Effect of Outliers: Simulated Results using Original Data (left) and Cleaned Data (right).

CONCLUSION

The application of Gaussian simulation techniques requires that model variables be multivariate Gaussian. The conventional method involves independently transforming each variable into normal score values, modeling the variograms and the cross variograms, and performing sequential cosimulation. This implicitly assumes that the multivariate distribution of all variables is Gaussian (homoscedastic, linear, and characterized by elliptical probability contours - no mineralogical constraints).

The stepwise conditional technique is a powerful and robust multivariate transformation technique. For nickel laterite deposits, complex multivariate features such as non-linear relations, heteroscedasticity and constraints are not uncommon due to the complex depositional environment. Since the ability to handle these problematic distributions is automatically built into the transformation process, the stepwise conditional transformation lends itself well to application to nickel laterite data.

Furthermore, cosimulation can be avoided by employing the stepwise conditional transformation prior to variography. This transformation removes the correlation between the model variables producing independent model variables. Each variable can be simulated independently, thereby simplifying the simulation process. The correlation between the variables is injected back during back transformation.

ACKNOWLEDGEMENTS

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