The Place of Geostatistical Simulation in Resource/Reserve Estimation

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Abstract

This paper will review the current practice of kriging block models within geological domains for resources and reserves. The problems of smoothing, conditional bias, and data values-independent error variance are well understood. The promise of simulation to overcome these problems has not been fulfilled for a number of reasons including the requirement for more complete input parameters, a greater reliance on stationarity, more complex multivariate modeling, increased computer demands, and difficulty checking the reasonableness of the resulting models.

Geostatistical simulation should not be used until these concerns are addressed. The specific procedures and implementation considerations to address these problems will be discussed with examples. Declustering and debiasing are discussed for the input distribution. Trend modeling and stepwise transformation are discussed for stationarity. Cosimulation by various schemes is discussed for multivariate modeling. The computer requirements for realistic 3-D multiple rock type modeling is discussed. Checking procedures are reviewed. We see that the procedures are available to address the concerns regarding simulation, but they involve more technical input and professional time that required by kriging.

The advantages of spending the additional time on simulation will be reviewed. The promises of unbiased recoverable reserves and a measure of uncertainty at any scale are very attractive. Now, senior technical staff and management must demonstrate the courage to explore geostatistical simulation in a controlled fashion.

Introduction

Geostatistical simulation is the next frontier of geostatistical ore resource/reserve estimation. The basic simulation paradigm has existed for many years. There is a widespread acceptance of the usefulness of the simulation approach to numerical modeling of variability and uncertainty. Nevertheless, few ore reserve estimates are made on the basis of simulated realizations. Mine planning and decision making is simpler with only one estimate per grid block; however, the slow adoption of simulation is for more important reasons than simplicity. This review paper will address some important practical reasons why simulation should be undertaken with care. Recommendations will be given and illustrated with a small dataset.

Problems with Estimated Models

Estimation refers to the calculation of a single predicted value at each location that minimizes some measure of error. Kriging is a proven estimation technique that minimizes the expected error variance subject to constraints. Ordinary kriging within rock types is a robust estimator that has been used for many reserve/resource estimates. Despite the robustness of kriging and the many successful mining ventures based on kriging estimates, there are some well known concerns with

estimated models: (1) unrealistic smoothing, particularly when the drillhole spacing is large, (2) conditional bias in local estimates and failure to account for the information effect, and (3) an incorrect, data values independent, measure of uncertainty.

Regardless of the kriging plan, kriged estimates are smooth relative to the underlying mineralization. Kriging assigns weights to local data and there is no ability to predict high and low areas away from the available data. One significant problem with smoothing is the perception that professionals develop of the variability in the deposit. Estimated models created by kriging appear more continuous than the true variability and there is a danger of unrealistic expectations with respect to selectivity. The second problem with smoothing is that the resource and reserve estimates can be biased. More information, such as blasthole samples, at the time of mining can lead to better selectivity than predicted by kriging. Dilution and lost ore due to underestimated variability can lead to reduced selectivity than predicted by kriging.

Fig. 1 shows data that will be used to illustrate some points in this paper. I commonly use this released data in training courses. The data set consists of thickness, Au grade, Ag grade, Cu grade, and Zn grade. Most of the figures will show thickness and gold grade; however, all variables will be considered in the multivariate simulation. Fig. 1a shows the reference distribution of thickness based on a large number of data. Fig. 1b shows the available data at the (pre)feasibility stage, Fig. 1(c) shows the histogram of thickness. The fitted omnidirectional semivariogram is shown on Fig. 2. There were too few data for reliable variogram inference.

Fig. 3 illustrates the first two problems with estimated models. The kriged map (Fig. 3a) does not reflect the variability shown by the reference data (compare with Fig 1a). The smoothness is illustrated by the histogram of the estimate (Fig. 3b) and the histogram of true block values and estimated block values (Fig. 3c). Of course, the true block values are not accessible in practice, but more information is likely available at the time of mining leading to estimated values closer to the true values than currently estimated. It is difficult to account for this in estimation. One approach is to estimate the recoverable reserves using an analytical volume variance correction scheme of some kind and modify the kriging plan to reproduce the anticipated reserves. This may correct the global resource bias, but the spatial distribution will be wrong. There will be too much high grade near the high-grade drillholes and too much low grade near the low grade drillholes. This conditional bias could lead to a serious bias in the economics if the mine plan unwittingly targets the high grades early in the mine plan. The proportion of high grade may be correctly predicted, but it is *not* in the contiguous regions as predicted by kriging with a restrictive search.

Fig. 4 illustrates the third problem with estimation by kriging: the incorrectness of local measures of errors. Fig 4a shows the kriging variances that depend solely on the data configuration. A cross plot of the local mean versus variance; note the strong dependence between them.

Among different interpolation methods, Kriging has some unique mathematical properties. Kriging estimates are the best linear unbiased estimates; they minimize a well-defined measure of error variance. The covariance between kriging estimates and all data used in kriging matches the specified covariance model. The covariance between the kriging estimates is also correct. The kriging variance provides a measure of error in the estimates. The kriging estimate and variance are the mean and variance of the local distribution of uncertainty in the case that the multivariate spatial distribution is Gaussian. The idea of extending kriging to create simulated realizations has been proposed for more than twenty years.

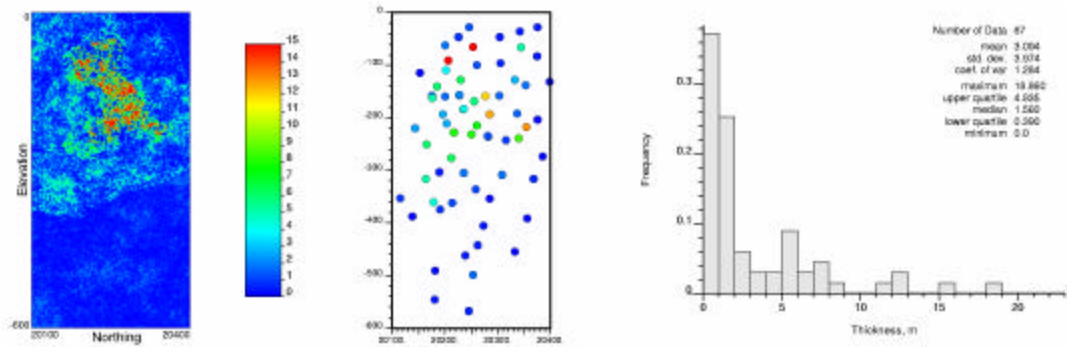

Fig. 1. Example data: (a) illustration of a reference distribution based on data and simulation, (b) available data, and (c) some basic statistics.

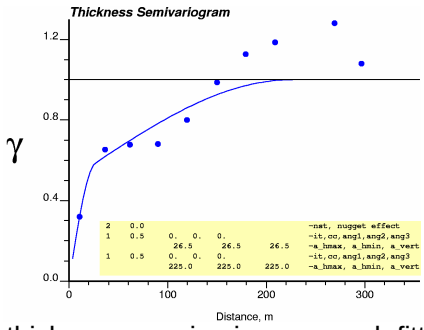

Fig. 2. Omnidirectional thickness semivariogram and fitted model. The limited number of drillholes led to unreliable directional semivariogram estimates.

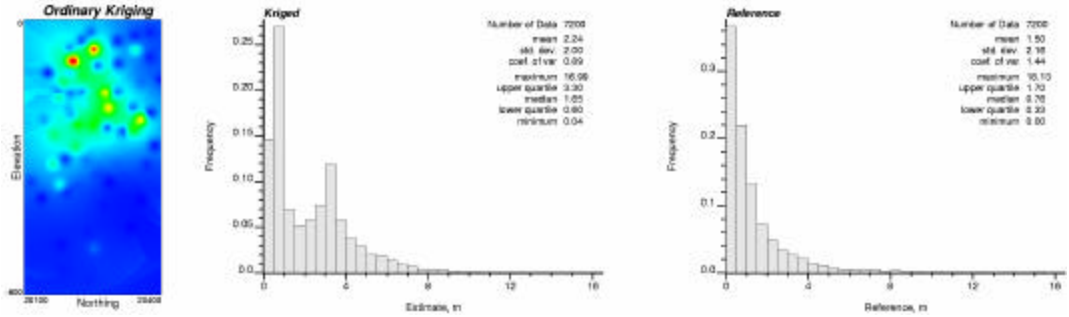

Fig. 3. Kriging 5m blocks: (a) map of the estimates, (b) histogram of the kriging estimates, and (c) histogram of the block values from the reference model.

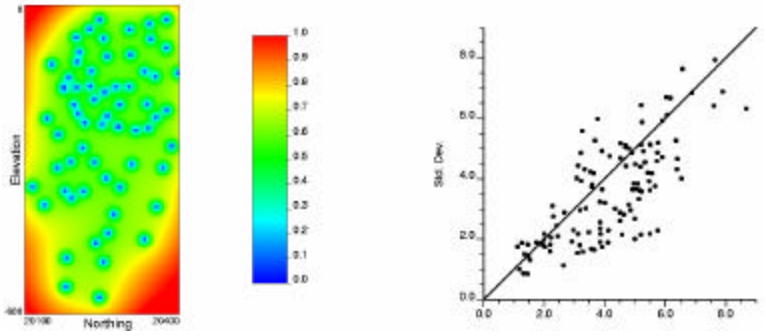

Fig. 4. (a) map of kriging standardized kriging variances, (b) plot of standard deviation versus local mean values.

Promise of Simulation

The central ideal of simulation is to correct the smoothing of kriging by drawing realizations of the multivariate spatial distribution of grades. Multiple realizations provide a quantification of joint uncertainty over all locations simultaneously. The first approach to simulation was based on unconditional simulation and a post-processing of the realizations to make them conditional to local data. Sequential techniques have gained in popularity because they simulate in a one-step procedure.

It is possible to interpret kriging in many ways: optimal interpolation under a geologically-based measure of distance, a projection onto the space of linear combinations of the data, and so on. There is no explicit need to assume a particular multivariate distribution model. The *multivariate distribution* here is the spatial distribution of the grades at all *N* locations taken simultaneously. Simulation, however, requires explicit specification of a multivariate distribution. Determination of a non-parametric multivariate distribution model is virtually impossible in real cases. Indicator methods permit non-parametric inference of the bivariate spatial distribution model; however, the trivariate and higher distributions are not controlled; thus, tend toward Gaussianity because of the central limit. The average of simulated realizations tends to Gaussianity. Most geostatistical simulation techniques transform the univariate distribution to a standard normal distribution, then assume a multivariate Gaussian distribution. Sequential Gaussian simulation, turning bands, matrix methods including LU simulation, and spectral simulation techniques are all Gaussian. There is little point in debating their pros and cons. They all generate realizations with the same statistical properties. These techniques are discussed in a number of textbooks and papers.

Fig. 5 shows two realizations out of 200 that were created based on the data presented in figures 1 through 3. Note that the variability is similar to that of the reference values (Fig 1a). Simulation gives up uniqueness and local accuracy for a realistic model of variability and a model of uncertainty. This uncertainty can be calculated over any spatial or temporal scale.

Consider the top 300m of the vein and the possibility of mining 36-50m by 50m stopes starting from the lower left (bottom South) and moving northward and then upward. A minimum mining thickness of 2m will be considered; unmineralized waste will be added if the thickness is less than 2m. Fig 6. shows the mining sequence and the relationship between the gold grade and the thickness. The gold grades were cosimulated using thickness. Some more comments on this are given below. Multiple realizations are passed through the simplistic mine plan. The results are shown on Figure 7. The kriged thickness (red line) and simulated thickness values for 100 realizations are shown. The histogram of the tonnage for 100 realizations is shown at the right. The black dot corresponds to the value from the kriged model. Note how there is a bias because the thickness cutoff was below cutoff and the kriging smooths more values to be above cutoff.

The promises (and place) of simulation are (1) numerical models of mineral deposits that more closely reflect the true expected variability of the mineralization, (2) a measure of variability and uncertainty at any spatial or temporal scale, and (3) an opportunity to transfer the variability and uncertainty through optimal decision making regarding blending, mine planning, economics and classification. These promises have been known for some time. A number of practical problems have slowed the adoption of simulation for these problems. The remainder of this paper will cover some of the essential implementation details that must be considered for the promises of simulation to be fulfilled.

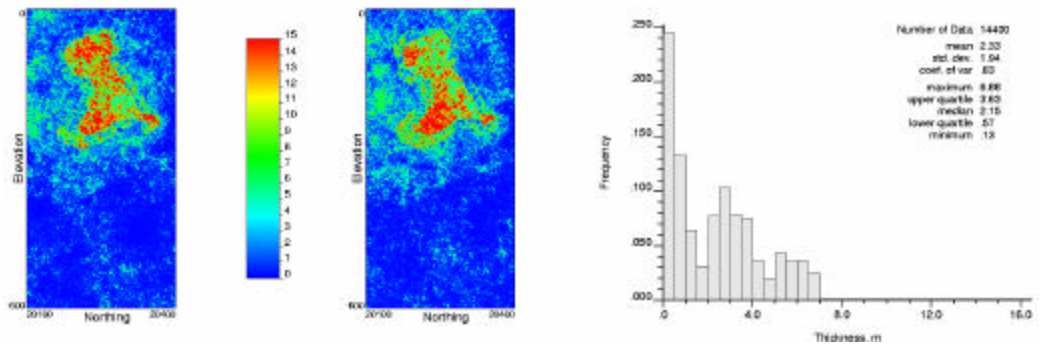

Fig. 5. (a) two simulated realizations accounting for the vertical trend, and (b) the histogram of block values based on the simulated realizations.

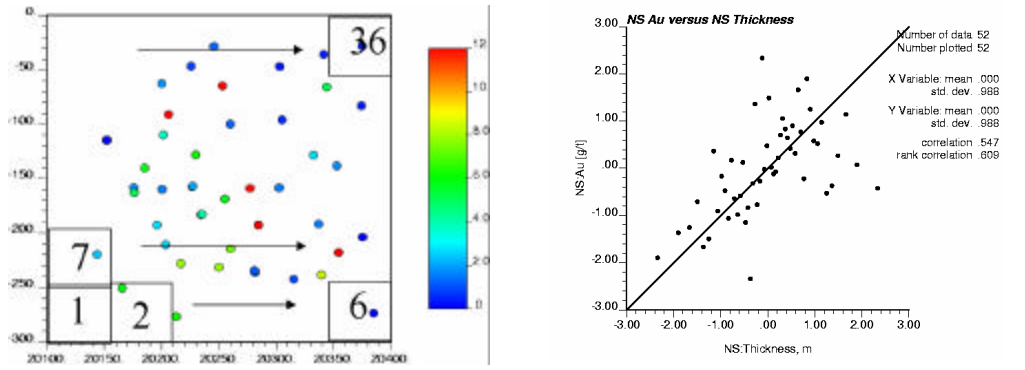

Fig. 6. (a) example mining sequence for the top 300m, and (b) scatterplot of gold grade versus thickness (in Gaussian units).

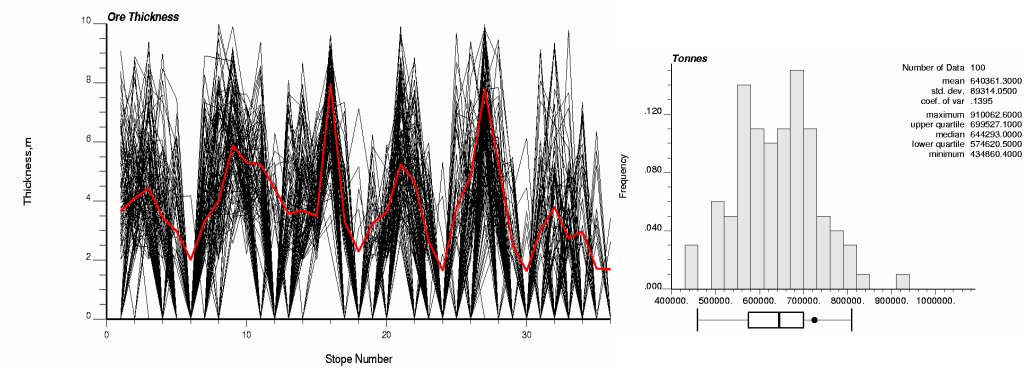

Fig. 7. Kriged thickness (red line) and simulated thickness values for 100 realizations. The zero thickness values correspond to situations where the stope does not meet the cutoff grade. The histogram of the tonnage for 100 realizations is shown at the right. The black dot corresponds to the value from the kriged model.

Trend Model

The spatial variability of mineral deposits is partly structured and partly random. For this reason, ordinary kriging (OK) is the most popular flavor of kriging. The kriging weights are constrained to sum to one, which amounts to estimate the mean locally instead of assuming a known and stationary mean. Provided there is enough data, the estimates of OK adapt well to low- and high-grade areas. The OK variance is not a good measure of local error since the variance depends the magnitude of the grades. It is problematic to implement OK or an equivalent approach in simulation. The estimates *and* the variance must account for low- and high- grade regions. The multivariate Gaussian distribution has strong assumptions of homoscedasticity, that is, the variance is independent of the grades. Ad-hoc solutions around this assumption inevitably lead to biased predictions and other problems. There is a need to (1) model the trend in a reliable manner, and (2) remove this trend prior to Gaussian simulation.

Modeling geological rock types mitigates the problems associated with trends. Often, the grades are more homogeneous within rock types. Rock types or geological units must be modeled prior to grade modeling. The focus of this paper is on the geostatistical simulation of continuous variables within pre-defined rock types. Despite the modeling of rock types, there are often trends within individual rock types.

An initial OK run or hand contouring can detect large-scale spatial features. Sometimes a simple cross plot of the grades versus direction may show a trend. To visualize trends, a moving window average of the data can be calculated to determine if local means and/or variances are indeed stationary. The size of these windows will depend on the number of data available. Notable changes in the local mean and variance within the domain lead to trend modeling. Although the identification of a trend is subjective, it is widely accepted that the trend is essentially deterministic and should not have short scale variability. Any feature that is not significantly larger than the data spacing should probably be left for stochastic modeling.

The mean component is defined at all locations via a 3D trend model, while the residual values are only defined at data locations. Geostatistical modeling is then only performed on the residuals that are considered to be stationary. Multiple realizations of the residuals are generated and added back to the single trend model to produce multiple realizations of the original RV.

The problem remains as to how the trend should be modeled so as to obtain a stationary residual random function (RF) for geostatistics. The idea is to obtain a model that accounts for large-scale features. There are several trend modeling approaches that have gained popularity in practice: (1) hand contouring of geologic sections accounting for drill hole data and geological interpretation, (2) simple moving average or inverse distance estimates, or (3) OK of blocks with a variogram with some nugget effect. The grade within a geological unit with a trend can be decomposed into a mean and residual: $z(\mathbf{u}) = m(\mathbf{u})+r(\mathbf{u})$, where the deterministic trend is $m(\mathbf{u})$ and the residual value $r(\mathbf{u})$ is to be modeled geostatistically.

The $r(\mathbf{u})$ values must be modeled accounting for the grid of $m(\mathbf{u})$ values since these two intermediary variables are related together. The figure below shows the residual versus trend for a particular geological unit of a porphyry copper deposit in Canada. Note the “forbidden triangle” in the lower left – any residual in this area would lead to a negative grade, that is, $r(\mathbf{u})<m(\mathbf{u})$ implies negative $z(\mathbf{u})$. The relatively small

correlation between r(**u**) and m(**u**) may not be important, but the constraint for negative grades and the dependence of the variability in r(**u**)/z(**u**) on trend m(**u**) is important. Stepwise conditional transformation (SCT) will be proposed below in the Multivariate Simulation section as a simple and effective approach to account for this relationship.

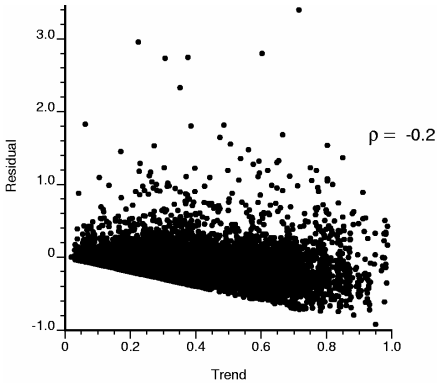

Fig. 8. Scatterplot of the residual after trend modeling versus the trend. Note the non-negative constraint expressed as a line with a slope of -1.

Declustering and Debiasing

Simple kriging requires the specification of a mean value, but in general Kriging does not require an input histogram. All simulation techniques, however, require specification of a global distribution that will be reproduced with statistical fluctuations. This global distribution is often used to transform the grades to a Gaussian distribution. Reproduction of the standard Gaussian distribution by most simulation techniques ensures that the specified distribution will be reproduced. Taking the equal weighted distribution of the data within the rock type as the representative distribution is usually a bad idea. Drillholes are rarely uniformly spaced and there is often a spatial sampling bias toward high grades (or perhaps low grades). Declustering and debiasing techniques must be considered to arrive at a representative distribution.

The representative distribution is partially addressed when a trend has been modeled. The only constraint then is that the residual distribution be representative, but the overall mean will be corrected through the trend model. Declustering and debiasing is particularly important when the trend has not been modeled.

Multivariate Simulation

Virtually all geostatistical simulation is done in a multivariate setting. Even with a single grade, there is often a trend, which makes it a multivariate problem. The stepwise conditional transformation technique is a multivariate Gaussian transformation, that is, multiple variables are simultaneously transformed to independent Gaussian variables that can be simulated independently. The values are back transformed to reintroduce the multivariate relationships that are removed in the forward transformation.

This technique, first introduced by Rosenblatt in 1952, is equivalent to the normal score transform in the case of one variable. In a bivariate case, the normal transformation of the second variable is conditional to the probability class of the first variable. Correspondingly, for *k*-variate problems, the k^{th} variable is conditionally transformed based on the (k-1) first variables:

$$Y_1 = G^{-1}[F_1^{-1}(Z_1)]$$
$$Y_2 = G^{-1}[F_{2|1}^{-1}(Z_2|Z_1)]$$
$$\bullet \quad \bullet \quad \bullet$$
$$Y_K = G^{-1}[F_{K|1,\ldots K-1}^{-1}(Z_K| Z_1, Z_2, \ldots Z_{K-1})]$$

$G^{-1}(\bullet)$ is the inverse of the standard normal distribution function. The conventional normal scores transformation is performed on conditional distributions with increasing levels of conditioning. The number of required data increases significantly, but we commonly consider sets of 2 or 3 variables at a time. Fig N. shows the steps to accomplish this conditional transformation in the case where the first variable is the mean/trend and the second variable is the variable. Once the data are binned based on their conditional probabilities, each group of data is normally transformed. Simulation is then performed on the normal score values of the Y_i's and back transformation is performed. For example, Z_1 can be determined from Y_1 with the correct conditional distribution; from Z_1 and the simulated value of Y_2, Z_2 can be calculated; and so forth.

Fig. 9 illustrates the transformation with the notion of transforming the residual from the trend using the trend as the primary variable. This transformation is very effective at removing correlation between variables, permitting independent simulation, and the reinjecting the correlation in the back transformation.

Some Common Issues with Simulation

Block kriging accounts for the volume scale between the data and the volume being estimated. In practice, block kriging is of little value since the smoothing of the kriging estimates is more important than estimation of blocks and not points. In simulation, however, it is not possible to directly simulated block values. The kriging within the simulation could easily be modified to account for blocks, but there would be an implicit assumption of linear averaging after normal-score transformation. This would lead to a bias in the simulated values. Standard practice is to simulate points and then average them up to the scale of interest. Simulating 9 or more point values per SMU block leads to reasonably stable results. Some practitioners prefer even more stable results such as 20 or so points per block.

Another issue with simulation is the quantification of uncertainty at a very large scale, for example, the deposit scale. Simply running multiple realizations does not lead to a reliable measure of uncertainty at a large scale. Areas of low- and high-grades tend to average out and all realizations lead to similar resource/reserve estimates. Uncertainty is systematically underestimated unless the input statistics are considered uncertain (through the use of a spatial bootstrap) and alternative scenarios are considered.

Mine planning is awkward with multiple realizations. The full engineering design could be considered on a few different models, but it is unlikely that pit smoothing, scheduling and detailed analysis could be considered on a 100 realizations. It is common to rank the realizations and target realizations such as the P_{10}, P_{50}, and P_{90} ones for detailed analysis. Ranking requires specification of a scalar number for ranking. Our first intuition would be to use quantity of metal for each realization. Interestingly, realizations with the same quantity of metal could be quite different. A realization with relatively few tones and high grade could be equivalent to a realization with more tones, but lower grade. By chance, the realizations ranked on quantity of metal can be quite unstable.

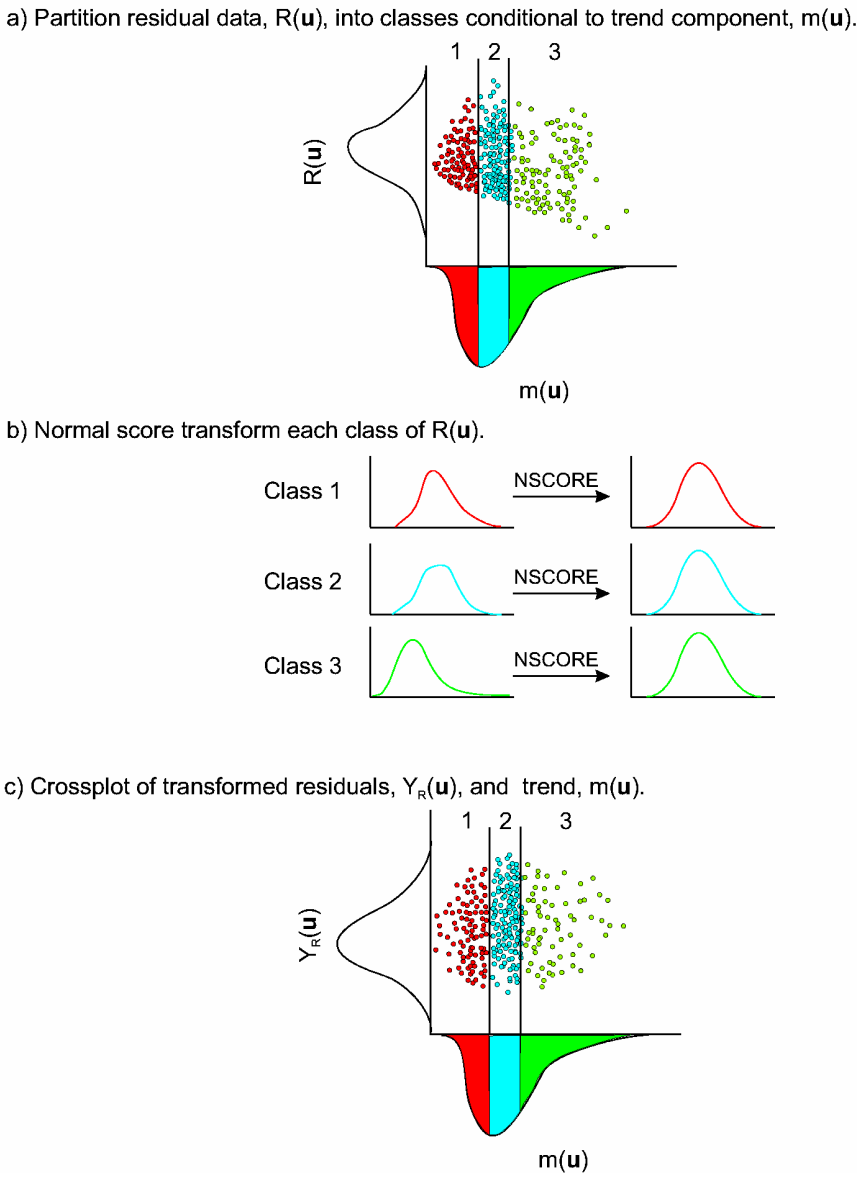

Fig. 9. Normal score transform of residuals conditioned to trend component: (a) partition residuals into classes based on its trend component, (b) normal score transform each residual class, and (c) assemble all transformed residuals (from all classes) and plot against the trend to show bivariate distribution with homoscedasticity and approximately zero correlation. Note that the marginal distribution of $Y_R(\mathbf{u})$ is Gaussian.

Checking

Estimation models are easier to visually check than geostatistical simulation; all features on a map of kriging estimates are either justified by the available data or are erroneous. Simulated realizations present stochastic features that have to appear correct on each realization and appear correct in expected value over multiple realizations. This requires more extensive checking than estimation. Also, simulation is more sensitive to the input statistics and departures from stationarity. Most problems translate to a bias in the final histogram, which leads to a direct bias in the predicted resources/reserves.

There are some very basic checks that should be performed on all simulated realizations. All numerical simulations should honor the input information including the geological interpretation, the data values at their locations, the data distribution, and the correlation structure, within acceptable statistical fluctuations. Moreover, the uncertainty measured by the differences between simulated realizations should be a reasonable measure of uncertainty. Confirming that the realizations reproduce the input statistics does not necessarily imply that the model is geologically realistic or good for production forecasting. These are minimum criteria that must be met. These checks should be an integral part of any geostatistical simulation modeling workflow. Fig. 10 below shows a standard display where the reference input statistic is shown in red, the result of all realizations combined is shown in blue, and 10 realizations are shown as gray lines. The variogram model is not reproduced in this case because the variogram was calculated on the realizations after they were back transformed to original units.

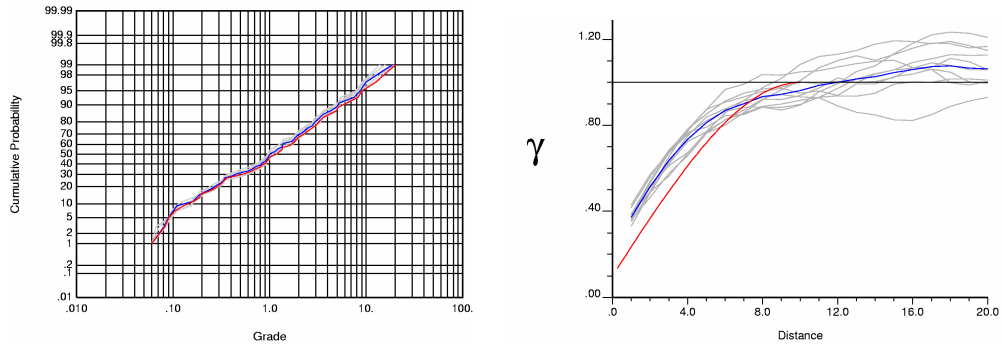

Fig. 10. Example checks of histogram and variogram reproduction. Although difficult to see, the probability plots for 10 realizations are shown as gray lines, all of the realizations are shown as blue, and the reference distribution is shown in red. The variogram reproduction is not particularly good.

As simple as this sounds, the first check should be a visualization of the realizations (in 3-D if possible). This visualization should highlight low and high valued areas. The project geologist should be satisfied with the variability of the high and low values and their overall distribution. The variability or uncertainty should be reasonable and plausible, for example, there should be no high values in clearly low areas and vice versa. Comparisons against simple geologic contours of trends, generated by methods such as hand contouring, inverse distance and other common estimation techniques, would also provide a level of comfort and confidence in the simulation models. The geologist should be neither intimidated by the geostatistical procedures nor swayed into accepting any strange results.

Once the realizations are deemed geologically plausible, common validation tools, such as cross validation and the jackknife, could be used. The basic idea is to estimate an attribute at a location where the true value is known. In cross validation, a data value is removed and the location is estimated using all other neighbouring data. Conversely, the jackknife refers to resampling without replacement. As a result, cross validation is commonly known as the *leave one out* approach, and the jackknife approach is known as the *keep some back* approach. The kriging methodology underlying the simulation algorithm should be checked. Cross validation in estimation mode should yield (a) a cross plot of the estimate versus the true value with a high correlation coefficient, (b) a distribution of errors that is symmetric with a

mean of zero and a low variance, and (c) a cross plot of the error versus the estimate should be centered about zero error, that is, conditionally unbiased estimates.

Cross validation in simulation mode is more difficult. Each realization should appear visually reasonable, but the additional variability of the simulation leads to less precise prediction. A reasonably large number of realizations (L>50) can be averaged and checked against a basic kriging model. There is no need that the average of multiple realizations be exactly kriging (that would only be true in the space of Gaussian transforms), but they should be reasonably close (0.9 correlation or higher). The distributions of uncertainty can also be checked for accuracy, that is, 80% of the true values should be within the P_{10} and P_{90} values of each distribution. The actual fraction of true values in different probability intervals can be checked with simulation.

Conclusions

Estimation has suffered longstanding concerns related to smoothing and potential bias in recoverable reserves. The promise of simulation to overcome these concerns is becoming a reality. This paper has focused primarily on a number of implementation concerns that must be addressed. The implementation and checking of simulation is much more difficult than conventional kriging, but the advantages of a realistic model of heterogeneity and uncertainty at any scale are compelling reasons to consider simulation.

Mineralization is partly structured and partly stochastic. Careful trend modeling is required for the structured aspect of the variability. Simulation of the residual is required for the stochastic component. Stepwise conditional transformation is useful to overcome the unavoidable relationship between the trend and residual. The same transformation can be used to simplify multivariate simulation of multiple grade values.

Checking the reproduction of input data, the deemed-representative histogram, and the specified variograms ensures consistency of the geostatistical realizations. Validation with actual production data is desirable. Overall, the average of the realizations should match the produced tones, grade, and metal. Moreover, the probabilities should be fair, for example, the produced values should be within the P_{10} and P_{90} 80% of the time.

References

Deutsch, C.V., 2002, *Geostatistical Reservoir Modeling*, Oxford University Press, New York, NY, .382 p.

Deutsch, C.V. and Journel, A.G., 1998. *GSLIB: Geostatistical Software Library and User's Guide*. Oxford University Press, New York, 2nd Edition, 379 p.

Journel, A.G. and Huijbregts, Ch. J., 1978. *Mining Geostatistics*. Academic Press, London, 600 p.

Leuangthong, O., and Deutsch, C.V., "Stepwise Conditional Transformation for Simulation of Multiple Variables," Mathematical Geology, 35(2), February 2003, p. 155-173.